Computer algebra independent integration tests

5-Inverse-trig-functions/5.6-Inverse-cosecant/5.6.1-u-a+b-arccsc-c-x-^n

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July 22, 2021 Compiled on July 22, 2021 at 5:32am

Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [178]. This is test number [158].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (178)	% 0.00 (0)
Mathematica	% 97.19 (173)	% 2.81 (5)
Maple	% 80.90 (144)	% 19.10 (34)
Maxima	% 46.07 (82)	% 53.93 (96)
Fricas	% 61.80 (110)	% 38.20 (68)
Sympy	% 35.39 (63)	% 64.61 (115)
Giac	% 52.81 (94)	% 47.19 (84)
Mupad	% 32.02 (57)	% 67.98 (121)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

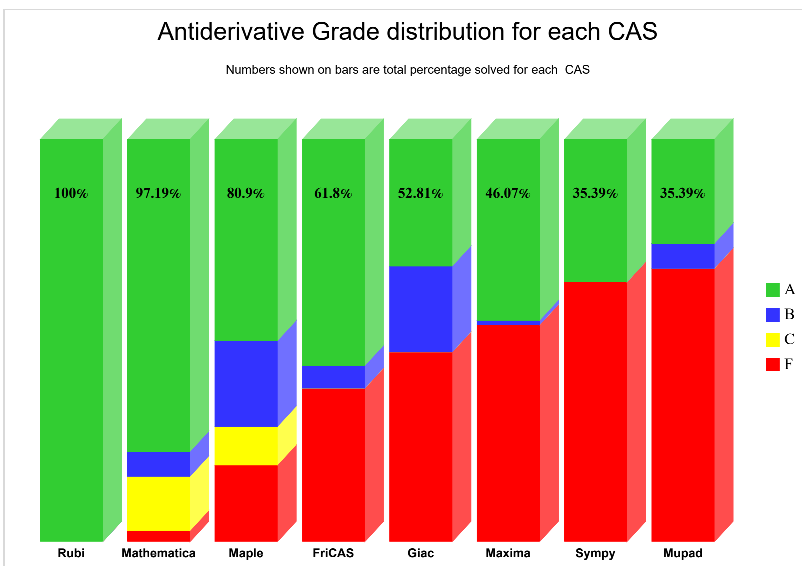
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

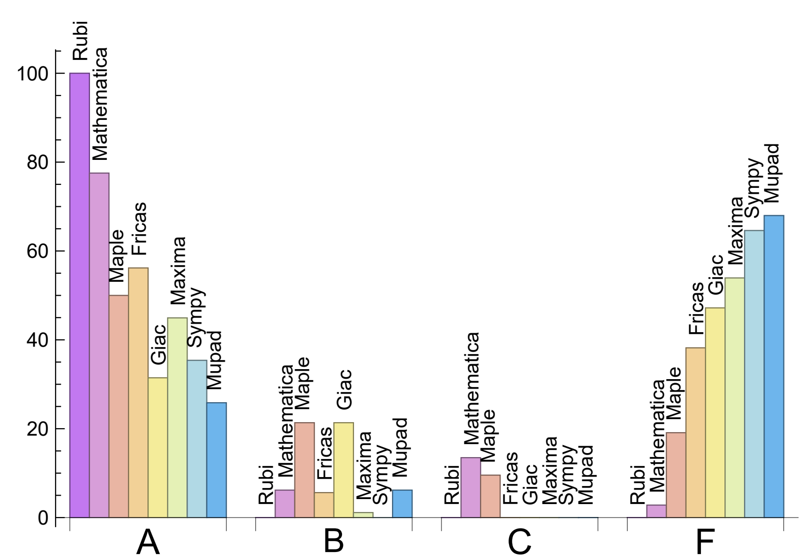
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	77.53	6.18	13.48	2.81
Maple	50.00	21.35	9.55	19.10
Maxima	44.94	1.12	0.00	53.93
Fricas	56.18	5.62	0.00	38.20
Sympy	35.39	0.00	0.00	64.61
Giac	31.46	21.35	0.00	47.19
Mupad	25.84	6.18	0.00	67.98

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	5	100.00 %	0.00 %	0.00 %
Maple	34	91.18 %	0.00 %	8.82 %
Maxima	96	75.00 %	7.29 %	17.71 %
Fricas	68	76.47 %	19.12 %	4.41 %
Sympy	115	53.04 %	46.96 %	0.00 %
Giac	84	65.48 %	7.14 %	27.38 %
Mupad	121	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

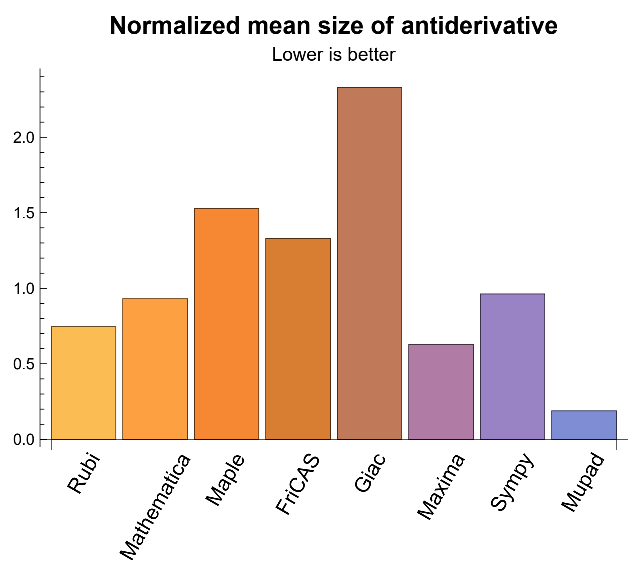
1.3 Performance

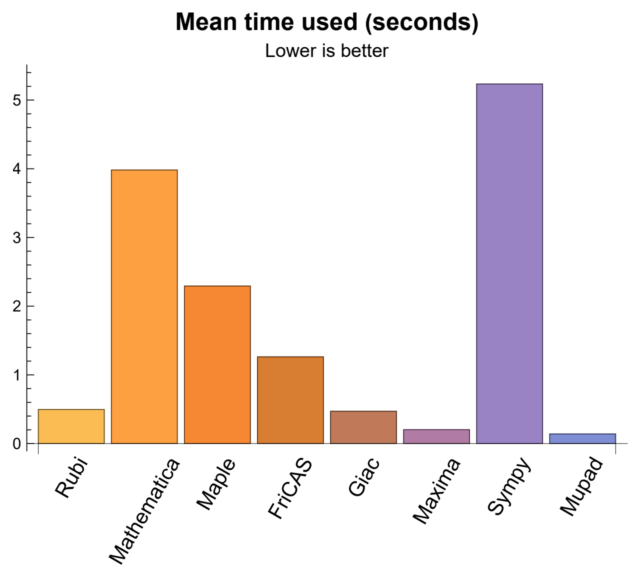
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.50	206.92	0.75	134.50	1.00
Mathematica	3.98	283.19	0.93	125.00	0.91
Maple	2.29	446.28	1.53	179.50	1.24
Maxima	0.20	73.04	0.63	0.00	0.00
Fricas	1.26	238.35	1.33	62.50	0.67
Sympy	5.23	126.87	0.96	58.00	1.16
Giac	0.47	311.95	2.33	48.00	1.22
Mupad	0.14	13.42	0.19	-1.00	-0.04

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{33, 34, 35, 39, 40, 42, 43, 54, 55, 61, 62, 67, 68, 73, 74, 121, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 177, 178}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {8, 16, 18, 19, 24, 25, 26, 27, 28, 48, 51, 53, 57, 60, 75, 86, 87, 96, 97, 98, 99, 100, 101, 102, 103, 104, 107, 108, 109, 110, 111, 115, 116, 117, 118, 128}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
```

```
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

    except Exception as ee:
           leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

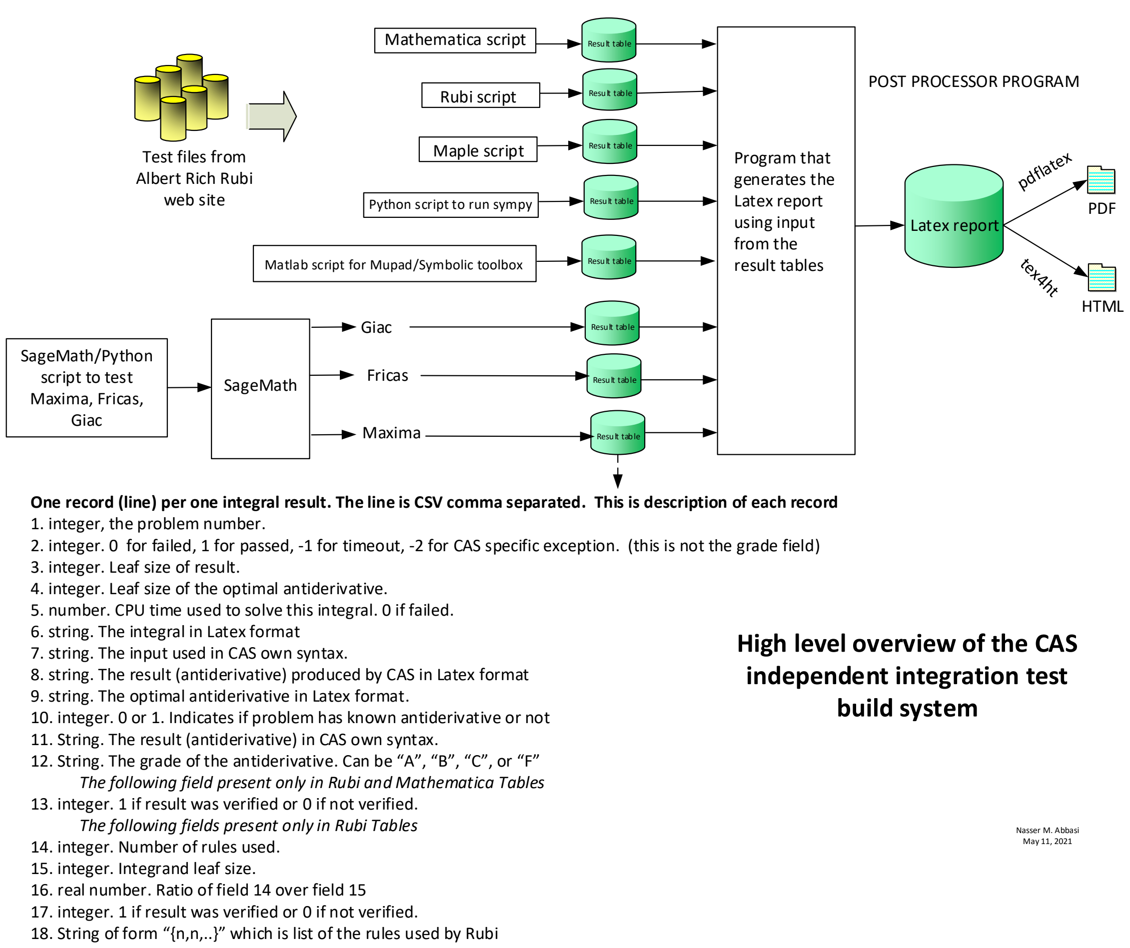
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 54, 55, 60, 61, 62, 66, 67, 68, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 101, 108, 109, 110, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 129, 130, 131, 132, 133, 134, 135, 138, 139, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 156, 157, 158, 159, 160, 161, 162, 163, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178 }

B grade: { 25, 53, 72, 98, 99, 100, 102, 103, 104, 107, 111 }

C grade: { 51, 52, 56, 57, 58, 59, 63, 64, 65, 69, 70, 71, 105, 112, 113, 118, 126, 127, 128, 136, 137, 146, 155, 164 }

F grade: { 106, 114, 165, 166, 167 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 6, 7, 8, 9, 11, 13, 14, 18, 22, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 46, 47, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 76, 79, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91, 92, 93, 94, 95, 96, 97, 121, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 177, 178 }

B grade: { 5, 10, 12, 15, 16, 17, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 44, 45, 48, 49, 50, 51, 52, 56, 70, 71, 72, 75, 77, 78, 88, 89, 105, 112, 113 }

C grade: { 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117 }

F grade: { 41, 68, 73, 74, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 174, 175, 176 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 17, 20, 29, 33, 34, 35, 39, 42, 43, 44, 45, 46, 47, 54, 61, 62, 67, 68, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95, 121, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 177, 178 }

B grade: { 15, 22 }

C grade: { }

F grade: { 8, 16, 18, 19, 21, 23, 24, 25, 26, 27, 28, 30, 31, 32, 36, 37, 38, 40, 41, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 73, 74, 75, 86, 87, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 174, 175, 176 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 14, 15, 20, 21, 22, 23, 29, 30, 31, 32, 33, 34, 35, 39, 40, 42, 43, 44, 45, 46, 54, 55, 61, 62, 67, 68, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95, 105, 118, 119, 120, 121, 122, 123, 124, 125, 128, 129, 130, 131, 132, 133, 134, 135, 138, 139, 140, 141, 142, 143, 144, 147, 148, 149, 150, 151, 152, 153, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 177, 178 }

B grade: { 7, 17, 47, 49, 50, 112, 113, 156, 157, 158 }

C grade: { }

F grade: { 8, 16, 18, 19, 24, 25, 26, 27, 28, 36, 37, 38, 41, 48, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 86, 87, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117, 126, 127, 136, 137, 145, 146, 154, 155, 163, 164, 165, 166, 167, 174, 175, 176 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 33, 34, 35, 39, 40, 42, 43, 44, 45, 46, 47, 62, 68, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95, 121, 122, 123, 124, 125, 130, 131, 133, 134, 135, 141, 142, 143, 153, 168, 171, 172, 177, 178 }

B grade: { }

C grade: { }

F grade: { 8, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 41, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 69, 70, 71, 72, 73, 74, 75, 86, 87, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 126, 127, 128, 129, 132, 136, 137, 138, 139, 140, 144, 145, 146, 147, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 169, 170, 173, 174, 175, 176 }

2.1.7 Giac

A grade: { 9, 10, 11, 12, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 54, 55, 61, 62, 67, 68, 73, 74, 80, 81, 92, 121, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 177, 178 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 13, 14, 15, 17, 20, 21, 22, 23, 29, 30, 31, 32, 44, 45, 46, 47, 76, 77, 78, 79, 82, 83, 84, 85, 88, 89, 90, 91, 93, 94, 95 }

C grade: { }

F grade: { 8, 16, 18, 19, 24, 25, 26, 27, 28, 41, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 86, 87, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 174, 175, 176 }

2.1.8 Mupad

A grade: { 33, 34, 35, 39, 40, 42, 43, 54, 55, 61, 62, 67, 68, 73, 74, 121, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 177, 178 }

B grade: { 6, 7, 8, 9, 10, 20, 29, 47, 79, 86, 87 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 36, 37, 38, 41, 44, 45, 46, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 174, 175, 176 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	107	177	161	115	221	646	-1
normalized size	1	1.00	0.94	1.55	1.41	1.01	1.94	5.67	-0.01
time (sec)	N/A	0.061	0.144	0.054	0.338	1.042	8.916	1.009	0.000

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	72	83	80	62	153	518	-1
normalized size	1	1.00	0.81	0.93	0.90	0.70	1.72	5.82	-0.01
time (sec)	N/A	0.042	0.130	0.051	0.346	1.389	4.425	0.213	0.000

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	97	150	132	106	175	480	-1
normalized size	1	1.00	1.09	1.69	1.48	1.19	1.97	5.39	-0.01
time (sec)	N/A	0.048	0.076	0.050	0.351	1.588	5.603	0.761	0.000

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	62	74	59	52	107	352	-1
normalized size	1	1.00	0.97	1.16	0.92	0.81	1.67	5.50	-0.02
time (sec)	N/A	0.027	0.104	0.049	0.341	1.509	3.061	0.166	0.000

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	85	123	97	94	107	310	-1
normalized size	1	1.00	1.33	1.92	1.52	1.47	1.67	4.84	-0.02
time (sec)	N/A	0.036	0.055	0.051	0.356	0.638	3.469	0.498	0.000

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	50	65	36	39	58	182	40
normalized size	1	1.00	1.28	1.67	0.92	1.00	1.49	4.67	1.03
time (sec)	N/A	0.012	0.023	0.049	0.345	1.017	2.143	0.164	0.653

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	58	37	53	64	32	62	33
normalized size	1	1.00	1.87	1.19	1.71	2.06	1.03	2.00	1.06
time (sec)	N/A	0.021	0.066	0.048	0.357	2.105	2.443	0.146	0.872

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	53	140	0	0	0	0	65
normalized size	1	1.00	0.83	2.19	0.00	0.00	0.00	0.00	1.02
time (sec)	N/A	0.085	0.033	0.184	0.000	0.637	0.000	0.000	0.763

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	41	63	33	26	37	42	37
normalized size	1	1.00	1.28	1.97	1.03	0.81	1.16	1.31	1.16
time (sec)	N/A	0.020	0.029	0.049	0.354	1.226	2.132	0.156	0.636

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	66	118	83	40	121	66	50
normalized size	1	1.00	1.29	2.31	1.63	0.78	2.37	1.29	0.98
time (sec)	N/A	0.032	0.037	0.050	0.467	0.863	3.356	0.150	0.726

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	59	75	58	39	112	87	-1
normalized size	1	1.00	0.98	1.25	0.97	0.65	1.87	1.45	-0.02
time (sec)	N/A	0.038	0.068	0.049	0.346	2.037	3.045	0.135	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	78	147	125	53	194	117	-1
normalized size	1	1.00	1.03	1.93	1.64	0.70	2.55	1.54	-0.01
time (sec)	N/A	0.045	0.060	0.049	0.453	1.016	5.502	0.154	0.000

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	69	83	76	50	158	149	-1
normalized size	1	1.00	0.84	1.01	0.93	0.61	1.93	1.82	-0.01
time (sec)	N/A	0.049	0.087	0.049	0.356	0.905	7.226	0.139	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	88	174	165	63	243	174	-1
normalized size	1	1.00	0.87	1.72	1.63	0.62	2.41	1.72	-0.01
time (sec)	N/A	0.062	0.105	0.052	0.459	0.872	8.826	0.135	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	124	208	197	146	0	811	-1
normalized size	1	1.00	1.16	1.94	1.84	1.36	0.00	7.58	-0.01
time (sec)	N/A	0.106	0.225	0.281	0.769	0.588	0.000	0.391	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	210	327	0	0	0	0	-1
normalized size	1	1.00	1.51	2.35	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	1.365	1.255	0.000	1.400	0.000	0.000	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	89	133	84	111	0	427	-1
normalized size	1	1.00	1.62	2.42	1.53	2.02	0.00	7.76	-0.02
time (sec)	N/A	0.070	0.151	0.265	0.360	2.498	0.000	0.248	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	147	196	0	0	0	0	-1
normalized size	1	1.00	1.75	2.33	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.210	0.264	0.000	0.944	0.000	0.000	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	137	361	0	0	0	0	-1
normalized size	1	1.00	1.51	3.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.155	0.219	0.000	0.685	0.000	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	71	118	79	57	0	104	88
normalized size	1	1.00	1.42	2.36	1.58	1.14	0.00	2.08	1.76
time (sec)	N/A	0.061	0.149	0.179	0.363	2.272	0.000	0.155	0.812

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	102	191	0	82	0	163	-1
normalized size	1	1.00	1.16	2.17	0.00	0.93	0.00	1.85	-0.01
time (sec)	N/A	0.078	0.120	0.148	0.000	1.108	0.000	0.181	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	108	154	197	93	0	224	-1
normalized size	1	1.00	1.06	1.51	1.93	0.91	0.00	2.20	-0.01
time (sec)	N/A	0.096	0.235	0.566	1.950	0.971	0.000	0.172	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	148	265	0	120	0	304	-1
normalized size	1	1.00	1.10	1.98	0.00	0.90	0.00	2.27	-0.01
time (sec)	N/A	0.112	0.183	0.464	0.000	0.963	0.000	0.167	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	285	510	0	0	0	0	-1
normalized size	1	1.00	1.38	2.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.217	0.905	1.064	0.000	0.970	0.000	0.000	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	580	648	0	0	0	0	-1
normalized size	1	1.00	2.64	2.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.188	7.818	1.547	0.000	1.917	0.000	0.000	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	182	349	0	0	0	0	-1
normalized size	1	1.00	1.44	2.77	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.486	0.875	0.000	0.578	0.000	0.000	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	265	437	0	0	0	0	-1
normalized size	1	1.00	1.84	3.03	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.297	0.763	0.000	2.194	0.000	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	242	666	0	0	0	0	-1
normalized size	1	1.00	1.95	5.37	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.143	0.232	0.265	0.000	1.995	0.000	0.000	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	135	199	147	98	0	195	155
normalized size	1	1.00	1.69	2.49	1.84	1.22	0.00	2.44	1.94
time (sec)	N/A	0.086	0.211	0.219	0.686	1.111	0.000	0.191	0.792

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	186	315	0	150	0	302	-1
normalized size	1	1.00	1.49	2.52	0.00	1.20	0.00	2.42	-0.01
time (sec)	N/A	0.105	0.220	0.383	0.000	1.555	0.000	0.179	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	204	299	0	173	0	428	-1
normalized size	1	1.00	1.20	1.76	0.00	1.02	0.00	2.52	-0.01
time (sec)	N/A	0.150	0.326	0.594	0.000	0.716	0.000	0.182	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	283	472	0	225	0	576	-1
normalized size	1	1.00	1.36	2.27	0.00	1.08	0.00	2.77	-0.00
time (sec)	N/A	0.176	0.360	0.757	0.000	0.932	0.000	0.181	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.015	2.913	2.294	0.000	1.837	0.000	0.000	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.006	3.489	0.901	0.000	1.075	0.000	0.000	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.024	0.299	1.046	0.000	0.446	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	43	48	0	0	0	54	-1
normalized size	1	1.00	0.91	1.02	0.00	0.00	0.00	1.15	-0.02
time (sec)	N/A	0.106	0.079	0.178	0.000	0.411	0.000	0.123	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	56	58	0	0	0	95	-1
normalized size	1	1.00	0.89	0.92	0.00	0.00	0.00	1.51	-0.02
time (sec)	N/A	0.134	0.079	0.098	0.000	0.435	0.000	0.163	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	91	102	0	0	0	200	-1
normalized size	1	1.00	0.78	0.87	0.00	0.00	0.00	1.71	-0.01
time (sec)	N/A	0.229	0.176	0.099	0.000	0.415	0.000	0.148	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.024	7.352	4.296	0.000	0.449	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.024	4.863	3.973	0.000	0.455	0.000	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	83	0	0	0	0	0	-1
normalized size	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.043	0.187	3.873	0.000	0.434	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.026	0.878	2.826	0.000	0.408	0.000	0.000	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.026	1.775	2.918	0.000	0.422	0.000	0.000	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	165	485	269	290	362	1112	-1
normalized size	1	1.00	0.99	2.90	1.61	1.74	2.17	6.66	-0.01
time (sec)	N/A	0.402	0.314	0.056	0.382	0.615	8.418	2.008	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	122	361	198	209	228	595	-1
normalized size	1	1.00	0.99	2.93	1.61	1.70	1.85	4.84	-0.01
time (sec)	N/A	0.277	0.192	0.054	0.371	0.548	7.027	2.928	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	113	140	92	129	104	349	-1
normalized size	1	1.00	1.36	1.69	1.11	1.55	1.25	4.20	-0.01
time (sec)	N/A	0.167	0.203	0.053	0.401	0.459	4.737	0.451	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	58	37	53	64	32	62	33
normalized size	1	1.00	1.87	1.19	1.71	2.06	1.03	2.00	1.06
time (sec)	N/A	0.023	0.048	0.045	0.351	0.472	2.431	0.128	0.002

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	411	869	0	0	0	0	-1
normalized size	1	1.00	1.60	3.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.406	0.694	1.114	0.000	0.432	0.000	0.000	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	141	214	0	475	0	0	-1
normalized size	1	1.00	1.38	2.10	0.00	4.66	0.00	0.00	-0.01
time (sec)	N/A	0.155	0.223	0.063	0.000	0.500	0.000	0.000	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	250	1007	0	1111	0	0	-1
normalized size	1	1.00	1.45	5.85	0.00	6.46	0.00	0.00	-0.01
time (sec)	N/A	0.292	0.506	0.067	0.000	0.952	0.000	0.000	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	496	693	870	1222	0	0	0	0	-1
normalized size	1	1.40	1.75	2.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.827	13.934	0.086	0.000	1.432	0.000	0.000	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	404	502	368	836	0	0	0	0	-1
normalized size	1	1.24	0.91	2.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.195	1.655	0.075	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	657	388	0	0	0	0	-1
normalized size	1	1.00	2.09	1.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.485	13.196	0.073	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.071	18.518	8.853	0.000	1.268	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.076	8.307	10.087	0.000	1.798	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	333	810	0	0	0	0	-1
normalized size	1	1.00	0.90	2.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.755	1.498	0.073	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	714	714	873	1248	0	0	0	0	-1
normalized size	1	1.00	1.22	1.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.589	13.848	0.083	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	530	530	784	862	0	0	0	0	-1
normalized size	1	1.00	1.48	1.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.907	13.671	0.076	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	289	412	0	0	0	0	-1
normalized size	1	1.00	0.84	1.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.600	1.509	0.073	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	243	254	0	0	0	0	-1
normalized size	1	1.00	1.15	1.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.319	3.690	0.069	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.072	6.640	4.565	0.000	0.837	0.000	0.000	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.079	9.214	7.832	0.000	2.940	0.000	0.000	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	551	551	814	890	0	0	0	0	-1
normalized size	1	1.00	1.48	1.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.474	14.016	0.082	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	750	441	0	0	0	0	-1
normalized size	1	1.00	2.03	1.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.814	13.839	0.076	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	226	277	0	0	0	0	-1
normalized size	1	1.00	0.95	1.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.575	1.883	0.074	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	124	217	0	0	0	0	-1
normalized size	1	1.00	1.04	1.82	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.225	0.216	0.063	0.000	2.914	0.000	0.000	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.089	11.783	7.378	0.000	1.462	0.000	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.094	14.909	180.000	0.000	0.977	0.000	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	602	602	887	1077	0	0	0	0	-1
normalized size	1	1.00	1.47	1.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.896	14.101	0.086	0.000	1.488	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	440	440	856	1040	0	0	0	0	-1
normalized size	1	1.00	1.95	2.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.185	13.952	0.080	0.000	0.984	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	345	913	0	0	0	0	-1
normalized size	1	1.00	1.10	2.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.001	1.750	0.079	0.000	1.032	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	725	886	0	0	0	0	-1
normalized size	1	1.00	2.43	2.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.409	13.636	0.079	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	F(-1)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.088	29.957	180.000	0.000	1.457	0.000	0.000	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	F(-1)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.101	26.677	180.000	0.000	2.498	0.000	0.000	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	540	637	1002	1640	0	0	0	0	-1
normalized size	1	1.18	1.86	3.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.752	13.985	0.081	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	140	338	296	191	408	1190	-1
normalized size	1	1.00	0.68	1.64	1.44	0.93	1.98	5.78	-0.00
time (sec)	N/A	0.125	0.230	0.056	0.343	1.771	12.254	2.124	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	121	282	232	170	294	840	-1
normalized size	1	1.00	0.75	1.75	1.44	1.06	1.83	5.22	-0.01
time (sec)	N/A	0.108	0.176	0.055	0.326	1.610	7.471	1.444	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	149	194	153	141	153	485	-1
normalized size	1	1.00	1.37	1.78	1.40	1.29	1.40	4.45	-0.01
time (sec)	N/A	0.051	0.276	0.055	0.322	0.707	6.027	1.232	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	104	136	89	125	73	1062	75
normalized size	1	1.00	1.20	1.56	1.02	1.44	0.84	12.21	0.86
time (sec)	N/A	0.065	0.124	0.060	0.320	0.548	5.433	0.458	0.833

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	69	121	94	66	151	139	-1
normalized size	1	1.00	0.66	1.15	0.90	0.63	1.44	1.32	-0.01
time (sec)	N/A	0.078	0.115	0.062	0.331	0.937	4.728	0.145	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	94	140	137	88	280	250	-1
normalized size	1	1.00	0.62	0.92	0.90	0.58	1.84	1.64	-0.01
time (sec)	N/A	0.092	0.170	0.062	0.336	0.535	9.748	0.171	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	110	158	172	109	372	374	-1
normalized size	1	1.00	0.56	0.80	0.87	0.55	1.89	1.90	-0.01
time (sec)	N/A	0.118	0.173	0.064	0.336	0.605	56.150	0.158	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	115	152	183	127	364	1270	-1
normalized size	1	1.00	0.59	0.78	0.93	0.65	1.86	6.48	-0.01
time (sec)	N/A	0.150	0.249	0.053	0.342	0.710	8.357	0.322	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	97	134	142	106	272	920	-1
normalized size	1	1.00	0.63	0.88	0.93	0.69	1.78	6.01	-0.01
time (sec)	N/A	0.122	0.219	0.053	0.332	0.597	5.549	0.264	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	78	115	98	85	177	570	-1
normalized size	1	1.00	0.57	0.83	0.71	0.62	1.28	4.13	-0.01
time (sec)	N/A	0.096	0.104	0.053	0.335	0.891	3.779	0.223	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	108	198	0	0	0	0	111
normalized size	1	1.00	0.87	1.60	0.00	0.00	0.00	0.00	0.90
time (sec)	N/A	0.286	0.088	3.304	0.000	1.007	0.000	0.000	0.925

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	125	185	0	0	0	0	124
normalized size	1	1.00	0.91	1.35	0.00	0.00	0.00	0.00	0.91
time (sec)	N/A	0.302	0.101	0.776	0.000	0.610	0.000	0.000	0.991

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	184	494	404	273	542	1573	-1
normalized size	1	1.00	0.73	1.96	1.60	1.08	2.15	6.24	-0.00
time (sec)	N/A	0.237	0.373	0.056	0.338	1.186	14.163	4.954	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	151	371	296	237	355	1027	-1
normalized size	1	1.00	0.79	1.94	1.55	1.24	1.86	5.38	-0.01
time (sec)	N/A	0.114	0.257	0.056	0.336	2.602	10.120	3.200	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	134	286	197	232	207	2494	-1
normalized size	1	1.00	0.82	1.75	1.21	1.42	1.27	15.30	-0.01
time (sec)	N/A	0.130	0.210	0.061	0.347	1.158	8.825	2.154	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	125	254	158	222	211	4280	-1
normalized size	1	1.00	0.80	1.62	1.01	1.41	1.34	27.26	-0.01
time (sec)	N/A	0.132	0.290	0.067	0.327	0.513	8.426	14.422	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	127	191	181	126	335	316	-1
normalized size	1	1.00	0.69	1.04	0.99	0.69	1.83	1.73	-0.01
time (sec)	N/A	0.160	0.278	0.064	0.350	0.694	10.894	0.158	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	153	223	241	158	510	493	-1
normalized size	1	1.00	0.63	0.93	1.00	0.66	2.12	2.05	-0.00
time (sec)	N/A	0.197	0.278	0.066	0.339	0.694	59.500	0.174	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	159	214	253	186	493	1700	-1
normalized size	1	1.00	0.66	0.88	1.05	0.77	2.04	7.02	-0.00
time (sec)	N/A	0.223	0.338	0.056	0.348	0.748	9.219	0.446	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	124	182	189	152	352	1154	-1
normalized size	1	1.00	0.64	0.93	0.97	0.78	1.81	5.92	-0.01
time (sec)	N/A	0.144	0.294	0.056	0.344	0.476	6.382	0.321	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	157	301	0	0	0	0	-1
normalized size	1	1.00	0.84	1.62	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.422	0.387	6.183	0.000	0.531	0.000	0.000	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	187	276	0	0	0	0	-1
normalized size	1	1.00	0.99	1.46	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.430	0.678	4.297	0.000	0.561	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	565	565	1260	407	0	0	0	0	-1
normalized size	1	1.00	2.23	0.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.257	1.819	4.272	0.000	0.611	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	507	507	1123	401	0	0	0	0	-1
normalized size	1	1.00	2.21	0.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.139	0.427	1.124	0.000	0.669	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	529	529	1068	272	0	0	0	0	-1
normalized size	1	1.00	2.02	0.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.870	0.438	2.036	0.000	0.478	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	479	479	407	2875	0	0	0	0	-1
normalized size	1	1.00	0.85	6.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.900	0.940	1.073	0.000	0.452	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	572	572	1241	332	0	0	0	0	-1
normalized size	1	1.00	2.17	0.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.100	1.750	3.983	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	628	628	1480	729	0	0	0	0	-1
normalized size	1	1.00	2.36	1.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.321	6.236	1.828	0.000	1.122	0.000	0.000	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	593	590	1442	541	0	0	0	0	-1
normalized size	1	0.99	2.43	0.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.238	2.255	1.364	0.000	0.642	0.000	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	131	286	354	0	385	0	0	-1
normalized size	1	0.98	2.13	2.64	0.00	2.87	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.783	0.081	0.000	0.613	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	566	566	0	3036	0	0	0	0	-1
normalized size	1	1.00	0.00	5.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.161	43.452	2.613	0.000	0.627	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	803	803	1634	1888	0	0	0	0	-1
normalized size	1	1.00	2.03	2.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.400	6.165	19.149	0.000	0.615	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	765	765	1482	1722	0	0	0	0	-1
normalized size	1	1.00	1.94	2.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.255	2.801	4.792	0.000	0.698	0.000	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	762	759	1477	1716	0	0	0	0	-1
normalized size	1	1.00	1.94	2.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.288	2.873	5.046	0.000	0.504	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	806	806	1525	1784	0	0	0	0	-1
normalized size	1	1.00	1.89	2.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.364	2.713	19.918	0.000	0.647	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	727	727	2053	1498	0	0	0	0	-1
normalized size	1	1.00	2.82	2.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.442	7.763	2.030	0.000	0.799	0.000	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	390	1870	0	1015	0	0	-1
normalized size	1	1.00	2.48	11.91	0.00	6.46	0.00	0.00	-0.01
time (sec)	N/A	0.182	1.117	0.087	0.000	1.389	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	385	1840	0	890	0	0	-1
normalized size	1	1.00	1.99	9.53	0.00	4.61	0.00	0.00	-0.01
time (sec)	N/A	0.191	0.942	0.076	0.000	1.084	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	704	704	0	5294	0	0	0	0	-1
normalized size	1	1.00	0.00	7.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.318	64.962	7.102	0.000	0.503	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1144	1144	2067	3175	0	0	0	0	-1
normalized size	1	1.00	1.81	2.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.600	6.199	5.009	0.000	0.493	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1144	1144	2075	2329	0	0	0	0	-1
normalized size	1	1.00	1.81	2.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.069	6.143	7.794	0.000	0.771	0.000	0.000	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1134	1134	2060	3166	0	0	0	0	-1
normalized size	1	1.00	1.82	2.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.049	6.064	10.036	0.000	1.967	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	365	0	0	1699	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	4.22	0.00	0.00	-0.00
time (sec)	N/A	1.312	0.637	7.372	0.000	7.328	0.000	0.000	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	326	0	0	1379	0	0	-1
normalized size	1	1.00	1.11	0.00	0.00	4.69	0.00	0.00	-0.00
time (sec)	N/A	0.409	0.745	6.635	0.000	2.216	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	265	0	0	1098	0	0	-1
normalized size	1	1.00	1.36	0.00	0.00	5.63	0.00	0.00	-0.01
time (sec)	N/A	0.212	0.483	5.056	0.000	2.837	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.102	5.216	4.387	0.000	0.758	0.000	0.000	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.107	4.815	1.296	0.000	0.705	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.096	9.571	6.372	0.000	0.853	0.000	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.034	6.674	4.379	0.000	0.432	0.000	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.087	1.741	3.382	0.000	0.434	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	247	0	0	0	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.432	0.651	6.424	0.000	0.510	0.000	0.000	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	453	453	325	0	0	0	0	0	-1
normalized size	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.620	0.721	8.948	0.000	0.730	0.000	0.000	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	339	0	0	1697	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	4.54	0.00	0.00	-0.00
time (sec)	N/A	0.538	0.617	6.428	0.000	9.246	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	304	0	0	1375	0	0	-1
normalized size	1	1.00	1.16	0.00	0.00	5.25	0.00	0.00	-0.00
time (sec)	N/A	0.288	0.758	4.740	0.000	4.496	0.000	0.000	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.116	6.085	3.975	0.000	0.880	0.000	0.000	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.122	6.056	1.069	0.000	2.942	0.000	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.124	9.914	5.865	0.000	0.859	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.045	8.473	4.244	0.000	1.636	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.106	13.649	3.707	0.000	1.238	0.000	0.000	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.107	6.488	6.641	0.000	1.467	0.000	0.000	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	303	0	0	0	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.573	0.687	9.023	0.000	2.034	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	554	554	383	0	0	0	0	0	-1
normalized size	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.891	0.875	7.491	0.000	0.991	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	328	0	0	1383	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	4.31	0.00	0.00	-0.00
time (sec)	N/A	1.048	0.748	7.873	0.000	4.800	0.000	0.000	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	271	0	0	1107	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	4.92	0.00	0.00	-0.00
time (sec)	N/A	0.327	0.528	7.302	0.000	2.401	0.000	0.000	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	210	0	0	870	0	0	-1
normalized size	1	1.00	1.59	0.00	0.00	6.59	0.00	0.00	-0.01
time (sec)	N/A	0.151	0.225	5.435	0.000	1.653	0.000	0.000	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.097	1.710	4.178	0.000	0.558	0.000	0.000	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.111	12.333	1.440	0.000	0.589	0.000	0.000	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.092	16.601	7.451	0.000	0.589	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.031	1.024	4.686	0.000	0.636	0.000	0.000	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	140	0	0	0	0	0	-1
normalized size	1	1.00	0.57	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.266	0.199	4.445	0.000	0.662	0.000	0.000	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	249	0	0	0	0	0	-1
normalized size	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.569	0.664	8.422	0.000	0.816	0.000	0.000	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	302	0	0	1480	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	5.87	0.00	0.00	-0.00
time (sec)	N/A	1.074	0.637	7.404	0.000	1.678	0.000	0.000	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	220	0	0	1072	0	0	-1
normalized size	1	1.00	1.41	0.00	0.00	6.87	0.00	0.00	-0.01
time (sec)	N/A	0.321	0.361	6.681	0.000	0.957	0.000	0.000	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	96	0	0	283	0	0	-1
normalized size	1	1.00	1.22	0.00	0.00	3.58	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.155	4.786	0.000	0.706	0.000	0.000	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.125	19.369	3.842	0.000	0.678	0.000	0.000	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.136	25.009	1.339	0.000	0.763	0.000	0.000	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.113	9.537	6.200	0.000	0.753	0.000	0.000	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.102	4.828	6.918	0.000	0.603	0.000	0.000	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	112	0	0	0	0	0	-1
normalized size	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.207	4.350	0.000	0.761	0.000	0.000	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	213	0	0	0	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.318	0.492	4.026	0.000	0.778	0.000	0.000	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	311	0	0	2119	0	0	-1
normalized size	1	1.00	1.28	0.00	0.00	8.72	0.00	0.00	-0.00
time (sec)	N/A	1.057	0.538	7.342	0.000	1.378	0.000	0.000	0.000

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	173	0	0	663	0	0	-1
normalized size	1	1.00	1.06	0.00	0.00	4.07	0.00	0.00	-0.01
time (sec)	N/A	0.246	0.268	6.727	0.000	1.095	0.000	0.000	0.000

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	157	0	0	573	0	0	-1
normalized size	1	1.00	1.14	0.00	0.00	4.15	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.208	4.939	0.000	1.056	0.000	0.000	0.000

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.123	37.842	3.726	0.000	0.818	0.000	0.000	0.000

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.140	46.557	1.347	0.000	0.797	0.000	0.000	0.000

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.121	14.827	8.811	0.000	0.802	0.000	0.000	0.000

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.110	12.520	6.298	0.000	0.807	0.000	0.000	0.000

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	185	0	0	0	0	0	-1
normalized size	1	1.00	0.67	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.283	0.286	6.914	0.000	0.657	0.000	0.000	0.000

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	249	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.238	0.560	4.315	0.000	0.731	0.000	0.000	0.000

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	585	566	0	0	0	0	0	0	-1
normalized size	1	0.97	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.405	0.223	12.128	0.000	0.761	0.000	0.000	0.000

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	371	352	0	0	0	0	0	0	-1
normalized size	1	0.95	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.432	0.145	9.316	0.000	0.624	0.000	0.000	0.000

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-1)	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	215	202	0	0	0	0	0	0	-1
normalized size	1	0.94	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.194	0.107	7.525	0.000	0.639	0.000	0.000	0.000

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.075	1.990	3.590	0.000	0.605	0.000	0.000	0.000

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.074	6.445	6.050	0.000	0.626	0.000	0.000	0.000

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.115	1.156	4.785	0.000	0.684	0.000	0.000	0.000

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.097	0.115	4.877	0.000	0.636	0.000	0.000	0.000

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.100	1.485	6.561	0.000	0.777	0.000	0.000	0.000

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.117	1.671	6.271	0.000	0.858	0.000	0.000	0.000

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	401	401	194	0	0	0	0	0	-1
normalized size	1	1.00	0.48	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.527	0.295	13.067	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	159	0	0	0	0	0	-1
normalized size	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.027	0.408	9.292	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	135	138	0	0	0	0	0	-1
normalized size	1	1.07	1.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.212	0.179	6.284	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.089	0.411	6.198	0.000	0.610	0.000	0.000	0.000

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.101	3.582	4.244	0.000	0.766	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [69] had the largest ratio of [.8571]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	5	1.00	12	0.417
2	A	4	3	1.00	12	0.250
3	A	6	5	1.00	12	0.417
4	A	3	3	1.00	12	0.250
5	A	5	5	1.00	12	0.417
6	A	2	2	1.00	10	0.200
7	A	5	4	1.00	8	0.500
8	A	6	6	1.00	12	0.500
9	A	2	2	1.00	12	0.167
10	A	4	4	1.00	12	0.333
11	A	4	3	1.00	12	0.250
12	A	5	4	1.00	12	0.333
13	A	4	3	1.00	12	0.250
14	A	6	4	1.00	12	0.333
15	A	5	5	1.00	14	0.357
16	A	8	6	1.00	14	0.429
17	A	4	4	1.00	12	0.333
18	A	7	5	1.00	10	0.500
19	A	6	6	1.00	14	0.429
20	A	4	3	1.00	14	0.214
21	A	4	3	1.00	14	0.214
22	A	5	5	1.00	14	0.357
23	A	5	3	1.00	14	0.214
24	A	10	10	1.00	14	0.714
25	A	11	8	1.00	14	0.571
26	A	7	7	1.00	12	0.583
27	A	9	6	1.00	10	0.600
28	A	7	7	1.00	14	0.500
29	A	5	3	1.00	14	0.214
30	A	6	6	1.00	14	0.429
31	A	8	6	1.00	14	0.429
32	A	10	6	1.00	14	0.429
33	A	0	0	0.00	0	0.000
34	A	0	0	0.00	0	0.000
35	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
36	A	4	4	1.00	14	0.286
37	A	6	6	1.00	14	0.429
38	A	9	5	1.00	14	0.357
39	A	0	0	0.00	0	0.000
40	A	0	0	0.00	0	0.000
41	A	3	3	1.00	14	0.214
42	A	0	0	0.00	0	0.000
43	A	0	0	0.00	0	0.000
44	A	11	9	1.00	16	0.562
45	A	10	9	1.00	16	0.562
46	A	9	9	1.00	14	0.643
47	A	5	4	1.00	8	0.500
48	A	4	2	1.00	16	0.125
49	A	7	7	1.00	16	0.438
50	A	8	8	1.00	16	0.500
51	A	31	16	1.40	21	0.762
52	A	24	15	1.24	19	0.790
53	A	15	11	1.00	18	0.611
54	A	0	0	0.00	0	0.000
55	A	0	0	0.00	0	0.000
56	A	22	13	1.00	18	0.722
57	A	27	17	1.00	21	0.810
58	A	20	15	1.00	21	0.714
59	A	14	13	1.00	19	0.684
60	A	9	9	1.00	18	0.500
61	A	0	0	0.00	0	0.000
62	A	0	0	0.00	0	0.000
63	A	23	15	1.00	21	0.714
64	A	16	13	1.00	21	0.619
65	A	11	11	1.00	19	0.579
66	A	6	6	1.00	18	0.333
67	A	0	0	0.00	0	0.000
68	A	0	0	0.00	0	0.000
69	A	31	18	1.00	21	0.857
70	A	25	17	1.00	21	0.810
71	A	19	14	1.00	19	0.737

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
72	A	12	11	1.00	18	0.611
73	A	0	0	0.00	0	0.000
74	A	0	0	0.00	0	0.000
75	A	19	14	1.18	18	0.778
76	A	7	7	1.00	19	0.368
77	A	6	7	1.00	19	0.368
78	A	5	5	1.00	16	0.312
79	A	4	5	1.00	19	0.263
80	A	4	5	1.00	19	0.263
81	A	5	6	1.00	19	0.316
82	A	6	6	1.00	19	0.316
83	A	5	5	1.00	19	0.263
84	A	5	5	1.00	19	0.263
85	A	6	5	1.00	17	0.294
86	A	11	11	1.00	19	0.579
87	A	13	13	1.00	19	0.684
88	A	7	8	1.00	21	0.381
89	A	6	7	1.00	18	0.389
90	A	6	7	1.00	21	0.333
91	A	6	7	1.00	21	0.333
92	A	5	6	1.00	21	0.286
93	A	6	7	1.00	21	0.333
94	A	5	6	1.00	21	0.286
95	A	6	5	1.00	19	0.263
96	A	12	13	1.00	21	0.619
97	A	14	15	1.00	21	0.714
98	A	25	12	1.00	21	0.571
99	A	26	9	1.00	19	0.474
100	A	19	7	1.00	18	0.389
101	A	19	7	1.00	21	0.333
102	A	24	10	1.00	21	0.476
103	A	31	14	1.00	21	0.667
104	A	29	12	0.99	21	0.571
105	A	7	5	0.98	19	0.263
106	A	24	10	1.00	21	0.476
107	A	51	15	1.00	21	0.714

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
108	A	27	10	1.00	21	0.476
109	A	47	11	1.00	18	0.611
110	A	50	13	1.00	21	0.619
111	A	33	13	1.00	21	0.619
112	A	6	7	1.00	21	0.333
113	A	8	6	1.00	19	0.316
114	A	28	11	1.00	21	0.524
115	A	35	11	1.00	21	0.524
116	A	63	12	1.00	21	0.571
117	A	81	12	1.00	18	0.667
118	A	12	12	1.00	23	0.522
119	A	11	12	1.00	23	0.522
120	A	9	9	1.00	21	0.429
121	A	0	0	0.00	0	0.000
122	A	0	0	0.00	0	0.000
123	A	0	0	0.00	0	0.000
124	A	0	0	0.00	0	0.000
125	A	0	0	0.00	0	0.000
126	A	11	11	1.00	23	0.478
127	A	12	12	1.00	23	0.522
128	A	12	12	1.00	23	0.522
129	A	10	10	1.00	21	0.476
130	A	0	0	0.00	0	0.000
131	A	0	0	0.00	0	0.000
132	A	0	0	0.00	0	0.000
133	A	0	0	0.00	0	0.000
134	A	0	0	0.00	0	0.000
135	A	0	0	0.00	0	0.000
136	A	12	12	1.00	23	0.522
137	A	13	12	1.00	23	0.522
138	A	11	12	1.00	23	0.522
139	A	10	12	1.00	23	0.522
140	A	8	8	1.00	21	0.381
141	A	0	0	0.00	0	0.000
142	A	0	0	0.00	0	0.000
143	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
144	A	0	0	0.00	0	0.000
145	A	11	11	1.00	23	0.478
146	A	11	12	1.00	23	0.522
147	A	10	11	1.00	23	0.478
148	A	9	11	1.00	23	0.478
149	A	4	4	1.00	21	0.190
150	A	0	0	0.00	0	0.000
151	A	0	0	0.00	0	0.000
152	A	0	0	0.00	0	0.000
153	A	0	0	0.00	0	0.000
154	A	5	5	1.00	20	0.250
155	A	10	11	1.00	23	0.478
156	A	10	11	1.00	23	0.478
157	A	7	8	1.00	23	0.348
158	A	5	5	1.00	21	0.238
159	A	0	0	0.00	0	0.000
160	A	0	0	0.00	0	0.000
161	A	0	0	0.00	0	0.000
162	A	0	0	0.00	0	0.000
163	A	10	10	1.00	23	0.435
164	A	10	11	1.00	20	0.550
165	A	6	7	0.97	23	0.304
166	A	6	7	0.95	23	0.304
167	A	5	6	0.94	21	0.286
168	A	0	0	0.00	0	0.000
169	A	0	0	0.00	0	0.000
170	A	0	0	0.00	0	0.000
171	A	0	0	0.00	0	0.000
172	A	0	0	0.00	0	0.000
173	A	0	0	0.00	0	0.000
174	A	16	11	1.00	26	0.423
175	A	13	11	1.00	26	0.423
176	A	8	9	1.07	26	0.346
177	A	0	0	0.00	0	0.000
178	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

3.1 $\int x^6 \left(a + b\csc^{-1}(cx)\right) dx$

Optimal. Leaf size=114

$$\frac{1}{7}x^7\left(a+b\csc^{-1}(cx)\right)+\frac{bx^6\sqrt{1-\frac{1}{c^2x^2}}}{42c}+\frac{5b\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{112c^7}+\frac{5bx^2\sqrt{1-\frac{1}{c^2x^2}}}{112c^5}+\frac{5bx^4\sqrt{1-\frac{1}{c^2x^2}}}{168c^3}$$

```
[Out] 1/7*x^7*(a+b*arccsc(c*x))+5/112*b*arctanh((1-1/c^2/x^2)^(1/2))/c^7+5/112*b*
x^2*(1-1/c^2/x^2)^(1/2)/c^5+5/168*b*x^4*(1-1/c^2/x^2)^(1/2)/c^3+1/42*b*x^6*
(1-1/c^2/x^2)^(1/2)/c
```

Rubi [A] time = 0.06, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5221, 266, 51, 63, 208}

$$\frac{1}{7}x^7\left(a+b\csc^{-1}(cx)\right)+\frac{bx^6\sqrt{1-\frac{1}{c^2x^2}}}{42c}+\frac{5bx^4\sqrt{1-\frac{1}{c^2x^2}}}{168c^3}+\frac{5bx^2\sqrt{1-\frac{1}{c^2x^2}}}{112c^5}+\frac{5b\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{112c^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (5*b*Sqrt[1 - 1/(c^2*x^2)]*x^2)/(112*c^5) + (5*b*Sqrt[1 - 1/(c^2*x^2)]*x^4)
/(168*c^3) + (b*Sqrt[1 - 1/(c^2*x^2)]*x^6)/(42*c) + (x^7*(a + b*ArcCsc[c*x]
))/7 + (5*b*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/(112*c^7)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5221

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((d*x)^(m + 1)*(a + b*ArcCsc[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m +
1)), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^6 \left(a+b\csc^{-1}(cx)\right) dx = \frac{1}{7}x^7\left(a+b\csc^{-1}(cx)\right)+\frac{b\int \frac{x^5}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{7c}$$

$$=\frac{1}{7}x^7\left(a+b\csc^{-1}(cx)\right)-\frac{b\,\text{Subst}\left(\int \frac{1}{x^4\sqrt{1-\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{14c}$$

$$=\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^6}{42c}+\frac{1}{7}x^7\left(a+b\csc^{-1}(cx)\right)-\frac{(5b)\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{1-\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{84c^3}$$

$$=\frac{5b\sqrt{1-\frac{1}{c^2x^2}}\,x^4}{168c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^6}{42c}+\frac{1}{7}x^7\left(a+b\csc^{-1}(cx)\right)-\frac{(5b)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x}{c^2}}}\,dx\right.}{112c^5}$$

$$=\frac{5b\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{112c^5}+\frac{5b\sqrt{1-\frac{1}{c^2x^2}}\,x^4}{168c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^6}{42c}+\frac{1}{7}x^7\left(a+b\csc^{-1}(cx)\right)-\frac{(5b)\,\text{Su}}{}$$

$$=\frac{5b\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{112c^5}+\frac{5b\sqrt{1-\frac{1}{c^2x^2}}\,x^4}{168c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^6}{42c}+\frac{1}{7}x^7\left(a+b\csc^{-1}(cx)\right)+\frac{(5b)\,\text{Su}}{}$$

$$=\frac{5b\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{112c^5}+\frac{5b\sqrt{1-\frac{1}{c^2x^2}}\,x^4}{168c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^6}{42c}+\frac{1}{7}x^7\left(a+b\csc^{-1}(cx)\right)+\frac{5b\,\text{tanh}}{}$$

Mathematica [A] time = 0.14, size = 107, normalized size = 0.94

$$\frac{ax^7}{7}+\frac{5b\log\left(x\left(\sqrt{\frac{c^2x^2-1}{c^2x^2}}+1\right)\right)}{112c^7}+b\sqrt{\frac{c^2x^2-1}{c^2x^2}}\left(\frac{5x^2}{112c^5}+\frac{5x^4}{168c^3}+\frac{x^6}{42c}\right)+\frac{1}{7}bx^7\csc^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (a*x^7)/7 + b*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)]*((5*x^2)/(112*c^5) + (5*x^4)/(
168*c^3) + x^6/(42*c)) + (b*x^7*ArcCsc[c*x])/7 + (5*b*Log[x*(1 + Sqrt[(-1 +
 c^2*x^2)/(c^2*x^2)])])/(112*c^7)
```

fricas [A] time = 1.04, size = 115, normalized size = 1.01

$$\frac{48\,ac^7x^7 - 96\,bc^7\arctan\left(-cx + \sqrt{c^2x^2-1}\right) + 48\left(bc^7x^7 - bc^7\right)\operatorname{arccsc}(cx) - 15\,b\log\left(-cx + \sqrt{c^2x^2-1}\right) + (8}{336\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/336*(48*a*c^7*x^7 - 96*b*c^7*arctan(-c*x + sqrt(c^2*x^2 - 1)) + 48*(b*c^7
*x^7 - b*c^7)*arccsc(c*x) - 15*b*log(-c*x + sqrt(c^2*x^2 - 1)) + (8*b*c^5*x
^5 + 10*b*c^3*x^3 + 15*b*c*x)*sqrt(c^2*x^2 - 1))/c^7
```

giac [B] time = 1.01, size = 646, normalized size = 5.67

$$\frac{1}{2688}\left(\frac{3\,bx^7\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^7\arcsin\left(\frac{1}{cx}\right)}{c} + \frac{3\,ax^7\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^7}{c} + \frac{bx^6\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^6}{c^2} + \frac{21\,bx^5\left(\sqrt{-\right.}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/2688*(3*b*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7*arcsin(1/(c*x))/c + 3*a*x^7*
(sqrt(-1/(c^2*x^2) + 1) + 1)^7/c + b*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6/c^2
 + 21*b*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*arcsin(1/(c*x))/c^3 + 21*a*x^5*(
sqrt(-1/(c^2*x^2) + 1) + 1)^5/c^3 + 9*b*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/
c^4 + 63*b*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x))/c^5 + 63*a*x^
3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^5 + 45*b*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1
)^2/c^6 + 105*b*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^7 + 105*a*
x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^7 + 120*b*log(sqrt(-1/(c^2*x^2) + 1) + 1)/
c^8 - 120*b*log(1/(abs(c)*abs(x)))/c^8 + 105*b*arcsin(1/(c*x))/(c^9*x*(sqrt
(-1/(c^2*x^2) + 1) + 1)) + 105*a/(c^9*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) - 45*
b/(c^10*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 63*b*arcsin(1/(c*x))/(c^11*x^
3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 63*a/(c^11*x^3*(sqrt(-1/(c^2*x^2) + 1)
+ 1)^3) - 9*b/(c^12*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) + 21*b*arcsin(1/(c*
x))/(c^13*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 21*a/(c^13*x^5*(sqrt(-1/(c^
2*x^2) + 1) + 1)^5) - b/(c^14*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6) + 3*b*arc
sin(1/(c*x))/(c^15*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7) + 3*a/(c^15*x^7*(sqr
t(-1/(c^2*x^2) + 1) + 1)^7))*c
```

maple [A] time = 0.05, size = 177, normalized size = 1.55

$$\frac{x^7a}{7} + \frac{b\,x^7\operatorname{arccsc}(cx)}{7} + \frac{b\,x^6}{42c\sqrt{\frac{c^2x^2-1}{c^2x^2}}} + \frac{b\,x^4}{168c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}} + \frac{5b\,x^2}{336c^5\sqrt{\frac{c^2x^2-1}{c^2x^2}}} - \frac{5b}{112c^7\sqrt{\frac{c^2x^2-1}{c^2x^2}}} + \frac{5b\sqrt{c^2x^2-1}\,\ln\left(cx + \sqrt{c\right.}}{112c^8\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a+b*arccsc(c*x)),x)
```

```
[Out] 1/7*x^7*a+1/7*b*x^7*arccsc(c*x)+1/42/c*b/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^6+1/
168/c^3*b/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^4+5/336/c^5*b/((c^2*x^2-1)/c^2/x^2)
^(1/2)*x^2-5/112/c^7*b/((c^2*x^2-1)/c^2/x^2)^(1/2)+5/112/c^8*b*(c^2*x^2-1)^
(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*ln(c*x+(c^2*x^2-1)^(1/2))
```

maxima [A] time = 0.34, size = 161, normalized size = 1.41

$$\frac{1}{7}\,ax^7+\frac{1}{672}\left(96\,x^7\operatorname{arccsc}(cx)+\frac{\frac{2\left(15\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-40\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+33\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^6\left(\frac{1}{c^2x^2}-1\right)^3+3\,c^6\left(\frac{1}{c^2x^2}-1\right)^2+3\,c^6\left(\frac{1}{c^2x^2}-1\right)+c^6}+\frac{15\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^6}-\frac{15\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^6}}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*x^7 + 1/672*(96*x^7*arccsc(c*x) + (2*(15*(-1/(c^2*x^2) + 1)^(5/2) - 4
0*(-1/(c^2*x^2) + 1)^(3/2) + 33*sqrt(-1/(c^2*x^2) + 1))/(c^6*(1/(c^2*x^2) -
 1)^3 + 3*c^6*(1/(c^2*x^2) - 1)^2 + 3*c^6*(1/(c^2*x^2) - 1) + c^6) + 15*log
(sqrt(-1/(c^2*x^2) + 1) + 1)/c^6 - 15*log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^6)/
c)*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^6\left(a+b\operatorname{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*asin(1/(c*x))),x)
```

```
[Out] int(x^6*(a + b*asin(1/(c*x))), x)
```

sympy [A] time = 8.92, size = 221, normalized size = 1.94

$$\frac{ax^7}{7}+\frac{bx^7\operatorname{acsc}(cx)}{7}+\frac{b\left(\begin{cases}\frac{cx^7}{6\sqrt{c^2x^2-1}}+\frac{x^5}{24c\sqrt{c^2x^2-1}}+\frac{5x^3}{48c^3\sqrt{c^2x^2-1}}-\frac{5x}{16c^5\sqrt{c^2x^2-1}}+\frac{5\operatorname{acosh}(cx)}{16c^6} & \text{for } \left|c^2x^2\right|>1\\ -\frac{icx^7}{6\sqrt{-c^2x^2+1}}-\frac{ix^5}{24c\sqrt{-c^2x^2+1}}-\frac{5ix^3}{48c^3\sqrt{-c^2x^2+1}}+\frac{5ix}{16c^5\sqrt{-c^2x^2+1}}-\frac{5i\operatorname{asin}(cx)}{16c^6} & \text{otherwise}\end{cases}\right)}{7c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(a+b*acsc(c*x)),x)
```

```
[Out] a*x**7/7 + b*x**7*acsc(c*x)/7 + b*Piecewise((c*x**7/(6*sqrt(c**2*x**2 - 1))
 + x**5/(24*c*sqrt(c**2*x**2 - 1)) + 5*x**3/(48*c**3*sqrt(c**2*x**2 - 1)) -
 5*x/(16*c**5*sqrt(c**2*x**2 - 1)) + 5*acosh(c*x)/(16*c**6), Abs(c**2*x**2)
 > 1), (-I*c*x**7/(6*sqrt(-c**2*x**2 + 1)) - I*x**5/(24*c*sqrt(-c**2*x**2 +
 1)) - 5*I*x**3/(48*c**3*sqrt(-c**2*x**2 + 1)) + 5*I*x/(16*c**5*sqrt(-c**2*
x**2 + 1)) - 5*I*asin(c*x)/(16*c**6), True))/(7*c)
```

3.2 $\int x^5 \left(a+b\csc^{-1}(cx)\right) dx$

Optimal. Leaf size=89

$$\frac{1}{6}x^6\left(a+b\csc^{-1}(cx)\right)+\frac{bx^5\sqrt{1-\frac{1}{c^2x^2}}}{30c}+\frac{4bx\sqrt{1-\frac{1}{c^2x^2}}}{45c^5}+\frac{2bx^3\sqrt{1-\frac{1}{c^2x^2}}}{45c^3}$$

```
[Out] 1/6*x^6*(a+b*arccsc(c*x))+4/45*b*x*(1-1/c^2/x^2)^(1/2)/c^5+2/45*b*x^3*(1-1/
c^2/x^2)^(1/2)/c^3+1/30*b*x^5*(1-1/c^2/x^2)^(1/2)/c
```

Rubi [A] time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5221, 271, 191}

$$\frac{1}{6}x^6\left(a+b\csc^{-1}(cx)\right)+\frac{bx^5\sqrt{1-\frac{1}{c^2x^2}}}{30c}+\frac{2bx^3\sqrt{1-\frac{1}{c^2x^2}}}{45c^3}+\frac{4bx\sqrt{1-\frac{1}{c^2x^2}}}{45c^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (4*b*Sqrt[1 - 1/(c^2*x^2)]*x)/(45*c^5) + (2*b*Sqrt[1 - 1/(c^2*x^2)]*x^3)/(4
5*c^3) + (b*Sqrt[1 - 1/(c^2*x^2)]*x^5)/(30*c) + (x^6*(a + b*ArcCsc[c*x]))/6
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 5221

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((d*x)^(m + 1)*(a + b*ArcCsc[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m +
1)), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^5\left(a+b\csc^{-1}(cx)\right)dx &= \frac{1}{6}x^6\left(a+b\csc^{-1}(cx)\right)+\frac{b\int\frac{x^4}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{6c}\\
&= \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^5}{30c}+\frac{1}{6}x^6\left(a+b\csc^{-1}(cx)\right)+\frac{(2b)\int\frac{x^2}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{15c^3}\\
&= \frac{2b\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{45c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^5}{30c}+\frac{1}{6}x^6\left(a+b\csc^{-1}(cx)\right)+\frac{(4b)\int\frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{45c^5}\\
&= \frac{4b\sqrt{1-\frac{1}{c^2x^2}}\,x}{45c^5}+\frac{2b\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{45c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^5}{30c}+\frac{1}{6}x^6\left(a+b\csc^{-1}(cx)\right)
\end{aligned}$$

Mathematica [A] time = 0.13, size = 72, normalized size = 0.81

$$\frac{ax^6}{6}+b\sqrt{\frac{c^2x^2-1}{c^2x^2}}\left(\frac{4x}{45c^5}+\frac{2x^3}{45c^3}+\frac{x^5}{30c}\right)+\frac{1}{6}bx^6\csc^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (a*x^6)/6 + b*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)]*((4*x)/(45*c^5) + (2*x^3)/(45*
c^3) + x^5/(30*c)) + (b*x^6*ArcCsc[c*x])/6
```

fricas [A] time = 1.39, size = 62, normalized size = 0.70

$$\frac{15\,bc^6x^6\,\mathrm{arccsc}\,(cx)+15\,ac^6x^6+\left(3\,bc^4x^4+4\,bc^2x^2+8\,b\right)\sqrt{c^2x^2-1}}{90\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/90*(15*b*c^6*x^6*arccsc(c*x) + 15*a*c^6*x^6 + (3*b*c^4*x^4 + 4*b*c^2*x^2
+ 8*b)*sqrt(c^2*x^2 - 1))/c^6
```

giac [B] time = 0.21, size = 518, normalized size = 5.82

$$\frac{1}{5760}\left(\frac{15\,bx^6\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^6\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{15\,ax^6\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^6}{c}+\frac{6\,bx^5\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^5}{c^2}+\frac{90\,bx^4\left(\sqrt{-\right.}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/5760*(15*b*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*arcsin(1/(c*x))/c + 15*a*x^
6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6/c + 6*b*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5
/c^2 + 90*b*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))/c^3 + 90*a*x
^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c^3 + 50*b*x^3*(sqrt(-1/(c^2*x^2) + 1) +
1)^3/c^4 + 225*b*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c^5 + 2
25*a*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^5 + 300*b*x*(sqrt(-1/(c^2*x^2) +
1) + 1)/c^6 + 300*b*arcsin(1/(c*x))/c^7 + 300*a/c^7 - 300*b/(c^8*x*(sqrt(-1
/(c^2*x^2) + 1) + 1)) + 225*b*arcsin(1/(c*x))/(c^9*x^2*(sqrt(-1/(c^2*x^2) +
 1) + 1)^2) + 225*a/(c^9*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) - 50*b/(c^10*x
^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 90*b*arcsin(1/(c*x))/(c^11*x^4*(sqrt(-
1/(c^2*x^2) + 1) + 1)^4) + 90*a/(c^11*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) -
 6*b/(c^12*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 15*b*arcsin(1/(c*x))/(c^13
*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6) + 15*a/(c^13*x^6*(sqrt(-1/(c^2*x^2) +
1) + 1)^6))*c
```

maple [A] time = 0.05, size = 83, normalized size = 0.93

$$\frac{\frac{c^6x^6a}{6}+b\left(\frac{c^6x^6\mathrm{arccsc}(cx)}{6}+\frac{\left(c^2x^2-1\right)\left(3c^4x^4+4c^2x^2+8\right)}{90\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccsc(c*x)),x)
```

```
[Out] 1/c^6*(1/6*c^6*x^6*a+b*(1/6*c^6*x^6*arccsc(c*x)+1/90*(c^2*x^2-1)*(3*c^4*x^4
+4*c^2*x^2+8)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x))
```

maxima [A] time = 0.35, size = 80, normalized size = 0.90

$$\frac{1}{6}\,ax^6+\frac{1}{90}\left(15\,x^6\,\mathrm{arccsc}\left(cx\right)+\frac{3\,c^4x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+10\,c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^5}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6 + 1/90*(15*x^6*arccsc(c*x) + (3*c^4*x^5*(-1/(c^2*x^2) + 1)^(5/2)
+ 10*c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 15*x*sqrt(-1/(c^2*x^2) + 1))/c^5)*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*asin(1/(c*x))),x)
```

```
[Out] int(x^5*(a + b*asin(1/(c*x))), x)
```

sympy [A] time = 4.42, size = 153, normalized size = 1.72

$$\frac{ax^6}{6}+\frac{bx^6\,\mathrm{acsc}\,(cx)}{6}+\frac{b\left(\begin{cases}\frac{x^4\sqrt{c^2x^2-1}}{5c}+\frac{4x^2\sqrt{c^2x^2-1}}{15c^3}+\frac{8\sqrt{c^2x^2-1}}{15c^5} & \text{for } \left|c^2x^2\right|>1\\ \frac{ix^4\sqrt{-c^2x^2+1}}{5c}+\frac{4ix^2\sqrt{-c^2x^2+1}}{15c^3}+\frac{8i\sqrt{-c^2x^2+1}}{15c^5} & \text{otherwise}\end{cases}\right)}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acsc(c*x)),x)
```

```
[Out] a*x**6/6 + b*x**6*acsc(c*x)/6 + b*Piecewise((x**4*sqrt(c**2*x**2 - 1)/(5*c)
 + 4*x**2*sqrt(c**2*x**2 - 1)/(15*c**3) + 8*sqrt(c**2*x**2 - 1)/(15*c**5),
Abs(c**2*x**2) > 1), (I*x**4*sqrt(-c**2*x**2 + 1)/(5*c) + 4*I*x**2*sqrt(-c*
*2*x**2 + 1)/(15*c**3) + 8*I*sqrt(-c**2*x**2 + 1)/(15*c**5), True))/(6*c)
```

3.3 $\int x^4 \left(a+b\csc^{-1}(cx)\right) dx$

Optimal. Leaf size=89

$$\frac{1}{5}x^5\left(a+b\csc^{-1}(cx)\right)+\frac{bx^4\sqrt{1-\frac{1}{c^2x^2}}}{20c}+\frac{3b\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{40c^5}+\frac{3bx^2\sqrt{1-\frac{1}{c^2x^2}}}{40c^3}$$

```
[Out] 1/5*x^5*(a+b*arccsc(c*x))+3/40*b*arctanh((1-1/c^2/x^2)^(1/2))/c^5+3/40*b*x^
2*(1-1/c^2/x^2)^(1/2)/c^3+1/20*b*x^4*(1-1/c^2/x^2)^(1/2)/c
```

Rubi [A] time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5221, 266, 51, 63, 208}

$$\frac{1}{5}x^5\left(a+b\csc^{-1}(cx)\right)+\frac{bx^4\sqrt{1-\frac{1}{c^2x^2}}}{20c}+\frac{3bx^2\sqrt{1-\frac{1}{c^2x^2}}}{40c^3}+\frac{3b\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{40c^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (3*b*Sqrt[1 - 1/(c^2*x^2)]*x^2)/(40*c^3) + (b*Sqrt[1 - 1/(c^2*x^2)]*x^4)/(2
0*c) + (x^5*(a + b*ArcCsc[c*x]))/5 + (3*b*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/(
40*c^5)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5221

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((d*x)^(m + 1)*(a + b*ArcCsc[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m +
1)), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^4\left(a+b\csc^{-1}(cx)\right)dx=\frac{1}{5}x^5\left(a+b\csc^{-1}(cx)\right)+\frac{b\int\frac{x^3}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{5c}$$
$$=\frac{1}{5}x^5\left(a+b\csc^{-1}(cx)\right)-\frac{b\,\text{Subst}\left(\int\frac{1}{x^3\sqrt{1-\frac{x}{c^2}}}dx,x,\frac{1}{x^2}\right)}{10c}$$
$$=\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^4}{20c}+\frac{1}{5}x^5\left(a+b\csc^{-1}(cx)\right)-\frac{(3b)\,\text{Subst}\left(\int\frac{1}{x^2\sqrt{1-\frac{x}{c^2}}}dx,x,\frac{1}{x^2}\right)}{40c^3}$$
$$=\frac{3b\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{40c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^4}{20c}+\frac{1}{5}x^5\left(a+b\csc^{-1}(cx)\right)-\frac{(3b)\,\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{c^2}}}\right.}{80c^5}$$
$$=\frac{3b\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{40c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^4}{20c}+\frac{1}{5}x^5\left(a+b\csc^{-1}(cx)\right)+\frac{(3b)\,\text{Subst}\left(\int\frac{1}{c^2-c^2x^2}\right.}{40c^3}$$
$$=\frac{3b\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{40c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^4}{20c}+\frac{1}{5}x^5\left(a+b\csc^{-1}(cx)\right)+\frac{3b\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right.}{40c^5}$$

Mathematica [A] time = 0.08, size = 97, normalized size = 1.09

$$\frac{ax^5}{5}+\frac{3b\log\left(x\left(\sqrt{\frac{c^2x^2-1}{c^2x^2}}+1\right)\right)}{40c^5}+b\sqrt{\frac{c^2x^2-1}{c^2x^2}}\left(\frac{3x^2}{40c^3}+\frac{x^4}{20c}\right)+\frac{1}{5}bx^5\csc^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (a*x^5)/5 + b*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)]*((3*x^2)/(40*c^3) + x^4/(20*c)
) + (b*x^5*ArcCsc[c*x])/5 + (3*b*Log[x*(1 + Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])
])/(40*c^5)
```

fricas [A] time = 1.59, size = 106, normalized size = 1.19

$$\frac{8\,ac^5x^5-16\,bc^5\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+8\left(bc^5x^5-bc^5\right)\operatorname{arccsc}(cx)-3\,b\log\left(-cx+\sqrt{c^2x^2-1}\right)+\left(2\,bc^3x\right.}{40\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/40*(8*a*c^5*x^5 - 16*b*c^5*arctan(-c*x + sqrt(c^2*x^2 - 1)) + 8*(b*c^5*x^
5 - b*c^5)*arccsc(c*x) - 3*b*log(-c*x + sqrt(c^2*x^2 - 1)) + (2*b*c^3*x^3 +
 3*b*c*x)*sqrt(c^2*x^2 - 1))/c^5
```

giac [B] time = 0.76, size = 480, normalized size = 5.39

$$\frac{1}{320}\left(\frac{2\,bx^5\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^5\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{2\,ax^5\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^5}{c}+\frac{bx^4\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^4}{c^2}+\frac{10\,bx^3\left(\sqrt{-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/320*(2*b*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*arcsin(1/(c*x))/c + 2*a*x^5*(
sqrt(-1/(c^2*x^2) + 1) + 1)^5/c + b*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c^2
+ 10*b*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x))/c^3 + 10*a*x^3*(s
qrt(-1/(c^2*x^2) + 1) + 1)^3/c^3 + 8*b*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c
^4 + 20*b*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^5 + 20*a*x*(sqrt
(-1/(c^2*x^2) + 1) + 1)/c^5 + 24*b*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^6 - 24
*b*log(1/(abs(c)*abs(x)))/c^6 + 20*b*arcsin(1/(c*x))/(c^7*x*(sqrt(-1/(c^2*x
^2) + 1) + 1)) + 20*a/(c^7*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) - 8*b/(c^8*x^2*(
sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 10*b*arcsin(1/(c*x))/(c^9*x^3*(sqrt(-1/(c^
2*x^2) + 1) + 1)^3) + 10*a/(c^9*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) - b/(c^
10*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) + 2*b*arcsin(1/(c*x))/(c^11*x^5*(sqr
t(-1/(c^2*x^2) + 1) + 1)^5) + 2*a/(c^11*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5)
)*c
```

maple [A] time = 0.05, size = 150, normalized size = 1.69

$$\frac{a\,x^{5}}{5}+\frac{x^{5} b\,\mathrm{arccsc}\left(c x\right)}{5}+\frac{b\,x^{4}}{20 c\sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}}+\frac{b\,x^{2}}{40 c^{3}\sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}}-\frac{3 b}{40 c^{5}\sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}}+\frac{3 b\sqrt{c^{2} x^{2}-1}\,\ln\left(c x+\sqrt{c^{2} x^{2}-1}\right)}{40 c^{6}\sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccsc(c*x)),x)
```

```
[Out] 1/5*a*x^5+1/5*x^5*b*arccsc(c*x)+1/20/c*b/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^4+1/
40/c^3*b/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^2-3/40/c^5*b/((c^2*x^2-1)/c^2/x^2)^(
1/2)+3/40/c^6*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*ln(c*x+(c^2
*x^2-1)^(1/2))
```

maxima [A] time = 0.35, size = 132, normalized size = 1.48

$$\frac{1}{5}\,a x^{5}+\frac{1}{80}\left(16\,x^{5}\,\mathrm{arccsc}\left(c x\right)-\frac{\frac{2\left(3\left(-\frac{1}{c^{2} x^{2}}+1\right)^{\frac{3}{2}}-5\,\sqrt{-\frac{1}{c^{2} x^{2}}+1}\right)}{c^{4}\left(\frac{1}{c^{2} x^{2}}-1\right)^{2}+2\,c^{4}\left(\frac{1}{c^{2} x^{2}}-1\right)+c^{4}}-\frac{3\,\log\left(\sqrt{-\frac{1}{c^{2} x^{2}}+1}+1\right)}{c^{4}}+\frac{3\,\log\left(\sqrt{-\frac{1}{c^{2} x^{2}}+1}-1\right)}{c^{4}}}{c}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5 + 1/80*(16*x^5*arccsc(c*x) - (2*(3*(-1/(c^2*x^2) + 1)^(3/2) - 5*s
qrt(-1/(c^2*x^2) + 1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(1/(c^2*x^2) - 1) +
 c^4) - 3*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 + 3*log(sqrt(-1/(c^2*x^2) + 1
) - 1)/c^4)/c)*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{4}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*asin(1/(c*x))),x)
```

```
[Out] int(x^4*(a + b*asin(1/(c*x))), x)
```

sympy [A] time = 5.60, size = 175, normalized size = 1.97

$$\frac{ax^5}{5} + \frac{bx^5 \operatorname{acsc}(cx)}{5} + \frac{b \left(\begin{cases} \frac{cx^5}{4\sqrt{c^2x^2-1}} + \frac{x^3}{8c\sqrt{c^2x^2-1}} - \frac{3x}{8c^3\sqrt{c^2x^2-1}} + \frac{3\operatorname{acosh}(cx)}{8c^4} & \text{for } |c^2x^2| > 1 \\ -\frac{icx^5}{4\sqrt{-c^2x^2+1}} - \frac{ix^3}{8c\sqrt{-c^2x^2+1}} + \frac{3ix}{8c^3\sqrt{-c^2x^2+1}} - \frac{3i\operatorname{asin}(cx)}{8c^4} & \text{otherwise} \end{cases} \right)}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acsc(c*x)),x)
```

```
[Out] a*x**5/5 + b*x**5*acsc(c*x)/5 + b*Piecewise((c*x**5/(4*sqrt(c**2*x**2 - 1))
 + x**3/(8*c*sqrt(c**2*x**2 - 1)) - 3*x/(8*c**3*sqrt(c**2*x**2 - 1)) + 3*ac
osh(c*x)/(8*c**4), Abs(c**2*x**2) > 1), (-I*c*x**5/(4*sqrt(-c**2*x**2 + 1))
 - I*x**3/(8*c*sqrt(-c**2*x**2 + 1)) + 3*I*x/(8*c**3*sqrt(-c**2*x**2 + 1))
- 3*I*asin(c*x)/(8*c**4), True))/(5*c)
```

3.4 $\int x^3 \left(a+b\csc^{-1}(cx)\right) dx$

Optimal. Leaf size=64

$$\frac{1}{4}x^4\left(a+b\csc^{-1}(cx)\right)+\frac{bx^3\sqrt{1-\frac{1}{c^2x^2}}}{12c}+\frac{bx\sqrt{1-\frac{1}{c^2x^2}}}{6c^3}$$

```
[Out] 1/4*x^4*(a+b*arccsc(c*x))+1/6*b*x*(1-1/c^2/x^2)^(1/2)/c^3+1/12*b*x^3*(1-1/c
^2/x^2)^(1/2)/c
```

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5221, 271, 191}

$$\frac{1}{4}x^4\left(a+b\csc^{-1}(cx)\right)+\frac{bx^3\sqrt{1-\frac{1}{c^2x^2}}}{12c}+\frac{bx\sqrt{1-\frac{1}{c^2x^2}}}{6c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*Sqrt[1 - 1/(c^2*x^2)]*x)/(6*c^3) + (b*Sqrt[1 - 1/(c^2*x^2)]*x^3)/(12*c)
+ (x^4*(a + b*ArcCsc[c*x]))/4
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 5221

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((d*x)^(m + 1)*(a + b*ArcCsc[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m +
1)), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+b\csc^{-1}(cx)\right)dx &= \frac{1}{4}x^4\left(a+b\csc^{-1}(cx)\right)+\frac{b\int\frac{x^2}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{4c}\\ &= \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c}+\frac{1}{4}x^4\left(a+b\csc^{-1}(cx)\right)+\frac{b\int\frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{6c^3}\\ &= \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x}{6c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c}+\frac{1}{4}x^4\left(a+b\csc^{-1}(cx)\right)\end{aligned}$$

Mathematica [A] time = 0.10, size = 62, normalized size = 0.97

$$\frac{ax^4}{4}+b\sqrt{\frac{c^2x^2-1}{c^2x^2}}\left(\frac{x}{6c^3}+\frac{x^3}{12c}\right)+\frac{1}{4}bx^4\csc^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (a*x^4)/4 + b*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)]*(x/(6*c^3) + x^3/(12*c)) + (b*
x^4*ArcCsc[c*x])/4
```

fricas [A] time = 1.51, size = 52, normalized size = 0.81

$$\frac{3\,bc^4x^4\,\mathrm{arccsc}\,(cx)+3\,ac^4x^4+\left(bc^2x^2+2\,b\right)\sqrt{c^2x^2-1}}{12\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*c^4*x^4*arccsc(c*x) + 3*a*c^4*x^4 + (b*c^2*x^2 + 2*b)*sqrt(c^2*x^
2 - 1))/c^4
```

giac [B] time = 0.17, size = 352, normalized size = 5.50

$$\frac{1}{192}\left(\frac{3\,bx^4\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^4\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{3\,ax^4\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^4}{c}+\frac{2\,bx^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3}{c^2}+\frac{12\,bx^2\left(\sqrt{-\right.}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/192*(3*b*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))/c + 3*a*x^4*(
sqrt(-1/(c^2*x^2) + 1) + 1)^4/c + 2*b*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^
2 + 12*b*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c^3 + 12*a*x^2*
(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^3 + 18*b*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^
4 + 18*b*arcsin(1/(c*x))/c^5 + 18*a/c^5 - 18*b/(c^6*x*(sqrt(-1/(c^2*x^2) +
1) + 1)) + 12*b*arcsin(1/(c*x))/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) +
12*a/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) - 2*b/(c^8*x^3*(sqrt(-1/(c^2*
x^2) + 1) + 1)^3) + 3*b*arcsin(1/(c*x))/(c^9*x^4*(sqrt(-1/(c^2*x^2) + 1) +
1)^4) + 3*a/(c^9*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4))*c
```

maple [A] time = 0.05, size = 74, normalized size = 1.16

$$\frac{\frac{a\,c^4x^4}{4}+b\left(\frac{c^4x^4\mathrm{arccsc}(cx)}{4}+\frac{\left(c^2x^2-1\right)\left(c^2x^2+2\right)}{12\sqrt{\frac{c^2x^2-1}{c^2x^2}}\;cx}\right)}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsc(c*x)),x)
```

```
[Out] 1/c^4*(1/4*a*c^4*x^4+b*(1/4*c^4*x^4*arccsc(c*x)+1/12*(c^2*x^2-1)*(c^2*x^2+2
)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x))
```

maxima [A] time = 0.34, size = 59, normalized size = 0.92

$$\frac{1}{4}\,ax^4+\frac{1}{12}\left(3\,x^4\,\mathrm{arccsc}\,(cx)+\frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+3\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/12*(3*x^4*arccsc(c*x) + (c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 3
*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^3 \left(a + b \operatorname{asin}\left(\frac{1}{c\,x}\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asin(1/(c*x))),x)
```

```
[Out] int(x^3*(a + b*asin(1/(c*x))), x)
```

sympy [A] time = 3.06, size = 107, normalized size = 1.67

$$\frac{ax^4}{4} + \frac{bx^4 \operatorname{acsc}(cx)}{4} + \frac{b\left(\begin{cases} \frac{x^2\sqrt{c^2x^2-1}}{3c} + \frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for } \left|c^2x^2\right| > 1 \\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c} + \frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise} \end{cases}\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsc(c*x)),x)
```

```
[Out] a*x**4/4 + b*x**4*acsc(c*x)/4 + b*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3*c)
 + 2*sqrt(c**2*x**2 - 1)/(3*c**3), Abs(c**2*x**2) > 1), (I*x**2*sqrt(-c**2*
x**2 + 1)/(3*c) + 2*I*sqrt(-c**2*x**2 + 1)/(3*c**3), True))/(4*c)
```

3.5 $\int x^2 \left(a + b\csc^{-1}(cx)\right) dx$

Optimal. Leaf size=64

$$\frac{1}{3}x^3\left(a + b\csc^{-1}(cx)\right) + \frac{bx^2\sqrt{1 - \frac{1}{c^2x^2}}}{6c} + \frac{b\tanh^{-1}\left(\sqrt{1 - \frac{1}{c^2x^2}}\right)}{6c^3}$$

```
[Out] 1/3*x^3*(a+b*arccsc(c*x))+1/6*b*arctanh((1-1/c^2/x^2)^(1/2))/c^3+1/6*b*x^2*
(1-1/c^2/x^2)^(1/2)/c
```

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5221, 266, 51, 63, 208}

$$\frac{1}{3}x^3\left(a + b\csc^{-1}(cx)\right) + \frac{bx^2\sqrt{1 - \frac{1}{c^2x^2}}}{6c} + \frac{b\tanh^{-1}\left(\sqrt{1 - \frac{1}{c^2x^2}}\right)}{6c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*Sqrt[1 - 1/(c^2*x^2)]*x^2)/(6*c) + (x^3*(a + b*ArcCsc[c*x]))/3 + (b*ArcT
anh[Sqrt[1 - 1/(c^2*x^2)]])/(6*c^3)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5221

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((d*x)^(m + 1)*(a + b*ArcCsc[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m +
1)), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^2\left(a+b\csc^{-1}(cx)\right)dx &= \frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)+\frac{b\int\frac{x}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{3c}\\
&= \frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)-\frac{b\,\text{Subst}\left(\int\frac{1}{x^2\sqrt{1-\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{6c}\\
&= \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{6c}+\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)-\frac{b\,\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{12c^3}\\
&= \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{6c}+\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)+\frac{b\,\text{Subst}\left(\int\frac{1}{c^2-c^2x^2}\,dx,x,\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c}\\
&= \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{6c}+\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)+\frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c^3}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 85, normalized size = 1.33

$$\frac{ax^3}{3}+\frac{bx^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{6c}+\frac{b\log\left(x\left(\sqrt{\frac{c^2x^2-1}{c^2x^2}}+1\right)\right)}{6c^3}+\frac{1}{3}bx^3\csc^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (a*x^3)/3 + (b*x^2*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(6*c) + (b*x^3*ArcCsc[c*
x])/3 + (b*Log[x*(1 + Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])])/(6*c^3)
```

fricas [A] time = 0.64, size = 94, normalized size = 1.47

$$\frac{2\,ac^3x^3-4\,bc^3\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+\sqrt{c^2x^2-1}\,bcx+2\left(bc^3x^3-bc^3\right)\operatorname{arccsc}(cx)-b\log\left(-cx+\sqrt{c^2x^2-1}\right.}{6\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*x^3 - 4*b*c^3*arctan(-c*x + sqrt(c^2*x^2 - 1)) + sqrt(c^2*x^2
- 1)*b*c*x + 2*(b*c^3*x^3 - b*c^3)*arccsc(c*x) - b*log(-c*x + sqrt(c^2*x^2
- 1)))/c^3
```

giac [B] time = 0.50, size = 310, normalized size = 4.84

$$\frac{1}{24}\left(\frac{bx^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{ax^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3}{c}+\frac{bx^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2}{c^2}+\frac{3\,bx\left(\sqrt{-\frac{1}{c^2x^2}+1}+\right.}{c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/24*(b*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x))/c + a*x^3*(sqrt(
-1/(c^2*x^2) + 1) + 1)^3/c + b*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^2 + 3*b
```

```
*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^3 + 3*a*x*(sqrt(-1/(c^2*x
^2) + 1) + 1)/c^3 + 4*b*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 - 4*b*log(1/(ab
s(c)*abs(x)))/c^4 + 3*b*arcsin(1/(c*x))/(c^5*x*(sqrt(-1/(c^2*x^2) + 1) + 1)
) + 3*a/(c^5*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) - b/(c^6*x^2*(sqrt(-1/(c^2*x^2
) + 1) + 1)^2) + b*arcsin(1/(c*x))/(c^7*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3)
 + a/(c^7*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3))*c
```

maple [B] time = 0.05, size = 123, normalized size = 1.92

$$\frac{x^3 a}{3}+\frac{x^3 b\,\mathrm{arccsc}\,(cx)}{3}+\frac{b\,x^2}{6c\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{b}{6c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{b\sqrt{c^2x^2-1}\,\ln\left(cx+\sqrt{c^2x^2-1}\right)}{6c^4\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsc(c*x)),x)
```

```
[Out] 1/3*x^3*a+1/3*x^3*b*arccsc(c*x)+1/6/c*b/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^2-1/6
/c^3*b/((c^2*x^2-1)/c^2/x^2)^(1/2)+1/6/c^4*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)
/c^2/x^2)^(1/2)/x*ln(c*x+(c^2*x^2-1)^(1/2))
```

maxima [A] time = 0.36, size = 97, normalized size = 1.52

$$\frac{1}{3}\,ax^3+\frac{1}{12}\left(4\,x^3\,\mathrm{arccsc}\,(cx)+\frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}-\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/12*(4*x^3*arccsc(c*x) + (2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(c^
2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c^2
*x^2) + 1) - 1)/c^2)/c)*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asin(1/(c*x))),x)
```

```
[Out] int(x^2*(a + b*asin(1/(c*x))), x)
```

sympy [A] time = 3.47, size = 107, normalized size = 1.67

$$\frac{ax^3}{3}+\frac{bx^3\,\mathrm{acsc}\,(cx)}{3}+\frac{b\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\mathrm{acosh}\,(cx)}{2c^2} & \text{for } \left|c^2x^2\right|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\,\mathrm{asin}\,(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsc(c*x)),x)
```

```
[Out] a*x**3/3 + b*x**3*acsc(c*x)/3 + b*Piecewise((x*sqrt(c**2*x**2 - 1)/(2*c) +
acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1), (-I*c*x**3/(2*sqrt(-c**2*x**2 + 1
)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*asin(c*x)/(2*c**2), True))/(3*c)
```

3.6 $\int x \left(a+b \csc^{-1}(cx)\right) dx$

Optimal. Leaf size=39

$$\frac{1}{2}x^2 \left(a+b \csc^{-1}(cx)\right)+\frac{bx\sqrt{1-\frac{1}{c^2x^2}}}{2c}$$

```
[Out] 1/2*x^2*(a+b*arccsc(c*x))+1/2*b*x*(1-1/c^2/x^2)^(1/2)/c
```

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5221, 191}

$$\frac{1}{2}x^2 \left(a+b \csc^{-1}(cx)\right)+\frac{bx\sqrt{1-\frac{1}{c^2x^2}}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*Sqrt[1 - 1/(c^2*x^2)]*x)/(2*c) + (x^2*(a + b*ArcCsc[c*x]))/2
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5221

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((d*x)^(m + 1)*(a + b*ArcCsc[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m +
1)), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int x \left(a+b \csc^{-1}(cx)\right) dx &= \frac{1}{2}x^2 \left(a+b \csc^{-1}(cx)\right)+\frac{b \int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}} \, dx}{2c} \\ &= \frac{b\sqrt{1-\frac{1}{c^2x^2}}\, x}{2c}+\frac{1}{2}x^2 \left(a+b \csc^{-1}(cx)\right) \end{aligned}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 1.28

$$\frac{ax^2}{2}+\frac{bx\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{2c}+\frac{1}{2}bx^2 \csc^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (a*x^2)/2 + (b*x*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(2*c) + (b*x^2*ArcCsc[c*x]
)/2
```

fricas [A] time = 1.02, size = 39, normalized size = 1.00

$$\frac{bc^2x^2 \operatorname{arccsc}(cx) + ac^2x^2 + \sqrt{c^2x^2-1}\, b}{2\, c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*c^2*x^2*arccsc(c*x) + a*c^2*x^2 + sqrt(c^2*x^2 - 1)*b)/c^2
```

giac [B] time = 0.16, size = 182, normalized size = 4.67

$$\frac{1}{8}\left(\frac{bx^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2 \arcsin\left(\frac{1}{cx}\right)}{c} + \frac{ax^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2}{c} + \frac{2\,bx\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2} + \frac{2\,b \arcsin\left(\frac{1}{cx}\right)}{c^3} + \cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/8*(b*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c + a*x^2*(sqrt(-
1/(c^2*x^2) + 1) + 1)^2/c + 2*b*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 + 2*b*ar
csin(1/(c*x))/c^3 + 2*a/c^3 - 2*b/(c^4*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + b*
arcsin(1/(c*x))/(c^5*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + a/(c^5*x^2*(sqrt
(-1/(c^2*x^2) + 1) + 1)^2))*c
```

maple [A] time = 0.05, size = 65, normalized size = 1.67

$$\frac{\frac{c^2x^2a}{2} + b\left(\frac{c^2x^2\operatorname{arccsc}(cx)}{2} + \frac{c^2x^2-1}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\; cx}\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsc(c*x)),x)
```

```
[Out] 1/c^2*(1/2*c^2*x^2*a+b*(1/2*c^2*x^2*arccsc(c*x)+1/2/((c^2*x^2-1)/c^2/x^2)^(
1/2)/c/x*(c^2*x^2-1)))
```

maxima [A] time = 0.34, size = 36, normalized size = 0.92

$$\frac{1}{2}\,ax^2 + \frac{1}{2}\left(x^2 \operatorname{arccsc}(cx) + \frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/2*(x^2*arccsc(c*x) + x*sqrt(-1/(c^2*x^2) + 1)/c)*b
```

mupad [B] time = 0.65, size = 40, normalized size = 1.03

$$\frac{a\,x^2}{2} + \frac{b\,x^2\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{2} + \frac{b\,x\,\sqrt{1-\frac{1}{c^2\,x^2}}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asin(1/(c*x))),x)
```

```
[Out] (a*x^2)/2 + (b*x^2*asin(1/(c*x)))/2 + (b*x*(1 - 1/(c^2*x^2))^(1/2))/(2*c)
```

sympy [**A**] time = 2.14, size = 58, normalized size = 1.49

$$\frac{ax^2}{2} + \frac{bx^2 \operatorname{acsc}(cx)}{2} + \frac{b\left(\begin{cases} \frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2| > 1 \\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise} \end{cases}\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsc(c*x)),x)
```

```
[Out] a*x**2/2 + b*x**2*acsc(c*x)/2 + b*Piecewise((sqrt(c**2*x**2 - 1)/c, Abs(c**
2*x**2) > 1), (I*sqrt(-c**2*x**2 + 1)/c, True))/(2*c)
```

3.7 $\int \left(a+b\csc^{-1}(cx)\right) dx$

Optimal. Leaf size=31

$$ax+\frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}+bx\csc^{-1}(cx)$$

```
[Out] a*x+b*x*arccsc(c*x)+b*arctanh((1-1/c^2/x^2)^(1/2))/c
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5215, 266, 63, 208}

$$ax+\frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}+bx\csc^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcCsc[c*x],x]
```

```
[Out] a*x + b*x*ArcCsc[c*x] + (b*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/c
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5215

```
Int[ArcCsc[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcCsc[c*x], x] + Dist[1/c, Int
[1/(x*Sqrt[1 - 1/(c^2*x^2)]), x], x] /; FreeQ[c, x]
```

Rubi steps

$$\int \left(a+b\csc^{-1}(cx)\right)\,dx = ax+b\int \csc^{-1}(cx)\,dx$$
$$= ax+bx\csc^{-1}(cx)+\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\,x}\,dx}{c}$$
$$= ax+bx\csc^{-1}(cx)-\frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{2c}$$
$$= ax+bx\csc^{-1}(cx)+(bc)\,\text{Subst}\left(\int \frac{1}{c^2-c^2x^2}\,dx,x,\sqrt{1-\frac{1}{c^2x^2}}\right)$$
$$= ax+bx\csc^{-1}(cx)+\frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}$$

Mathematica [A] time = 0.07, size = 58, normalized size = 1.87

$$ax+\frac{bx\sqrt{1-\frac{1}{c^2x^2}}\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{\sqrt{c^2x^2-1}}+bx\csc^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcCsc[c*x],x]
```

```
[Out] a*x + b*x*ArcCsc[c*x] + (b*Sqrt[1 - 1/(c^2*x^2)]*x*ArcTanh[(c*x)/Sqrt[-1 +
c^2*x^2]])/Sqrt[-1 + c^2*x^2]
```

fricas [B] time = 2.10, size = 64, normalized size = 2.06

$$\frac{acx-2\,bc\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+(bcx-bc)\,\text{arccsc}\,(cx)-b\log\left(-cx+\sqrt{c^2x^2-1}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccsc(c*x),x, algorithm="fricas")
```

```
[Out] (a*c*x - 2*b*c*arctan(-c*x + sqrt(c^2*x^2 - 1)) + (b*c*x - b*c)*arccsc(c*x)
 - b*log(-c*x + sqrt(c^2*x^2 - 1)))/c
```

giac [B] time = 0.15, size = 62, normalized size = 2.00

$$\frac{1}{2}\,bc\left(\frac{2\,x\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)-\log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}\right)+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccsc(c*x),x, algorithm="giac")
```

```
[Out] 1/2*b*c*(2*x*arcsin(1/(c*x))/c + (log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-sq
rt(-1/(c^2*x^2) + 1) + 1))/c^2) + a*x
```

maple [A] time = 0.05, size = 37, normalized size = 1.19

$$ax+bx\,\text{arccsc}\,(cx)+\frac{b\ln\left(cx+cx\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arccsc(c*x),x)
```

```
[Out] a*x+b*x*arccsc(c*x)+b/c*ln(c*x+c*x*(1-1/c^2/x^2)^(1/2))
```

maxima [A] time = 0.36, size = 53, normalized size = 1.71

$$ax + \frac{\left(2\,cx\,\operatorname{arccsc}\left(cx\right) + \log\left(\sqrt{-\frac{1}{c^2x^2} + 1} + 1\right) - \log\left(-\sqrt{-\frac{1}{c^2x^2} + 1} + 1\right)\right)b}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccsc(c*x),x, algorithm="maxima")
```

```
[Out] a*x + 1/2*(2*c*x*arccsc(c*x) + log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-sqrt(
-1/(c^2*x^2) + 1) + 1))*b/c
```

mupad [B] time = 0.87, size = 33, normalized size = 1.06

$$a\,x + b\,x\,\operatorname{asin}\left(\frac{1}{c\,x}\right) + \frac{b\,\operatorname{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{c^2\,x^2}}}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*asin(1/(c*x)),x)
```

```
[Out] a*x + b*x*asin(1/(c*x)) + (b*atanh(1/(1 - 1/(c^2*x^2))^(1/2)))/c
```

sympy [A] time = 2.44, size = 32, normalized size = 1.03

$$ax + b\left(x\operatorname{acsc}\left(cx\right) + \frac{\begin{cases}\operatorname{acosh}\left(cx\right) & \text{for}\ \left|c^2x^2\right| > 1\\ -i\operatorname{asin}\left(cx\right) & \text{otherwise}\end{cases}}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*acsc(c*x),x)
```

```
[Out] a*x + b*(x*acsc(c*x) + Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin
(c*x), True))/c)
```

3.8 $\int \frac{a+b\csc^{-1}(cx)}{x}\,dx$

Optimal. Leaf size=64

$$\frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2b}-\log\left(1-e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)+\frac{1}{2}ib\text{Li}_2\left(e^{2i\csc^{-1}(cx)}\right)$$

```
[Out] 1/2*I*(a+b*arccsc(c*x))^2/b-(a+b*arccsc(c*x))*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/
2))^2)+1/2*I*b*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)
```

Rubi [A] time = 0.08, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5219, 4625, 3717, 2190, 2279, 2391}

$$\frac{1}{2}ib\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)+\frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2b}-\log\left(1-e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/x,x]
```

```
[Out] ((I/2)*(a + b*ArcCsc[c*x])^2)/b - (a + b*ArcCsc[c*x])*Log[1 - E^((2*I)*ArcC
sc[c*x])] + (I/2)*b*PolyLog[2, E^((2*I)*ArcCsc[c*x])]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4625

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n/Tan[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 5219

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a + b
*ArcSin[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x}\,dx = -\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)$$
$$= -\text{Subst}\left(\int (a+bx)\cot(x)\,dx, x, \csc^{-1}(cx)\right)$$
$$= \frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2b} + 2i\,\text{Subst}\left(\int \frac{e^{2ix}(a+bx)}{1-e^{2ix}}\,dx, x, \csc^{-1}(cx)\right)$$
$$= \frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2b} - \left(a+b\csc^{-1}(cx)\right)\log\left(1-e^{2i\csc^{-1}(cx)}\right) + b\,\text{Subst}\left(\int \log\left(1-e^{2ix}\right.\right.$$
$$= \frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2b} - \left(a+b\csc^{-1}(cx)\right)\log\left(1-e^{2i\csc^{-1}(cx)}\right) - \frac{1}{2}(ib)\,\text{Subst}\left(\int \frac{\log(1-}{x}\right.$$
$$= \frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2b} - \left(a+b\csc^{-1}(cx)\right)\log\left(1-e^{2i\csc^{-1}(cx)}\right) + \frac{1}{2}ib\text{Li}_2\left(e^{2i\csc^{-1}(cx)}\right)$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.83

$$a\log(x) + \frac{1}{2}ib\left(\csc^{-1}(cx)^2 + \text{Li}_2\left(e^{2i\csc^{-1}(cx)}\right)\right) - b\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsc[c*x])/x,x]
[Out] -(b*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])]) + a*Log[x] + (I/2)*b*(ArcCs
c[c*x]^2 + PolyLog[2, E^((2*I)*ArcCsc[c*x])])
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arccsc}\,(cx)+a}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x,x, algorithm="fricas")
[Out] integral((b*arccsc(c*x) + a)/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x,x, algorithm="giac")
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(x)]Undef/Unsigne
d Inf encountered in limitLimit: Max order reached or unable to make series
 expansion Error: Bad Argument Value
```

maple [A] time = 0.18, size = 140, normalized size = 2.19

$$a\ln(cx) + \frac{ib\,\text{arccsc}\,(cx)^2}{2} - b\,\text{arccsc}\,(cx)\ln\left(1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right) - b\,\text{arccsc}\,(cx)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right) + ib\,\text{pc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x,x)
```

```
[Out] a*ln(c*x)+1/2*I*b*arccsc(c*x)^2-b*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2
))-b*arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))+I*b*polylog(2,-I/c/x-(1-1/
c^2/x^2)^(1/2))+I*b*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(c^2\int\frac{\sqrt{cx+1}\,\sqrt{cx-1}\,\log(x)}{c^4x^3-c^2x}\,dx+\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right)\log(x)\right)b+a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x,x, algorithm="maxima")
```

```
[Out] (c^2*integrate(sqrt(c*x + 1)*sqrt(c*x - 1)*log(x)/(c^4*x^3 - c^2*x), x) + a
rctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(x))*b + a*log(x)
```

mupad [B] time = 0.76, size = 65, normalized size = 1.02

$$\frac{b\,\mathrm{polylog}\left(2,\mathrm{e}^{\mathrm{asin}\left(\frac{1}{c\,x}\right)\,2{}\mathrm{i}}\right)\,1{}\mathrm{i}}{2}+\frac{b\,{\mathrm{asin}\left(\frac{1}{c\,x}\right)}^2\,1{}\mathrm{i}}{2}+a\,\ln(x)-b\,\ln\left(1-\mathrm{e}^{\mathrm{asin}\left(\frac{1}{c\,x}\right)\,2{}\mathrm{i}}\right)\,\mathrm{asin}\left(\frac{1}{c\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/x,x)
```

```
[Out] (b*polylog(2, exp(asin(1/(c*x))*2i))*1i)/2 + (b*asin(1/(c*x))^2*1i)/2 + a*l
og(x) - b*log(1 - exp(asin(1/(c*x))*2i))*asin(1/(c*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\,\mathrm{acsc}\,(cx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x,x)
```

```
[Out] Integral((a + b*acsc(c*x))/x, x)
```

3.9 $\int \frac{a+b\csc^{-1}(cx)}{x^2}\,dx$

Optimal. Leaf size=32

$$-\frac{a+b\csc^{-1}(cx)}{x}-bc\sqrt{1-\frac{1}{c^2x^2}}$$

[Out] (-a-b*arccsc(c*x))/x-b*c*(1-1/c^2/x^2)^(1/2)

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5221, 261}

$$-\frac{a+b\csc^{-1}(cx)}{x}-bc\sqrt{1-\frac{1}{c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcCsc[c*x])/x^2,x]

[Out] -(b*c*Sqrt[1 - 1/(c^2*x^2)]) - (a + b*ArcCsc[c*x])/x

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5221

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((d*x)^(m + 1)*(a + b*ArcCsc[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m +
1)), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\csc^{-1}(cx)}{x^2}\,dx &= -\frac{a+b\csc^{-1}(cx)}{x}-\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\,x^3}\,dx}{c}\\ &= -bc\sqrt{1-\frac{1}{c^2x^2}}-\frac{a+b\csc^{-1}(cx)}{x}\end{aligned}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 1.28

$$-\frac{a}{x}-bc\sqrt{\frac{c^2x^2-1}{c^2x^2}}-\frac{b\csc^{-1}(cx)}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcCsc[c*x])/x^2,x]

[Out] -(a/x) - b*c*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)] - (b*ArcCsc[c*x])/x

fricas [A] time = 1.23, size = 26, normalized size = 0.81

$$-\frac{b\,\text{arccsc}\,(cx)+\sqrt{c^2x^2-1}\,b+a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] -(b*arccsc(c*x) + sqrt(c^2*x^2 - 1)*b + a)/x
```

giac [A] time = 0.16, size = 42, normalized size = 1.31

$$-\left(b\sqrt{-\frac{1}{c^2x^2}+1}+\frac{b\arcsin\left(\frac{1}{cx}\right)}{cx}+\frac{a}{cx}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2,x, algorithm="giac")
```

```
[Out] -(b*sqrt(-1/(c^2*x^2) + 1) + b*arcsin(1/(c*x))/(c*x) + a/(c*x))*c
```

maple [A] time = 0.05, size = 63, normalized size = 1.97

$$c\left(-\frac{a}{cx}+b\left(-\frac{\operatorname{arccsc}(cx)}{cx}-\frac{c^2x^2-1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2x^2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x^2,x)
```

```
[Out] c*(-a/c/x+b*(-1/c/x*arccsc(c*x)-1/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^2/x^2*(c^2*
x^2-1)))
```

maxima [A] time = 0.35, size = 33, normalized size = 1.03

$$-\left(c\sqrt{-\frac{1}{c^2x^2}+1}+\frac{\operatorname{arccsc}(cx)}{x}\right)b-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] -(c*sqrt(-1/(c^2*x^2) + 1) + arccsc(c*x)/x)*b - a/x
```

mupad [B] time = 0.64, size = 37, normalized size = 1.16

$$-\frac{a}{x}-b\,c\,\sqrt{1-\frac{1}{c^2\,x^2}}-\frac{b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/x^2,x)
```

```
[Out] - a/x - b*c*(1 - 1/(c^2*x^2))^(1/2) - (b*asin(1/(c*x)))/x
```

sympy [A] time = 2.13, size = 37, normalized size = 1.16

$$\begin{cases} -\frac{a}{x}-bc\sqrt{1-\frac{1}{c^2x^2}}-\frac{b\operatorname{acsc}(cx)}{x} & \text{for } c\neq 0 \\ -\frac{a+\tilde{\infty}b}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x**2,x)
```

```
[Out] Piecewise((-a/x - b*c*sqrt(1 - 1/(c**2*x**2)) - b*acsc(c*x)/x, Ne(c, 0)), (
-(a + zoo*b)/x, True))
```

3.10 $\int \frac{a+b\csc^{-1}(cx)}{x^3}\,dx$

Optimal. Leaf size=51

$$-\frac{a+b\csc^{-1}(cx)}{2x^2}-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{4x}+\frac{1}{4}bc^2\csc^{-1}(cx)$$

```
[Out] 1/4*b*c^2*arccsc(c*x)+1/2*(-a-b*arccsc(c*x))/x^2-1/4*b*c*(1-1/c^2/x^2)^(1/2
)/x
```

Rubi [A] time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5221, 335, 321, 216}

$$-\frac{a+b\csc^{-1}(cx)}{2x^2}-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{4x}+\frac{1}{4}bc^2\csc^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/x^3,x]
```

```
[Out] -(b*c*Sqrt[1 - 1/(c^2*x^2)])/(4*x) + (b*c^2*ArcCsc[c*x])/4 - (a + b*ArcCsc[
c*x])/(2*x^2)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5221

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((d*x)^(m + 1)*(a + b*ArcCsc[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m +
1)), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x^3}\,dx = -\frac{a+b\csc^{-1}(cx)}{2x^2} - \frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\,x^4}\,dx}{2c}$$
$$= -\frac{a+b\csc^{-1}(cx)}{2x^2} + \frac{b\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2c}$$
$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{a+b\csc^{-1}(cx)}{2x^2} + \frac{1}{4}(bc)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{4x} + \frac{1}{4}bc^2\csc^{-1}(cx) - \frac{a+b\csc^{-1}(cx)}{2x^2}$$

Mathematica [A] time = 0.04, size = 66, normalized size = 1.29

$$-\frac{a}{2x^2} - \frac{bc\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{4x} + \frac{1}{4}bc^2\sin^{-1}\left(\frac{1}{cx}\right) - \frac{b\csc^{-1}(cx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])/x^3,x]
```

```
[Out] -1/2*a/x^2 - (b*c*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(4*x) - (b*ArcCsc[c*x])/(
2*x^2) + (b*c^2*ArcSin[1/(c*x)])/4
```

fricas [A] time = 0.86, size = 40, normalized size = 0.78

$$\frac{\left(bc^2x^2 - 2\,b\right)\operatorname{arccsc}(cx) - \sqrt{c^2x^2-1}\,b - 2\,a}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*((b*c^2*x^2 - 2*b)*arccsc(c*x) - sqrt(c^2*x^2 - 1)*b - 2*a)/x^2
```

giac [A] time = 0.15, size = 66, normalized size = 1.29

$$-\frac{1}{4}\left(2\,bc\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right) + 2\,ac\left(\frac{1}{c^2x^2}-1\right) + bc\arcsin\left(\frac{1}{cx}\right) + \frac{b\sqrt{-\frac{1}{c^2x^2}+1}}{x}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*b*c*(1/(c^2*x^2) - 1)*arcsin(1/(c*x)) + 2*a*c*(1/(c^2*x^2) - 1) + b
*c*arcsin(1/(c*x)) + b*sqrt(-1/(c^2*x^2) + 1)/x)*c
```

maple [B] time = 0.05, size = 118, normalized size = 2.31

$$-\frac{a}{2x^2} - \frac{b\operatorname{arccsc}(cx)}{2x^2} + \frac{cb\sqrt{c^2x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x} - \frac{cb}{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x} + \frac{b}{4c\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x^3,x)
```

```
[Out] -1/2*a/x^2-1/2*b/x^2*arccsc(c*x)+1/4*c*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2
/x^2)^(1/2)/x*arctan(1/(c^2*x^2-1)^(1/2))-1/4*c*b/((c^2*x^2-1)/c^2/x^2)^(1/
2)/x+1/4/c*b/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^3
```

maxima [A] time = 0.47, size = 83, normalized size = 1.63

$$\frac{1}{4}\,b\left(\frac{\frac{c^4x\sqrt{-\frac{1}{c^2x^2}+1}}{c^2x^2\left(\frac{1}{c^2x^2}-1\right)-1}-c^3\arctan\left(cx\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c}-\frac{2\,\mathrm{arccsc}\,(cx)}{x^2}\right)-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*b*((c^4*x*sqrt(-1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) - c^3*
arctan(c*x*sqrt(-1/(c^2*x^2) + 1)))/c - 2*arccsc(c*x)/x^2) - 1/2*a/x^2
```

mupad [B] time = 0.73, size = 50, normalized size = 0.98

$$-\frac{a}{2\,x^2}-\frac{b\,c^2\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\,\left(\frac{2}{c^2\,x^2}-1\right)}{4}-\frac{b\,c\,\sqrt{1-\frac{1}{c^2\,x^2}}}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/x^3,x)
```

```
[Out] - a/(2*x^2) - (b*c^2*asin(1/(c*x))*(2/(c^2*x^2) - 1))/4 - (b*c*(1 - 1/(c^2*
x^2))^(1/2))/(4*x)
```

sympy [A] time = 3.36, size = 121, normalized size = 2.37

$$-\frac{a}{2x^2}-\frac{b\,\mathrm{acsc}\,(cx)}{2x^2}-\frac{b\left(\begin{cases}\frac{ic^3\,\mathrm{acosh}\left(\frac{1}{cx}\right)}{2}+\frac{ic^2\sqrt{-1+\frac{1}{c^2x^2}}}{2x} & \text{for } \frac{1}{|c^2x^2|}>1\\ -\frac{c^3\,\mathrm{asin}\left(\frac{1}{cx}\right)}{2}+\frac{c^2}{2x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{1}{2x^3\sqrt{1-\frac{1}{c^2x^2}}} & \text{otherwise}\end{cases}\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x**3,x)
```

```
[Out] -a/(2*x**2) - b*acsc(c*x)/(2*x**2) - b*Piecewise((I*c**3*acosh(1/(c*x))/2 +
 I*c**2*sqrt(-1 + 1/(c**2*x**2))/(2*x), 1/Abs(c**2*x**2) > 1), (-c**3*asin(
1/(c*x))/2 + c**2/(2*x*sqrt(1 - 1/(c**2*x**2))) - 1/(2*x**3*sqrt(1 - 1/(c**
2*x**2))), True))/(2*c)
```

3.11 $\int \frac{a+b\csc^{-1}(cx)}{x^4}\,dx$

Optimal. Leaf size=60

$$-\frac{a+b\csc^{-1}(cx)}{3x^3}+\frac{1}{9}bc^3\left(1-\frac{1}{c^2x^2}\right)^{3/2}-\frac{1}{3}bc^3\sqrt{1-\frac{1}{c^2x^2}}$$

```
[Out] 1/9*b*c^3*(1-1/c^2/x^2)^(3/2)+1/3*(-a-b*arccsc(c*x))/x^3-1/3*b*c^3*(1-1/c^2
/x^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5221, 266, 43}

$$-\frac{a+b\csc^{-1}(cx)}{3x^3}+\frac{1}{9}bc^3\left(1-\frac{1}{c^2x^2}\right)^{3/2}-\frac{1}{3}bc^3\sqrt{1-\frac{1}{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/x^4,x]
```

```
[Out] -(b*c^3*Sqrt[1 - 1/(c^2*x^2)])/3 + (b*c^3*(1 - 1/(c^2*x^2))^(3/2))/9 - (a +
 b*ArcCsc[c*x])/(3*x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5221

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((d*x)^(m + 1)*(a + b*ArcCsc[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m +
1)), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x^4}\,dx = -\frac{a+b\csc^{-1}(cx)}{3x^3} - \frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\,x^5}\,dx}{3c}$$
$$= -\frac{a+b\csc^{-1}(cx)}{3x^3} + \frac{b\,\text{Subst}\left(\int \frac{x}{\sqrt{1-\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{6c}$$
$$= -\frac{a+b\csc^{-1}(cx)}{3x^3} + \frac{b\,\text{Subst}\left(\int \left(\frac{c^2}{\sqrt{1-\frac{x}{c^2}}} - c^2\sqrt{1-\frac{x}{c^2}}\right)dx, x, \frac{1}{x^2}\right)}{6c}$$
$$= -\frac{1}{3}bc^3\sqrt{1-\frac{1}{c^2x^2}} + \frac{1}{9}bc^3\left(1-\frac{1}{c^2x^2}\right)^{3/2} - \frac{a+b\csc^{-1}(cx)}{3x^3}$$

Mathematica [A] time = 0.07, size = 59, normalized size = 0.98

$$-\frac{a}{3x^3} + b\left(-\frac{2c^3}{9} - \frac{c}{9x^2}\right)\sqrt{\frac{c^2x^2-1}{c^2x^2}} - \frac{b\csc^{-1}(cx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])/x^4,x]
```

```
[Out] -1/3*a/x^3 + b*((-2*c^3)/9 - c/(9*x^2))*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)] - (b
*ArcCsc[c*x])/(3*x^3)
```

fricas [A] time = 2.04, size = 39, normalized size = 0.65

$$-\frac{3\,b\,\text{arccsc}\,(cx) + \left(2\,bc^2x^2 + b\right)\sqrt{c^2x^2-1} + 3\,a}{9\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/9*(3*b*arccsc(c*x) + (2*b*c^2*x^2 + b)*sqrt(c^2*x^2 - 1) + 3*a)/x^3
```

giac [A] time = 0.13, size = 87, normalized size = 1.45

$$\frac{1}{9}\left(bc^2\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 3\,bc^2\sqrt{-\frac{1}{c^2x^2}+1} - \frac{3\,bc\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)}{x} - \frac{3\,bc\arcsin\left(\frac{1}{cx}\right)}{x} - \frac{3\,a}{cx^3}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^4,x, algorithm="giac")
```

```
[Out] 1/9*(b*c^2*(-1/(c^2*x^2) + 1)^(3/2) - 3*b*c^2*sqrt(-1/(c^2*x^2) + 1) - 3*b*
c*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))/x - 3*b*c*arcsin(1/(c*x))/x - 3*a/(c*x^
3))*c
```

maple [A] time = 0.05, size = 75, normalized size = 1.25

$$c^3\left(-\frac{a}{3c^3x^3} + b\left(-\frac{\text{arccsc}\,(cx)}{3c^3x^3} - \frac{\left(c^2x^2-1\right)\left(2c^2x^2+1\right)}{9\sqrt{\frac{c^2x^2-1}{c^2x^2}}\;c^4x^4}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x^4,x)
```

```
[Out] c^3*(-1/3*a/c^3/x^3+b*(-1/3/c^3/x^3*arccsc(c*x)-1/9*(c^2*x^2-1)*(2*c^2*x^2+
1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^4/x^4))
```

maxima [A] time = 0.35, size = 58, normalized size = 0.97

$$\frac{1}{9}\,b\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c}-\frac{3\,\mathrm{arccsc}\,(cx)}{x^3}\right)-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 1/9*b*((c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c - 3*
arccsc(c*x)/x^3) - 1/3*a/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/x^4,x)
```

```
[Out] int((a + b*asin(1/(c*x)))/x^4, x)
```

sympy [A] time = 3.05, size = 112, normalized size = 1.87

$$-\frac{a}{3x^3}-\frac{b\,\mathrm{acsc}\,(cx)}{3x^3}-\frac{b\left(\begin{cases}\frac{2c^3\sqrt{c^2x^2-1}}{3x}+\frac{c\sqrt{c^2x^2-1}}{3x^3} & \text{for}\ \left|c^2x^2\right|>1\\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x}+\frac{ic\sqrt{-c^2x^2+1}}{3x^3} & \text{otherwise}\end{cases}\right)}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x**4,x)
```

```
[Out] -a/(3*x**3) - b*acsc(c*x)/(3*x**3) - b*Piecewise((2*c**3*sqrt(c**2*x**2 - 1
)/(3*x) + c*sqrt(c**2*x**2 - 1)/(3*x**3), Abs(c**2*x**2) > 1), (2*I*c**3*sq
rt(-c**2*x**2 + 1)/(3*x) + I*c*sqrt(-c**2*x**2 + 1)/(3*x**3), True))/(3*c)
```

3.12 $\int \frac{a+b\csc^{-1}(cx)}{x^5}\,dx$

Optimal. Leaf size=76

$$-\frac{a+b\csc^{-1}(cx)}{4x^4}+\frac{3}{32}bc^4\csc^{-1}(cx)-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16x^3}-\frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}}{32x}$$

```
[Out] 3/32*b*c^4*arccsc(c*x)+1/4*(-a-b*arccsc(c*x))/x^4-1/16*b*c*(1-1/c^2/x^2)^(1
/2)/x^3-3/32*b*c^3*(1-1/c^2/x^2)^(1/2)/x
```

Rubi [A] time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5221, 335, 321, 216}

$$-\frac{a+b\csc^{-1}(cx)}{4x^4}-\frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}}{32x}-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16x^3}+\frac{3}{32}bc^4\csc^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/x^5,x]
```

```
[Out] -(b*c*Sqrt[1 - 1/(c^2*x^2)])/(16*x^3) - (3*b*c^3*Sqrt[1 - 1/(c^2*x^2)])/(32
*x) + (3*b*c^4*ArcCsc[c*x])/32 - (a + b*ArcCsc[c*x])/(4*x^4)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5221

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((d*x)^(m + 1)*(a + b*ArcCsc[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m +
1)), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x^5}\,dx = -\frac{a+b\csc^{-1}(cx)}{4x^4} - \frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\,x^6}\,dx}{4c}$$

$$= -\frac{a+b\csc^{-1}(cx)}{4x^4} + \frac{b\,\text{Subst}\left(\int \frac{x^4}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{4c}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16x^3} - \frac{a+b\csc^{-1}(cx)}{4x^4} + \frac{1}{16}(3bc)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16x^3} - \frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}}{32x} - \frac{a+b\csc^{-1}(cx)}{4x^4} + \frac{1}{32}\left(3bc^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x,\right.$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16x^3} - \frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}}{32x} + \frac{3}{32}bc^4\csc^{-1}(cx) - \frac{a+b\csc^{-1}(cx)}{4x^4}$$

Mathematica [A] time = 0.06, size = 78, normalized size = 1.03

$$-\frac{a}{4x^4} + \frac{3}{32}bc^4\sin^{-1}\left(\frac{1}{cx}\right) + b\left(-\frac{3c^3}{32x} - \frac{c}{16x^3}\right)\sqrt{\frac{c^2x^2-1}{c^2x^2}} - \frac{b\csc^{-1}(cx)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])/x^5,x]
```

```
[Out] -1/4*a/x^4 + b*(-1/16*c/x^3 - (3*c^3)/(32*x))*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)
] - (b*ArcCsc[c*x])/(4*x^4) + (3*b*c^4*ArcSin[1/(c*x)])/32
```

fricas [A] time = 1.02, size = 53, normalized size = 0.70

$$\frac{\left(3\,bc^4x^4 - 8\,b\right)\text{arccsc}\,(cx) - \left(3\,bc^2x^2 + 2\,b\right)\sqrt{c^2x^2-1} - 8\,a}{32\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] 1/32*((3*b*c^4*x^4 - 8*b)*arccsc(c*x) - (3*b*c^2*x^2 + 2*b)*sqrt(c^2*x^2 -
1) - 8*a)/x^4
```

giac [A] time = 0.15, size = 117, normalized size = 1.54

$$-\frac{1}{32}\left(8\,bc^3\left(\frac{1}{c^2x^2}-1\right)^2\arcsin\left(\frac{1}{cx}\right) + 16\,bc^3\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right) + 5\,bc^3\arcsin\left(\frac{1}{cx}\right) - \frac{2\,bc^2\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}}{x} + \frac{5\,b}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^5,x, algorithm="giac")
```

```
[Out] -1/32*(8*b*c^3*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*x)) + 16*b*c^3*(1/(c^2*x^2)
- 1)*arcsin(1/(c*x)) + 5*b*c^3*arcsin(1/(c*x)) - 2*b*c^2*(-1/(c^2*x^2) + 1)
^(3/2)/x + 5*b*c^2*sqrt(-1/(c^2*x^2) + 1)/x + 8*a/(c*x^4))*c
```

maple [B] time = 0.05, size = 147, normalized size = 1.93

$$-\frac{a}{4x^4}-\frac{b\,\mathrm{arccsc}\,(cx)}{4x^4}+\frac{3c^3b\sqrt{c^2x^2-1}\ \arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}}\ x}-\frac{3c^3b}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}}\ x}+\frac{cb}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}}\ x^3}+\frac{b}{16c\sqrt{\frac{c^2x^2-1}{c^2x^2}}\ x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x^5,x)
```

```
[Out] -1/4*a/x^4-1/4*b/x^4*arccsc(c*x)+3/32*c^3*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/
c^2/x^2)^(1/2)/x*arctan(1/(c^2*x^2-1)^(1/2))-3/32*c^3*b/((c^2*x^2-1)/c^2/x^
2)^(1/2)/x+1/32*c*b/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^3+1/16/c*b/((c^2*x^2-1)/c
^2/x^2)^(1/2)/x^5
```

maxima [A] time = 0.45, size = 125, normalized size = 1.64

$$-\frac{1}{32}\,b\left(\frac{3\,c^5\arctan\left(cx\sqrt{-\frac{1}{c^2x^2}+1}\right)+\frac{3\,c^8x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+5\,c^6x\sqrt{-\frac{1}{c^2x^2}+1}}{c^4x^4\left(\frac{1}{c^2x^2}-1\right)^2-2\,c^2x^2\left(\frac{1}{c^2x^2}-1\right)+1}}{c}+\frac{8\,\mathrm{arccsc}\,(cx)}{x^4}\right)-\frac{a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] -1/32*b*((3*c^5*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)) + (3*c^8*x^3*(-1/(c^2*x^
2) + 1)^(3/2) + 5*c^6*x*sqrt(-1/(c^2*x^2) + 1))/(c^4*x^4*(1/(c^2*x^2) - 1)^
2 - 2*c^2*x^2*(1/(c^2*x^2) - 1) + 1))/c + 8*arccsc(c*x)/x^4) - 1/4*a/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/x^5,x)
```

```
[Out] int((a + b*asin(1/(c*x)))/x^5, x)
```

sympy [A] time = 5.50, size = 194, normalized size = 2.55

$$-\frac{a}{4x^4}-\frac{b\,\mathrm{acsc}\,(cx)}{4x^4}-\frac{b\left(\begin{cases}\frac{3ic^5\,\mathrm{acosh}\left(\frac{1}{cx}\right)}{8}-\frac{3ic^4}{8x\sqrt{-1+\frac{1}{c^2x^2}}}+\frac{ic^2}{8x^3\sqrt{-1+\frac{1}{c^2x^2}}}+\frac{i}{4x^5\sqrt{-1+\frac{1}{c^2x^2}}} & \text{for}\ \frac{1}{|c^2x^2|}>1\\ -\frac{3c^5\,\mathrm{asin}\left(\frac{1}{cx}\right)}{8}+\frac{3c^4}{8x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{c^2}{8x^3\sqrt{1-\frac{1}{c^2x^2}}}-\frac{1}{4x^5\sqrt{1-\frac{1}{c^2x^2}}} & \text{otherwise}\end{cases}\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x**5,x)
```

```
[Out] -a/(4*x**4) - b*acsc(c*x)/(4*x**4) - b*Piecewise((3*I*c**5*acosh(1/(c*x))/8
 - 3*I*c**4/(8*x*sqrt(-1 + 1/(c**2*x**2))) + I*c**2/(8*x**3*sqrt(-1 + 1/(c*
```

```
*2*x**2))) + I/(4*x**5*sqrt(-1 + 1/(c**2*x**2))), 1/Abs(c**2*x**2) > 1), (-
3*c**5*asin(1/(c*x))/8 + 3*c**4/(8*x*sqrt(1 - 1/(c**2*x**2))) - c**2/(8*x**
3*sqrt(1 - 1/(c**2*x**2))) - 1/(4*x**5*sqrt(1 - 1/(c**2*x**2))), True))/(4*
c)
```

3.13 $\int \frac{a+b\csc^{-1}(cx)}{x^6}\,dx$

Optimal. Leaf size=82

$$-\frac{a+b\csc^{-1}(cx)}{5x^5}-\frac{1}{25}bc^5\left(1-\frac{1}{c^2x^2}\right)^{5/2}+\frac{2}{15}bc^5\left(1-\frac{1}{c^2x^2}\right)^{3/2}-\frac{1}{5}bc^5\sqrt{1-\frac{1}{c^2x^2}}$$

```
[Out] 2/15*b*c^5*(1-1/c^2/x^2)^(3/2)-1/25*b*c^5*(1-1/c^2/x^2)^(5/2)+1/5*(-a-b*arc
csc(c*x))/x^5-1/5*b*c^5*(1-1/c^2/x^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5221, 266, 43}

$$-\frac{a+b\csc^{-1}(cx)}{5x^5}-\frac{1}{25}bc^5\left(1-\frac{1}{c^2x^2}\right)^{5/2}+\frac{2}{15}bc^5\left(1-\frac{1}{c^2x^2}\right)^{3/2}-\frac{1}{5}bc^5\sqrt{1-\frac{1}{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/x^6,x]
```

```
[Out] -(b*c^5*Sqrt[1 - 1/(c^2*x^2)])/5 + (2*b*c^5*(1 - 1/(c^2*x^2))^(3/2))/15 - (
b*c^5*(1 - 1/(c^2*x^2))^(5/2))/25 - (a + b*ArcCsc[c*x])/(5*x^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5221

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((d*x)^(m + 1)*(a + b*ArcCsc[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m +
1)), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x^6}\,dx = -\frac{a+b\csc^{-1}(cx)}{5x^5} - \frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\,x^7}\,dx}{5c}$$
$$= -\frac{a+b\csc^{-1}(cx)}{5x^5} + \frac{b\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{10c}$$
$$= -\frac{a+b\csc^{-1}(cx)}{5x^5} + \frac{b\,\mathrm{Subst}\left(\int \left(\frac{c^4}{\sqrt{1-\frac{x}{c^2}}} - 2c^4\sqrt{1-\frac{x}{c^2}} + c^4\left(1-\frac{x}{c^2}\right)^{3/2}\right) dx, x, \frac{1}{x^2}\right)}{10c}$$
$$= -\frac{1}{5}bc^5\sqrt{1-\frac{1}{c^2x^2}} + \frac{2}{15}bc^5\left(1-\frac{1}{c^2x^2}\right)^{3/2} - \frac{1}{25}bc^5\left(1-\frac{1}{c^2x^2}\right)^{5/2} - \frac{a+b\csc^{-1}(cx)}{5x^5}$$

Mathematica [A] time = 0.09, size = 69, normalized size = 0.84

$$-\frac{a}{5x^5} + b\left(-\frac{8c^5}{75} - \frac{4c^3}{75x^2} - \frac{c}{25x^4}\right)\sqrt{\frac{c^2x^2-1}{c^2x^2}} - \frac{b\csc^{-1}(cx)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])/x^6,x]
```

```
[Out] -1/5*a/x^5 + b*((-8*c^5)/75 - c/(25*x^4) - (4*c^3)/(75*x^2))*Sqrt[(-1 + c^2
*x^2)/(c^2*x^2)] - (b*ArcCsc[c*x])/(5*x^5)
```

fricas [A] time = 0.90, size = 50, normalized size = 0.61

$$-\frac{15\,b\,\mathrm{arccsc}\,(cx) + \left(8\,bc^4x^4 + 4\,bc^2x^2 + 3\,b\right)\sqrt{c^2x^2-1} + 15\,a}{75\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] -1/75*(15*b*arccsc(c*x) + (8*b*c^4*x^4 + 4*b*c^2*x^2 + 3*b)*sqrt(c^2*x^2 -
1) + 15*a)/x^5
```

giac [B] time = 0.14, size = 149, normalized size = 1.82

$$-\frac{1}{75}\left(3\,bc^4\left(\frac{1}{c^2x^2}-1\right)^2\sqrt{-\frac{1}{c^2x^2}+1} - 10\,bc^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + \frac{15\,bc^3\left(\frac{1}{c^2x^2}-1\right)^2\arcsin\left(\frac{1}{cx}\right)}{x} + 15\,bc^4\sqrt{-\frac{1}{c^2x^2}+1} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^6,x, algorithm="giac")
```

```
[Out] -1/75*(3*b*c^4*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1) - 10*b*c^4*(-1/(c
^2*x^2) + 1)^(3/2) + 15*b*c^3*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*x))/x + 15*b*
c^4*sqrt(-1/(c^2*x^2) + 1) + 30*b*c^3*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))/x +
 15*b*c^3*arcsin(1/(c*x))/x + 15*a/(c*x^5))*c
```

maple [A] time = 0.05, size = 83, normalized size = 1.01

$$c^5\left(-\frac{a}{5c^5x^5} + b\left(-\frac{\mathrm{arccsc}\,(cx)}{5c^5x^5} - \frac{\left(c^2x^2-1\right)\left(8c^4x^4+4c^2x^2+3\right)}{75\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^6x^6}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x^6,x)
```

```
[Out] c^5*(-1/5*a/c^5/x^5+b*(-1/5/c^5/x^5*arccsc(c*x)-1/75*(c^2*x^2-1)*(8*c^4*x^4
+4*c^2*x^2+3)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^6/x^6))
```

maxima [A] time = 0.36, size = 76, normalized size = 0.93

$$-\frac{1}{75}\,b\left(\frac{3\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,c^6\sqrt{-\frac{1}{c^2x^2}+1}}{c}+\frac{15\,\mathrm{arccsc}\,(cx)}{x^5}\right)-\frac{a}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] -1/75*b*((3*c^6*(-1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(-1/(c^2*x^2) + 1)^(3/2)
+ 15*c^6*sqrt(-1/(c^2*x^2) + 1))/c + 15*arccsc(c*x)/x^5) - 1/5*a/x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/x^6,x)
```

```
[Out] int((a + b*asin(1/(c*x)))/x^6, x)
```

sympy [A] time = 7.23, size = 158, normalized size = 1.93

$$-\frac{a}{5x^5}-\frac{b\,\mathrm{acsc}\,(cx)}{5x^5}-\frac{b\left(\begin{cases}\frac{8c^5\sqrt{c^2x^2-1}}{15x}+\frac{4c^3\sqrt{c^2x^2-1}}{15x^3}+\frac{c\sqrt{c^2x^2-1}}{5x^5} & \text{for}\ \left|c^2x^2\right|>1\\ \frac{8ic^5\sqrt{-c^2x^2+1}}{15x}+\frac{4ic^3\sqrt{-c^2x^2+1}}{15x^3}+\frac{ic\sqrt{-c^2x^2+1}}{5x^5} & \text{otherwise}\end{cases}\right)}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x**6,x)
```

```
[Out] -a/(5*x**5) - b*acsc(c*x)/(5*x**5) - b*Piecewise((8*c**5*sqrt(c**2*x**2 - 1
)/(15*x) + 4*c**3*sqrt(c**2*x**2 - 1)/(15*x**3) + c*sqrt(c**2*x**2 - 1)/(5*
x**5), Abs(c**2*x**2) > 1), (8*I*c**5*sqrt(-c**2*x**2 + 1)/(15*x) + 4*I*c**
3*sqrt(-c**2*x**2 + 1)/(15*x**3) + I*c*sqrt(-c**2*x**2 + 1)/(5*x**5), True)
)/(5*c)
```

3.14 $\int \frac{a+b\csc^{-1}(cx)}{x^7}\,dx$

Optimal. Leaf size=101

$$-\frac{a+b\csc^{-1}(cx)}{6x^6}+\frac{5}{96}bc^6\csc^{-1}(cx)-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{36x^5}-\frac{5bc^5\sqrt{1-\frac{1}{c^2x^2}}}{96x}-\frac{5bc^3\sqrt{1-\frac{1}{c^2x^2}}}{144x^3}$$

```
[Out] 5/96*b*c^6*arccsc(c*x)+1/6*(-a-b*arccsc(c*x))/x^6-1/36*b*c*(1-1/c^2/x^2)^(1
/2)/x^5-5/144*b*c^3*(1-1/c^2/x^2)^(1/2)/x^3-5/96*b*c^5*(1-1/c^2/x^2)^(1/2)/
x
```

Rubi [A] time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5221, 335, 321, 216}

$$-\frac{a+b\csc^{-1}(cx)}{6x^6}-\frac{5bc^5\sqrt{1-\frac{1}{c^2x^2}}}{96x}-\frac{5bc^3\sqrt{1-\frac{1}{c^2x^2}}}{144x^3}-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{36x^5}+\frac{5}{96}bc^6\csc^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/x^7,x]
```

```
[Out] -(b*c*Sqrt[1 - 1/(c^2*x^2)])/(36*x^5) - (5*b*c^3*Sqrt[1 - 1/(c^2*x^2)])/(14
4*x^3) - (5*b*c^5*Sqrt[1 - 1/(c^2*x^2)])/(96*x) + (5*b*c^6*ArcCsc[c*x])/96
- (a + b*ArcCsc[c*x])/(6*x^6)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5221

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((d*x)^(m + 1)*(a + b*ArcCsc[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m +
1)), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x^7}\,dx = -\frac{a+b\csc^{-1}(cx)}{6x^6} - \frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\,x^8}\,dx}{6c}$$

$$= -\frac{a+b\csc^{-1}(cx)}{6x^6} + \frac{b\,\text{Subst}\left(\int \frac{x^6}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{6c}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{36x^5} - \frac{a+b\csc^{-1}(cx)}{6x^6} + \frac{1}{36}(5bc)\,\text{Subst}\left(\int \frac{x^4}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{36x^5} - \frac{5bc^3\sqrt{1-\frac{1}{c^2x^2}}}{144x^3} - \frac{a+b\csc^{-1}(cx)}{6x^6} + \frac{1}{48}\left(5bc^3\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^2}{c^2}}}\,dx,\right.$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{36x^5} - \frac{5bc^3\sqrt{1-\frac{1}{c^2x^2}}}{144x^3} - \frac{5bc^5\sqrt{1-\frac{1}{c^2x^2}}}{96x} - \frac{a+b\csc^{-1}(cx)}{6x^6} + \frac{1}{96}\left(5bc^5\right)\text{Subs}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{36x^5} - \frac{5bc^3\sqrt{1-\frac{1}{c^2x^2}}}{144x^3} - \frac{5bc^5\sqrt{1-\frac{1}{c^2x^2}}}{96x} + \frac{5}{96}bc^6\csc^{-1}(cx) - \frac{a+b\csc^{-1}(cx)}{6x^6}$$

Mathematica [A] time = 0.10, size = 88, normalized size = 0.87

$$-\frac{a}{6x^6} + \frac{5}{96}bc^6\sin^{-1}\left(\frac{1}{cx}\right) + b\left(-\frac{5c^5}{96x} - \frac{5c^3}{144x^3} - \frac{c}{36x^5}\right)\sqrt{\frac{c^2x^2-1}{c^2x^2}} - \frac{b\csc^{-1}(cx)}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])/x^7,x]
```

```
[Out] -1/6*a/x^6 + b*(-1/36*c/x^5 - (5*c^3)/(144*x^3) - (5*c^5)/(96*x))*Sqrt[(-1
+ c^2*x^2)/(c^2*x^2)] - (b*ArcCsc[c*x])/(6*x^6) + (5*b*c^6*ArcSin[1/(c*x)])
/96
```

fricas [A] time = 0.87, size = 63, normalized size = 0.62

$$\frac{3\left(5\,bc^6x^6 - 16\,b\right)\text{arccsc}\,(cx) - \left(15\,bc^4x^4 + 10\,bc^2x^2 + 8\,b\right)\sqrt{c^2x^2-1} - 48\,a}{288\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^7,x, algorithm="fricas")
```

```
[Out] 1/288*(3*(5*b*c^6*x^6 - 16*b)*arccsc(c*x) - (15*b*c^4*x^4 + 10*b*c^2*x^2 +
8*b)*sqrt(c^2*x^2 - 1) - 48*a)/x^6
```

giac [B] time = 0.13, size = 174, normalized size = 1.72

$$-\frac{1}{288}\left(48\,bc^5\left(\frac{1}{c^2x^2}-1\right)^3\arcsin\left(\frac{1}{cx}\right) + 144\,bc^5\left(\frac{1}{c^2x^2}-1\right)^2\arcsin\left(\frac{1}{cx}\right) + 144\,bc^5\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right) + 33\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^7,x, algorithm="giac")
```

```
[Out] -1/288*(48*b*c^5*(1/(c^2*x^2) - 1)^3*arcsin(1/(c*x)) + 144*b*c^5*(1/(c^2*x^
2) - 1)^2*arcsin(1/(c*x)) + 144*b*c^5*(1/(c^2*x^2) - 1)*arcsin(1/(c*x)) + 3
3*b*c^5*arcsin(1/(c*x)) + 8*b*c^4*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1
)/x - 26*b*c^4*(-1/(c^2*x^2) + 1)^(3/2)/x + 33*b*c^4*sqrt(-1/(c^2*x^2) + 1)
/x + 48*a/(c*x^6))*c
```

maple [A] time = 0.05, size = 174, normalized size = 1.72

$$-\frac{a}{6x^6}-\frac{b\,\mathrm{arccsc}\,(cx)}{6x^6}+\frac{5c^5b\sqrt{c^2x^2-1}\ \arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{96\sqrt{\frac{c^2x^2-1}{c^2x^2}}\ x}-\frac{5c^5b}{96\sqrt{\frac{c^2x^2-1}{c^2x^2}}\ x}+\frac{5c^3b}{288\sqrt{\frac{c^2x^2-1}{c^2x^2}}\ x^3}+\frac{cb}{144\sqrt{\frac{c^2x^2-1}{c^2x^2}}\ x^5}+\frac{b}{36c\sqrt{\frac{c^2x^2-1}{c^2x^2}}\ x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x^7,x)
```

```
[Out] -1/6*a/x^6-1/6*b/x^6*arccsc(c*x)+5/96*c^5*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/
c^2/x^2)^(1/2)/x*arctan(1/(c^2*x^2-1)^(1/2))-5/96*c^5*b/((c^2*x^2-1)/c^2/x^
2)^(1/2)/x+5/288*c^3*b/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^3+1/144*c*b/((c^2*x^2-
1)/c^2/x^2)^(1/2)/x^5+1/36/c*b/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^7
```

maxima [A] time = 0.46, size = 165, normalized size = 1.63

$$-\frac{1}{288}\,b\left(\frac{15\,c^7\arctan\left(cx\sqrt{-\frac{1}{c^2x^2}+1}\right)-\frac{15\,c^{12}x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+40\,c^{10}x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+33\,c^8x\sqrt{-\frac{1}{c^2x^2}+1}}{c^6x^6\left(\frac{1}{c^2x^2}-1\right)^3-3\,c^4x^4\left(\frac{1}{c^2x^2}-1\right)^2+3\,c^2x^2\left(\frac{1}{c^2x^2}-1\right)-1}}{c}+\frac{48\,\mathrm{arccsc}\,(cx)}{x^6}\right)-\frac{a}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^7,x, algorithm="maxima")
```

```
[Out] -1/288*b*((15*c^7*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)) - (15*c^12*x^5*(-1/(c^
2*x^2) + 1)^(5/2) + 40*c^10*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 33*c^8*x*sqrt(-1
/(c^2*x^2) + 1))/(c^6*x^6*(1/(c^2*x^2) - 1)^3 - 3*c^4*x^4*(1/(c^2*x^2) - 1)
^2 + 3*c^2*x^2*(1/(c^2*x^2) - 1) - 1))/c + 48*arccsc(c*x)/x^6) - 1/6*a/x^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/x^7,x)
```

```
[Out] int((a + b*asin(1/(c*x)))/x^7, x)
```

sympy [A] time = 8.83, size = 243, normalized size = 2.41

$$-\frac{a}{6x^6}-\frac{b\,\mathrm{acsc}\,(cx)}{6x^6}-\frac{b\left(\begin{cases}\frac{5ic^7\,\mathrm{acosh}\left(\frac{1}{cx}\right)}{16}-\frac{5ic^6}{16x\sqrt{-1+\frac{1}{c^2x^2}}}+\frac{5ic^4}{48x^3\sqrt{-1+\frac{1}{c^2x^2}}}+\frac{ic^2}{24x^5\sqrt{-1+\frac{1}{c^2x^2}}}+\frac{i}{6x^7\sqrt{-1+\frac{1}{c^2x^2}}} & \text{for}\ \frac{1}{|c^2x^2|}>1\\ -\frac{5c^7\,\mathrm{asin}\left(\frac{1}{cx}\right)}{16}+\frac{5c^6}{16x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{5c^4}{48x^3\sqrt{1-\frac{1}{c^2x^2}}}-\frac{c^2}{24x^5\sqrt{1-\frac{1}{c^2x^2}}}-\frac{1}{6x^7\sqrt{1-\frac{1}{c^2x^2}}} & \text{otherwise}\end{cases}\right)}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x**7,x)
```

```
[Out] -a/(6*x**6) - b*acsc(c*x)/(6*x**6) - b*Piecewise((5*I*c**7*acosh(1/(c*x))/1
6 - 5*I*c**6/(16*x*sqrt(-1 + 1/(c**2*x**2))) + 5*I*c**4/(48*x**3*sqrt(-1 +
1/(c**2*x**2))) + I*c**2/(24*x**5*sqrt(-1 + 1/(c**2*x**2))) + I/(6*x**7*sqr
t(-1 + 1/(c**2*x**2))), 1/Abs(c**2*x**2) > 1), (-5*c**7*asin(1/(c*x))/16 +
5*c**6/(16*x*sqrt(1 - 1/(c**2*x**2))) - 5*c**4/(48*x**3*sqrt(1 - 1/(c**2*x*
*2))) - c**2/(24*x**5*sqrt(1 - 1/(c**2*x**2))) - 1/(6*x**7*sqrt(1 - 1/(c**2
*x**2))), True))/(6*c)
```

3.15 $\int x^3 \left(a+b\csc^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=107

$$\frac{bx^3\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{6c}+\frac{bx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{3c^3}+\frac{1}{4}x^4\left(a+b\csc^{-1}(cx)\right)^2+\frac{b^2\log(x)}{3c^4}+\frac{b^2x^2}{12c^2}$$

```
[Out] 1/12*b^2*x^2/c^2+1/4*x^4*(a+b*arccsc(c*x))^2+1/3*b^2*ln(x)/c^4+1/3*b*x*(a+b
*arccsc(c*x))*(1-1/c^2/x^2)^(1/2)/c^3+1/6*b*x^3*(a+b*arccsc(c*x))*(1-1/c^2/
x^2)^(1/2)/c
```

Rubi [A] time = 0.11, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5223, 4410, 4185, 4184, 3475}

$$\frac{bx^3\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{6c}+\frac{bx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{3c^3}+\frac{1}{4}x^4\left(a+b\csc^{-1}(cx)\right)^2+\frac{b^2x^2}{12c^2}+\frac{b^2\log(x)}{3c^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcCsc[c*x])^2,x]
```

```
[Out] (b^2*x^2)/(12*c^2) + (b*Sqrt[1 - 1/(c^2*x^2)]*x*(a + b*ArcCsc[c*x]))/(3*c^3
) + (b*Sqrt[1 - 1/(c^2*x^2)]*x^3*(a + b*ArcCsc[c*x]))/(6*c) + (x^4*(a + b*A
rcCsc[c*x])^2)/4 + (b^2*Log[x])/(3*c^4)
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5223

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\int x^3 \left(a+b\csc^{-1}(cx)\right)^2\,dx = -\frac{\text{Subst}\left(\int (a+bx)^2\cot(x)\csc^4(x)\,dx,x,\csc^{-1}(cx)\right)}{c^4}$$

$$= \frac{1}{4}x^4\left(a+b\csc^{-1}(cx)\right)^2 - \frac{b\,\text{Subst}\left(\int (a+bx)\csc^4(x)\,dx,x,\csc^{-1}(cx)\right)}{2c^4}$$

$$= \frac{b^2x^2}{12c^2} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^3\left(a+b\csc^{-1}(cx)\right)}{6c} + \frac{1}{4}x^4\left(a+b\csc^{-1}(cx)\right)^2 - \frac{b\,\text{Subst}\left(\int (a\right.}{}$$

$$= \frac{b^2x^2}{12c^2} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(a+b\csc^{-1}(cx)\right)}{3c^3} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^3\left(a+b\csc^{-1}(cx)\right)}{6c} + \frac{1}{4}x^4\left(\right.$$

$$= \frac{b^2x^2}{12c^2} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(a+b\csc^{-1}(cx)\right)}{3c^3} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^3\left(a+b\csc^{-1}(cx)\right)}{6c} + \frac{1}{4}x^4\left(\right.$$

Mathematica [A] time = 0.22, size = 124, normalized size = 1.16

$$\frac{cx\left(3a^2c^3x^3+2ab\sqrt{1-\frac{1}{c^2x^2}}\left(c^2x^2+2\right)+b^2cx\right)+2bcx\csc^{-1}(cx)\left(3ac^3x^3+b\sqrt{1-\frac{1}{c^2x^2}}\left(c^2x^2+2\right)\right)+3b^2c^4x^4\,c}{12c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcCsc[c*x])^2,x]
```

```
[Out] (c*x*(b^2*c*x + 3*a^2*c^3*x^3 + 2*a*b*Sqrt[1 - 1/(c^2*x^2)]*(2 + c^2*x^2))
+ 2*b*c*x*(3*a*c^3*x^3 + b*Sqrt[1 - 1/(c^2*x^2)]*(2 + c^2*x^2))*ArcCsc[c*x]
 + 3*b^2*c^4*x^4*ArcCsc[c*x]^2 + 4*b^2*Log[x])/(12*c^4)
```

fricas [A] time = 0.59, size = 146, normalized size = 1.36

$$\frac{3\,b^2c^4x^4\,\text{arccsc}\,(cx)^2+3\,a^2c^4x^4-12\,abc^4\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+b^2c^2x^2+4\,b^2\log(x)+6\left(abc^4x^4-abc^4\right.}{12\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*c^4*x^4*arccsc(c*x)^2 + 3*a^2*c^4*x^4 - 12*a*b*c^4*arctan(-c*x
+ sqrt(c^2*x^2 - 1)) + b^2*c^2*x^2 + 4*b^2*log(x) + 6*(a*b*c^4*x^4 - a*b*c^
4)*arccsc(c*x) + 2*(a*b*c^2*x^2 + 2*a*b + (b^2*c^2*x^2 + 2*b^2)*arccsc(c*x)
)*sqrt(c^2*x^2 - 1))/c^4
```

giac [B] time = 0.39, size = 811, normalized size = 7.58

$$\frac{1}{192}\left(\frac{3\,b^2x^4\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^4\arcsin\left(\frac{1}{cx}\right)^2}{c}+\frac{6\,abx^4\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^4\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{3\,a^2x^4\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right.}{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))^2,x, algorithm="giac")
```

```
[Out] 1/192*(3*b^2*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))^2/c + 6*a*b
*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))/c + 3*a^2*x^4*(sqrt(-1/
(c^2*x^2) + 1) + 1)^4/c + 4*b^2*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1
```

```
/(c*x))/c^2 + 4*a*b*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^2 + 12*b^2*x^2*(sq
rt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))^2/c^3 + 24*a*b*x^2*(sqrt(-1/(c^
2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c^3 + 12*a^2*x^2*(sqrt(-1/(c^2*x^2) + 1)
 + 1)^2/c^3 + 4*b^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^3 + 36*b^2*x*(sqrt
(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^4 + 36*a*b*x*(sqrt(-1/(c^2*x^2) +
 1) + 1)/c^4 + 18*b^2*arcsin(1/(c*x))^2/c^5 + 36*a*b*arcsin(1/(c*x))/c^5 -
128*b^2*log(2)/c^5 + 64*b^2*log(2*sqrt(-1/(c^2*x^2) + 1) + 2)/c^5 - 64*b^2*
log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^5 - 64*b^2*log(1/(abs(c)*abs(x)))/c^5 + 1
8*a^2/c^5 + 8*b^2/c^5 - 36*b^2*arcsin(1/(c*x))/(c^6*x*(sqrt(-1/(c^2*x^2) +
1) + 1)) - 36*a*b/(c^6*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 12*b^2*arcsin(1/(c
*x))^2/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 24*a*b*arcsin(1/(c*x))/(c
^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 12*a^2/(c^7*x^2*(sqrt(-1/(c^2*x^2)
 + 1) + 1)^2) + 4*b^2/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) - 4*b^2*arcs
in(1/(c*x))/(c^8*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) - 4*a*b/(c^8*x^3*(sqrt
(-1/(c^2*x^2) + 1) + 1)^3) + 3*b^2*arcsin(1/(c*x))^2/(c^9*x^4*(sqrt(-1/(c^2
*x^2) + 1) + 1)^4) + 6*a*b*arcsin(1/(c*x))/(c^9*x^4*(sqrt(-1/(c^2*x^2) + 1)
 + 1)^4) + 3*a^2/(c^9*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4))*c
```

maple [B] time = 0.28, size = 208, normalized size = 1.94

$$\frac{a^2x^4}{4}+\frac{b^2\operatorname{arccsc}(cx)^2x^4}{4}+\frac{b^2\operatorname{arccsc}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^3}{6c}+\frac{b^2x^2}{12c^2}+\frac{b^2\operatorname{arccsc}(cx)\,x\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3c^3}-\frac{b^2\ln\left(\frac{1}{cx}\right)}{3c^4}+\frac{ab\,x^4\operatorname{arccsc}(cx)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsc(c*x))^2,x)
```

```
[Out] 1/4*a^2*x^4+1/4*b^2*arccsc(c*x)^2*x^4+1/6/c*b^2*arccsc(c*x)*((c^2*x^2-1)/c^
2/x^2)^(1/2)*x^3+1/12*b^2*x^2/c^2+1/3/c^3*b^2*arccsc(c*x)*x*((c^2*x^2-1)/c^
2/x^2)^(1/2)-1/3/c^4*b^2*ln(1/c/x)+1/2*a*b*x^4*arccsc(c*x)+1/6/c*a*b/((c^2*
x^2-1)/c^2/x^2)^(1/2)*x^3+1/6/c^3*a*b/((c^2*x^2-1)/c^2/x^2)^(1/2)*x-1/3/c^5
*a*b/((c^2*x^2-1)/c^2/x^2)^(1/2)/x
```

maxima [B] time = 0.77, size = 197, normalized size = 1.84

$$\frac{1}{4}b^2x^4\operatorname{arccsc}(cx)^2+\frac{1}{4}a^2x^4+\frac{1}{6}\left(3x^4\operatorname{arccsc}(cx)+\frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+3x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)ab+\frac{\left(2c^4x^4\arctan\left(1,\sqrt{cx}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*arccsc(c*x)^2 + 1/4*a^2*x^4 + 1/6*(3*x^4*arccsc(c*x) + (c^2*x^3
*(-1/(c^2*x^2) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*a*b + 1/12*(2*
c^4*x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 2*c^2*x^2*arctan2(1, sqrt
(c*x + 1)*sqrt(c*x - 1)) + (c^2*x^2 + 2*log(x^2))*sqrt(c*x + 1)*sqrt(c*x -
1) - 4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*b^2/(sqrt(c*x + 1)*sqrt(c*x
 - 1)*c^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asin(1/(c*x)))^2,x)
```

```
[Out] int(x^3*(a + b*asin(1/(c*x)))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 (a + b \operatorname{acsc}(cx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsc(c*x))**2,x)
[Out] Integral(x**3*(a + b*acsc(c*x))**2, x)
```

3.16 $\int x^2 \left(a+b\csc^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=139

$$\frac{2b\tanh^{-1}\left(e^{i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)}{3c^3}+\frac{bx^2\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{3c}+\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)^2-\frac{ib^2\mathrm{Li}_2\left(-e^{i\csc^{-1}(}\right.}{3c^3}$$

```
[Out] 1/3*b^2*x/c^2+1/3*x^3*(a+b*arccsc(c*x))^2+2/3*b*(a+b*arccsc(c*x))*arctanh(I
/c/x+(1-1/c^2/x^2)^(1/2))/c^3-1/3*I*b^2*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2
))/c^3+1/3*I*b^2*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))/c^3+1/3*b*x^2*(a+b*ar
ccsc(c*x))*(1-1/c^2/x^2)^(1/2)/c
```

Rubi [A] time = 0.12, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5223, 4410, 4185, 4183, 2279, 2391}

$$-\frac{ib^2\text{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)}{3c^3}+\frac{ib^2\text{PolyLog}\left(2,e^{i\csc^{-1}(cx)}\right)}{3c^3}+\frac{bx^2\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{3c}+\frac{2b\tanh^{-1}\left(e^{i\csc^{-1}(cx)}\right)}{3c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcCsc[c*x])^2,x]
```

```
[Out] (b^2*x)/(3*c^2) + (b*Sqrt[1 - 1/(c^2*x^2)]*x^2*(a + b*ArcCsc[c*x]))/(3*c) +
 (x^3*(a + b*ArcCsc[c*x])^2)/3 + (2*b*(a + b*ArcCsc[c*x])*ArcTanh[E^(I*ArcC
sc[c*x])])/(3*c^3) - ((I/3)*b^2*PolyLog[2, -E^(I*ArcCsc[c*x])])/c^3 + ((I/3
)*b^2*PolyLog[2, E^(I*ArcCsc[c*x])])/c^3
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
```

```
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5223

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\int x^2\left(a+b\csc^{-1}(cx)\right)^2dx=-\frac{\text{Subst}\left(\int(a+bx)^2\cot(x)\csc^3(x)\,dx,x,\csc^{-1}(cx)\right)}{c^3}$$

$$=\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)^2-\frac{(2b)\,\text{Subst}\left(\int(a+bx)\csc^3(x)\,dx,x,\csc^{-1}(cx)\right)}{3c^3}$$

$$=\frac{b^2x}{3c^2}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^2\left(a+b\csc^{-1}(cx)\right)}{3c}+\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)^2-\frac{b\,\text{Subst}\left(\int(a-\right.}{}$$

$$=\frac{b^2x}{3c^2}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^2\left(a+b\csc^{-1}(cx)\right)}{3c}+\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)^2+\frac{2b\left(a+b\csc^{-}\right.}{}$$

$$=\frac{b^2x}{3c^2}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^2\left(a+b\csc^{-1}(cx)\right)}{3c}+\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)^2+\frac{2b\left(a+b\csc^{-}\right.}{}$$

$$=\frac{b^2x}{3c^2}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^2\left(a+b\csc^{-1}(cx)\right)}{3c}+\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)^2+\frac{2b\left(a+b\csc^{-}\right.}{}$$

Mathematica [A] time = 1.36, size = 210, normalized size = 1.51

$$\frac{1}{3}\left(a^2x^3+\frac{ab\left(c^3x^3+\sqrt{c^2x^2-1}\,\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)-cx\right)}{c^4x\sqrt{1-\frac{1}{c^2x^2}}}+2abx^3\csc^{-1}(cx)-\frac{ib^2\text{Li}_2\left(-e^{i\csc^{-1}(cx)}\right)}{c^3}+\frac{b^2\left(c^3x^3\text{cs}\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcCsc[c*x])^2,x]
```

```
[Out] (a^2*x^3 + 2*a*b*x^3*ArcCsc[c*x] + (a*b*(-(c*x) + c^3*x^3 + Sqrt[-1 + c^2*x
^2]*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]]))/(c^4*Sqrt[1 - 1/(c^2*x^2)]*x) - (I*
b^2*PolyLog[2, -E^(I*ArcCsc[c*x])])/c^3 + (b^2*(c*x + c^3*x^3*ArcCsc[c*x]^2
 + ArcCsc[c*x]*(c^2*Sqrt[1 - 1/(c^2*x^2)]*x^2 - Log[1 - E^(I*ArcCsc[c*x])]
+ Log[1 + E^(I*ArcCsc[c*x])]) + I*PolyLog[2, E^(I*ArcCsc[c*x])]))/c^3)/3
```

fricas [F] time = 1.40, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2x^2\,\text{arccsc}\,(cx)^2+2\,abx^2\,\text{arccsc}\,(cx)+a^2x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*arccsc(c*x)^2 + 2*a*b*x^2*arccsc(c*x) + a^2*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(b\,\text{arccsc}\,(cx)+a)^2x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)^2*x^2, x)
```

maple [B] time = 1.26, size = 327, normalized size = 2.35

$$\frac{x^3a^2}{3}+\frac{b^2x^3\mathrm{arccsc}\,(cx)^2}{3}+\frac{b^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,\mathrm{arccsc}\,(cx)\,x^2}{3c}+\frac{b^2x}{3c^2}+\frac{b^2\mathrm{arccsc}\,(cx)\ln\left(1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{3c^3}-\frac{ib^2\,\mathrm{polylog}\left(2,\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsc(c*x))^2,x)
```

```
[Out] 1/3*x^3*a^2+1/3*b^2*x^3*arccsc(c*x)^2+1/3/c*b^2*((c^2*x^2-1)/c^2/x^2)^(1/2)
*arccsc(c*x)*x^2+1/3*b^2*x/c^2+1/3/c^3*b^2*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/
x^2)^(1/2))-1/3*I*b^2*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))/c^3-1/3/c^3*b^2
*arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))+1/3*I*b^2*polylog(2,I/c/x+(1-1
/c^2/x^2)^(1/2))/c^3+2/3*x^3*a*b*arccsc(c*x)+1/3/c*a*b/((c^2*x^2-1)/c^2/x^2
)^(1/2)*x^2-1/3/c^3*a*b/((c^2*x^2-1)/c^2/x^2)^(1/2)+1/3/c^4*a*b*(c^2*x^2-1)
^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*ln(c*x+(c^2*x^2-1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}a^2x^3+\frac{1}{6}\left(4\,x^3\,\mathrm{arccsc}\,(cx)+\frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}-\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)ab+\frac{1}{12}\left(4\,x^3\arctan\left(1,\sqrt{cx+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 + 1/6*(4*x^3*arccsc(c*x) + (2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(c
^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c^
2*x^2) + 1) - 1)/c^2)/c)*a*b + 1/12*(4*x^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*
x - 1))^2 - x^3*log(c^2*x^2)^2 - 2*c^2*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x +
 1)/c^5 + 3*log(c*x - 1)/c^5)*log(c)^2 + 36*c^2*integrate(1/3*x^4*log(c^2*x
^2)/(c^2*x^2 - 1), x)*log(c) - 72*c^2*integrate(1/3*x^4*log(x)/(c^2*x^2 - 1
), x)*log(c) + 36*c^2*integrate(1/3*x^4*log(c^2*x^2)*log(x)/(c^2*x^2 - 1),
x) - 36*c^2*integrate(1/3*x^4*log(x)^2/(c^2*x^2 - 1), x) + 12*c^2*integrate
(1/3*x^4*log(c^2*x^2)/(c^2*x^2 - 1), x) + 6*(2*x/c^2 - log(c*x + 1)/c^3 + l
og(c*x - 1)/c^3)*log(c)^2 - 36*integrate(1/3*x^2*log(c^2*x^2)/(c^2*x^2 - 1)
, x)*log(c) + 72*integrate(1/3*x^2*log(x)/(c^2*x^2 - 1), x)*log(c) + 24*int
egrate(1/3*sqrt(c*x + 1)*sqrt(c*x - 1)*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x
 - 1)))/(c^2*x^2 - 1), x) - 36*integrate(1/3*x^2*log(c^2*x^2)*log(x)/(c^2*x
^2 - 1), x) + 36*integrate(1/3*x^2*log(x)^2/(c^2*x^2 - 1), x) - 12*integrat
e(1/3*x^2*log(c^2*x^2)/(c^2*x^2 - 1), x))*b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asin(1/(c*x)))^2,x)
```

```
[Out] int(x^2*(a + b*asin(1/(c*x)))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + b \operatorname{acsc}(cx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsc(c*x))**2,x)
[Out] Integral(x**2*(a + b*acsc(c*x))**2, x)
```

3.17 $\int x\left(a+b\csc^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=55

$$\frac{bx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{c}+\frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)^2+\frac{b^2\log(x)}{c^2}$$

```
[Out] 1/2*x^2*(a+b*arccsc(c*x))^2+b^2*ln(x)/c^2+b*x*(a+b*arccsc(c*x))*(1-1/c^2/x^
2)^(1/2)/c
```

Rubi [A] time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5223, 4410, 4184, 3475}

$$\frac{bx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{c}+\frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)^2+\frac{b^2\log(x)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcCsc[c*x])^2,x]
```

```
[Out] (b*Sqrt[1 - 1/(c^2*x^2)]*x*(a + b*ArcCsc[c*x]))/c + (x^2*(a + b*ArcCsc[c*x]
)^2)/2 + (b^2*Log[x])/c^2
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5223

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\int x\left(a+b\csc^{-1}(cx)\right)^2\,dx = -\frac{\text{Subst}\left(\int (a+bx)^2\cot(x)\csc^2(x)\,dx, x, \csc^{-1}(cx)\right)}{c^2}$$
$$= \frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)^2 - \frac{b\,\text{Subst}\left(\int (a+bx)\csc^2(x)\,dx, x, \csc^{-1}(cx)\right)}{c^2}$$
$$= \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(a+b\csc^{-1}(cx)\right)}{c} + \frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)^2 - \frac{b^2\,\text{Subst}\left(\int \cot(x)\,dx, x\right.}{c^2}$$
$$= \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(a+b\csc^{-1}(cx)\right)}{c} + \frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)^2 + \frac{b^2\log(x)}{c^2}$$

Mathematica [A] time = 0.15, size = 89, normalized size = 1.62

$$\frac{acx\left(acx+2b\sqrt{1-\frac{1}{c^2x^2}}\right)+2bcx\csc^{-1}(cx)\left(acx+b\sqrt{1-\frac{1}{c^2x^2}}\right)+b^2c^2x^2\csc^{-1}(cx)^2+2b^2\log(cx)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcCsc[c*x])^2,x]
```

```
[Out] (a*c*x*(2*b*Sqrt[1 - 1/(c^2*x^2)] + a*c*x) + 2*b*c*x*(b*Sqrt[1 - 1/(c^2*x^2
)] + a*c*x)*ArcCsc[c*x] + b^2*c^2*x^2*ArcCsc[c*x]^2 + 2*b^2*Log[c*x])/(2*c^
2)
```

fricas [B] time = 2.50, size = 111, normalized size = 2.02

$$\frac{b^2c^2x^2\,\text{arccsc}\,(cx)^2+a^2c^2x^2-4\,abc^2\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+2\,b^2\log(x)+2\left(abc^2x^2-abc^2\right)\text{arccsc}\,(cx)\cdots}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*c^2*x^2*arccsc(c*x)^2 + a^2*c^2*x^2 - 4*a*b*c^2*arctan(-c*x + sqrt
(c^2*x^2 - 1)) + 2*b^2*log(x) + 2*(a*b*c^2*x^2 - a*b*c^2)*arccsc(c*x) + 2*s
qrt(c^2*x^2 - 1)*(b^2*arccsc(c*x) + a*b))/c^2
```

giac [B] time = 0.25, size = 427, normalized size = 7.76

$$\frac{1}{8}\left(\frac{b^2x^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2\arcsin\left(\frac{1}{cx}\right)^2}{c}+\frac{2\,abx^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{a^2x^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2}{c}+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))^2,x, algorithm="giac")
```

```
[Out] 1/8*(b^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))^2/c + 2*a*b*x^2
*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c + a^2*x^2*(sqrt(-1/(c^2*x
^2) + 1) + 1)^2/c + 4*b^2*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^
2 + 4*a*b*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 + 2*b^2*arcsin(1/(c*x))^2/c^3
+ 4*a*b*arcsin(1/(c*x))/c^3 - 16*b^2*log(2)/c^3 + 8*b^2*log(2*sqrt(-1/(c^2*
x^2) + 1) + 2)/c^3 - 8*b^2*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^3 - 8*b^2*log(
1/(abs(c)*abs(x)))/c^3 + 2*a^2/c^3 - 4*b^2*arcsin(1/(c*x))/(c^4*x*(sqrt(-1/
(c^2*x^2) + 1) + 1)) - 4*a*b/(c^4*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + b^2*arc
sin(1/(c*x))^2/(c^5*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 2*a*b*arcsin(1/(c
```

```
*x))/(c^5*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + a^2/(c^5*x^2*(sqrt(-1/(c^2*
x^2) + 1) + 1)^2))*c
```

maple [B] time = 0.26, size = 133, normalized size = 2.42

$$\frac{a^2x^2}{2}+\frac{x^2b^2\mathrm{arccsc}\,(cx)^2}{2}+\frac{b^2\mathrm{arccsc}\,(cx)\,x\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{c}-\frac{b^2\ln\left(\frac{1}{cx}\right)}{c^2}+ab\,x^2\mathrm{arccsc}\,(cx)+\frac{abx}{c\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{ab}{c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsc(c*x))^2,x)
```

```
[Out] 1/2*a^2*x^2+1/2*x^2*b^2*arccsc(c*x)^2+1/c*b^2*arccsc(c*x)*x*((c^2*x^2-1)/c^
2/x^2)^(1/2)-1/c^2*b^2*ln(1/c/x)+a*b*x^2*arccsc(c*x)+1/c*a*b/((c^2*x^2-1)/c
^2/x^2)^(1/2)*x-1/c^3*a*b/((c^2*x^2-1)/c^2/x^2)^(1/2)/x
```

maxima [A] time = 0.36, size = 84, normalized size = 1.53

$$\frac{1}{2}\,b^2x^2\,\mathrm{arccsc}\,(cx)^2+\frac{1}{2}\,a^2x^2+\left(x^2\,\mathrm{arccsc}\,(cx)+\frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)ab+\left(\frac{x\sqrt{-\frac{1}{c^2x^2}+1}\,\mathrm{arccsc}\,(cx)}{c}+\frac{\log(x)}{c^2}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*arccsc(c*x)^2 + 1/2*a^2*x^2 + (x^2*arccsc(c*x) + x*sqrt(-1/(c^2
*x^2) + 1)/c)*a*b + (x*sqrt(-1/(c^2*x^2) + 1)*arccsc(c*x)/c + log(x)/c^2)*b
^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asin(1/(c*x)))^2,x)
```

```
[Out] int(x*(a + b*asin(1/(c*x)))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\,(a+b\,\mathrm{acsc}\,(cx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsc(c*x))**2,x)
```

```
[Out] Integral(x*(a + b*acsc(c*x))**2, x)
```

3.18 $\int \left(a+b\csc^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=84

$$x\left(a+b\csc^{-1}(cx)\right)^2+\frac{4b\tanh^{-1}\left(e^{i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)}{c}-\frac{2ib^2\text{Li}_2\left(-e^{i\csc^{-1}(cx)}\right)}{c}+\frac{2ib^2\text{Li}_2\left(e^{i\csc^{-1}(cx)}\right)}{c}$$

```
[Out] x*(a+b*arccsc(c*x))^2+4*b*(a+b*arccsc(c*x))*arctanh(I/c/x+(1-1/c^2/x^2)^(1/
2))/c-2*I*b^2*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))/c+2*I*b^2*polylog(2,I/c
/x+(1-1/c^2/x^2)^(1/2))/c
```

Rubi [A] time = 0.07, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5217, 4410, 4183, 2279, 2391}

$$-\frac{2ib^2\text{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)}{c}+\frac{2ib^2\text{PolyLog}\left(2,e^{i\csc^{-1}(cx)}\right)}{c}+x\left(a+b\csc^{-1}(cx)\right)^2+\frac{4b\tanh^{-1}\left(e^{i\csc^{-1}(cx)}\right)\left(a+b\right.}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])^2,x]
```

```
[Out] x*(a + b*ArcCsc[c*x])^2 + (4*b*(a + b*ArcCsc[c*x])*ArcTanh[E^(I*ArcCsc[c*x]
)])/c - ((2*I)*b^2*PolyLog[2, -E^(I*ArcCsc[c*x])])/c + ((2*I)*b^2*PolyLog[2
, E^(I*ArcCsc[c*x])])/c
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5217

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[c^(-1), Sub
st[Int[(a + b*x)^n*Csc[x]*Cot[x], x], x, ArcCsc[c*x]], x] /; FreeQ[{a, b, c
, n}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \left(a+b\csc^{-1}(cx)\right)^2 dx = -\frac{\text{Subst}\left(\int (a+bx)^2\cot(x)\csc(x)\,dx, x, \csc^{-1}(cx)\right)}{c}$$
$$= x\left(a+b\csc^{-1}(cx)\right)^2 - \frac{(2b)\,\text{Subst}\left(\int (a+bx)\csc(x)\,dx, x, \csc^{-1}(cx)\right)}{c}$$
$$= x\left(a+b\csc^{-1}(cx)\right)^2 + \frac{4b\left(a+b\csc^{-1}(cx)\right)\tanh^{-1}\left(e^{i\csc^{-1}(cx)}\right)}{c} + \frac{\left(2b^2\right)\text{Subst}\left(\int \log\left(1\right.\right.}{}$$
$$= x\left(a+b\csc^{-1}(cx)\right)^2 + \frac{4b\left(a+b\csc^{-1}(cx)\right)\tanh^{-1}\left(e^{i\csc^{-1}(cx)}\right)}{c} - \frac{\left(2ib^2\right)\text{Subst}\left(\int \frac{\log(1-}{x}\right.}{c}$$
$$= x\left(a+b\csc^{-1}(cx)\right)^2 + \frac{4b\left(a+b\csc^{-1}(cx)\right)\tanh^{-1}\left(e^{i\csc^{-1}(cx)}\right)}{c} - \frac{2ib^2\text{Li}_2\left(-e^{i\csc^{-1}(cx)}\right)}{c} -$$

Mathematica [A] time = 0.21, size = 147, normalized size = 1.75

$$\frac{a^2cx + 2abcx\csc^{-1}(cx) + 2ab\log\left(\cos\left(\frac{1}{2}\csc^{-1}(cx)\right)\right) - 2ab\log\left(\sin\left(\frac{1}{2}\csc^{-1}(cx)\right)\right) - 2ib^2\text{Li}_2\left(-e^{i\csc^{-1}(cx)}\right) + 2ib^2\text{I}}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsc[c*x])^2,x]
```

```
[Out] (a^2*c*x + 2*a*b*c*x*ArcCsc[c*x] + b^2*c*x*ArcCsc[c*x]^2 - 2*b^2*ArcCsc[c*x
]*Log[1 - E^(I*ArcCsc[c*x])] + 2*b^2*ArcCsc[c*x]*Log[1 + E^(I*ArcCsc[c*x])]
 + 2*a*b*Log[Cos[ArcCsc[c*x]/2]] - 2*a*b*Log[Sin[ArcCsc[c*x]/2]] - (2*I)*b^
2*PolyLog[2, -E^(I*ArcCsc[c*x])] + (2*I)*b^2*PolyLog[2, E^(I*ArcCsc[c*x])])
/c
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2\,\text{arccsc}\,(cx)^2 + 2\,ab\,\text{arccsc}\,(cx) + a^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*arccsc(c*x)^2 + 2*a*b*arccsc(c*x) + a^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{arccsc}\,(cx) + a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)^2, x)
```

maple [A] time = 0.26, size = 196, normalized size = 2.33

$$x\,b^2\text{arccsc}\,(cx)^2 + 2xab\,\text{arccsc}\,(cx) - \frac{2b^2\text{arccsc}\,(cx)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)}{c} + \frac{2b^2\text{arccsc}\,(cx)\ln\left(1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))^2,x)
```

```
[Out] x*b^2*arccsc(c*x)^2+2*x*a*b*arccsc(c*x)-2/c*b^2*arccsc(c*x)*ln(1-I/c/x-(1-1
/c^2/x^2)^(1/2))+2/c*b^2*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))-2*I/c*
dilog(1+I/c/x+(1-1/c^2/x^2)^(1/2))*b^2+2*I/c*dilog(1-I/c/x-(1-1/c^2/x^2)^(1
/2))*b^2+a^2*x+2/c*ln(c*x+c*x*(1-1/c^2/x^2)^(1/2))*a*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{4}\left(2\,c^2\left(\frac{2\,x}{c^2}-\frac{\log\left(cx+1\right)}{c^3}+\frac{\log\left(cx-1\right)}{c^3}\right)\log(c)^2-4\,c^2\int\frac{x^2\log\left(c^2x^2\right)}{c^2x^2-1}\,dx\log(c)+8\,c^2\int\frac{x^2\log(x)}{c^2x^2-1}\,dx\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*c^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3)*log(c)^2 - 4*c^
2*integrate(x^2*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) + 8*c^2*integrate(x^2
*log(x)/(c^2*x^2 - 1), x)*log(c) - 4*x*arctan2(1, sqrt(c*x + 1)*sqrt(c*x -
1))^2 - 4*c^2*integrate(x^2*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) + 4*c^2*i
ntegrate(x^2*log(x)^2/(c^2*x^2 - 1), x) - 4*c^2*integrate(x^2*log(c^2*x^2)/
(c^2*x^2 - 1), x) + x*log(c^2*x^2)^2 + 2*(log(c*x + 1)/c - log(c*x - 1)/c)*
log(c)^2 + 4*integrate(log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) - 8*integrate(
log(x)/(c^2*x^2 - 1), x)*log(c) - 8*integrate(sqrt(c*x + 1)*sqrt(c*x - 1)*a
rctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^2 - 1), x) + 4*integrate(log(
c^2*x^2)*log(x)/(c^2*x^2 - 1), x) - 4*integrate(log(x)^2/(c^2*x^2 - 1), x)
+ 4*integrate(log(c^2*x^2)/(c^2*x^2 - 1), x))*b^2 + a^2*x + (2*c*x*arccsc(c
*x) + log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-sqrt(-1/(c^2*x^2) + 1) + 1))*a
*b/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))^2,x)
```

```
[Out] int((a + b*asin(1/(c*x)))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{acsc}\left(cx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))**2,x)
```

```
[Out] Integral((a + b*acsc(c*x))**2, x)
```

3.19 $\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x}\,dx$

Optimal. Leaf size=91

$$ib\text{Li}_2\left(e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)+\frac{i\left(a+b\csc^{-1}(cx)\right)^3}{3b}-\log\left(1-e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)^2-\frac{1}{2}b^2\text{Li}_3\left(e^{2i\csc^{-1}(c\right.$$

```
[Out] 1/3*I*(a+b*arccsc(c*x))^3/b-(a+b*arccsc(c*x))^2*ln(1-(I/c/x+(1-1/c^2/x^2)^(
1/2))^2)+I*b*(a+b*arccsc(c*x))*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)-1/2
*b^2*polylog(3,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)
```

Rubi [A] time = 0.12, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5223, 3717, 2190, 2531, 2282, 6589}

$$ib\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)-\frac{1}{2}b^2\text{PolyLog}\left(3,e^{2i\csc^{-1}(cx)}\right)+\frac{i\left(a+b\csc^{-1}(cx)\right)^3}{3b}-\log\left(1-e^{2i\csc^{-1}(cx)}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])^2/x,x]
```

```
[Out] ((I/3)*(a + b*ArcCsc[c*x])^3)/b - (a + b*ArcCsc[c*x])^2*Log[1 - E^((2*I)*Ar
cCsc[c*x])] + I*b*(a + b*ArcCsc[c*x])*PolyLog[2, E^((2*I)*ArcCsc[c*x])] - (
b^2*PolyLog[3, E^((2*I)*ArcCsc[c*x])])/2
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 5223

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x}\,dx = -\text{Subst}\left(\int (a+bx)^2\cot(x)\,dx, x, \csc^{-1}(cx)\right)$$
$$= \frac{i\left(a+b\csc^{-1}(cx)\right)^3}{3b} + 2i\,\text{Subst}\left(\int \frac{e^{2ix}(a+bx)^2}{1-e^{2ix}}\,dx, x, \csc^{-1}(cx)\right)$$
$$= \frac{i\left(a+b\csc^{-1}(cx)\right)^3}{3b} - \left(a+b\csc^{-1}(cx)\right)^2\log\left(1-e^{2i\csc^{-1}(cx)}\right) + (2b)\,\text{Subst}\left(\int (a+b\right.$$
$$= \frac{i\left(a+b\csc^{-1}(cx)\right)^3}{3b} - \left(a+b\csc^{-1}(cx)\right)^2\log\left(1-e^{2i\csc^{-1}(cx)}\right) + ib\left(a+b\csc^{-1}(cx)\right)\text{I}$$
$$= \frac{i\left(a+b\csc^{-1}(cx)\right)^3}{3b} - \left(a+b\csc^{-1}(cx)\right)^2\log\left(1-e^{2i\csc^{-1}(cx)}\right) + ib\left(a+b\csc^{-1}(cx)\right)\text{I}$$
$$= \frac{i\left(a+b\csc^{-1}(cx)\right)^3}{3b} - \left(a+b\csc^{-1}(cx)\right)^2\log\left(1-e^{2i\csc^{-1}(cx)}\right) + ib\left(a+b\csc^{-1}(cx)\right)\text{I}$$

Mathematica [A] time = 0.16, size = 137, normalized size = 1.51

$$a^2\log(cx)+iab\left(\csc^{-1}(cx)^2+\text{Li}_2\left(e^{2i\csc^{-1}(cx)}\right)\right)-2ab\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)+\frac{1}{24}ib^2\left(-24\csc^{-1}(cx)\text{Li}_2\left(e^{-}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsc[c*x])^2/x,x]
```

```
[Out] -2*a*b*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] + a^2*Log[c*x] + I*a*b*(A
rcCsc[c*x]^2 + PolyLog[2, E^((2*I)*ArcCsc[c*x])]) + (I/24)*b^2*(Pi^3 - 8*Ar
cCsc[c*x]^3 + (24*I)*ArcCsc[c*x]^2*Log[1 - E^((-2*I)*ArcCsc[c*x])] - 24*Arc
Csc[c*x]*PolyLog[2, E^((-2*I)*ArcCsc[c*x])] + (12*I)*PolyLog[3, E^((-2*I)*A
rcCsc[c*x])])
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\,\text{arccsc}\,(cx)^2 + 2\,ab\,\text{arccsc}\,(cx) + a^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arccsc(c*x)^2 + 2*a*b*arccsc(c*x) + a^2)/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^2/x,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(x)]ln of unsigne
d or minus infinity Error: Bad Argument Value
```

maple [B] time = 0.22, size = 361, normalized size = 3.97

$$a^2 \ln(cx) + \frac{ib^2 \operatorname{arccsc}(cx)^3}{3} - b^2 \operatorname{arccsc}(cx)^2 \ln\left(1 + \frac{i}{cx} + \sqrt{1 - \frac{1}{c^2x^2}}\right) + 2ib^2 \operatorname{arccsc}(cx) \operatorname{polylog}\left(2, -\frac{i}{cx} - \sqrt{1 - \frac{1}{c^2x^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))^2/x,x)
```

```
[Out] a^2*ln(c*x)+1/3*I*b^2*arccsc(c*x)^3-b^2*arccsc(c*x)^2*ln(1+I/c/x+(1-1/c^2/x
^2)^(1/2))+2*I*b^2*arccsc(c*x)*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))-2*b^2*
polylog(3,-I/c/x-(1-1/c^2/x^2)^(1/2))-b^2*arccsc(c*x)^2*ln(1-I/c/x-(1-1/c^2
/x^2)^(1/2))+2*I*b^2*arccsc(c*x)*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))-2*b^2
*polylog(3,I/c/x+(1-1/c^2/x^2)^(1/2))+I*a*b*arccsc(c*x)^2-2*a*b*arccsc(c*x)
*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))-2*a*b*arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)
^(1/2))+2*I*a*b*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))+2*I*a*b*polylog(2,I/c
/x+(1-1/c^2/x^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2} b^2 c^2 \left(\frac{\log(cx+1)}{c^2} + \frac{\log(cx-1)}{c^2}\right) \log(c)^2 + b^2 c^2 \int \frac{x^2 \log\left(c^2x^2\right)}{c^2x^3 - x}\,dx \log(c) - 2\,b^2 c^2 \int \frac{x^2 \log(x)}{c^2x^3 - x}\,dx \log(c) + 2\,b^2 c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] -1/2*b^2*c^2*(log(c*x + 1)/c^2 + log(c*x - 1)/c^2)*log(c)^2 + b^2*c^2*integ
rate(x^2*log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) - 2*b^2*c^2*integrate(x^2*lo
g(x)/(c^2*x^3 - x), x)*log(c) + 2*b^2*c^2*integrate(x^2*log(c^2*x^2)*log(x)
/(c^2*x^3 - x), x) - b^2*c^2*integrate(x^2*log(x)^2/(c^2*x^3 - x), x) + 2*a
*b*c^2*integrate(x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^3 - x),
 x) + 1/2*b^2*(log(c*x + 1) + log(c*x - 1) - 2*log(x))*log(c)^2 + b^2*arcta
n2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2*log(x) - 1/4*b^2*log(c^2*x^2)^2*log(x)
 - b^2*integrate(log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) + 2*b^2*integrate(lo
g(x)/(c^2*x^3 - x), x)*log(c) + 2*b^2*integrate(sqrt(c*x + 1)*sqrt(c*x - 1)
*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)/(c^2*x^3 - x), x) - 2*b^2*i
ntegrate(log(c^2*x^2)*log(x)/(c^2*x^3 - x), x) + b^2*integrate(log(x)^2/(c^
2*x^3 - x), x) - 2*a*b*integrate(arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c
^2*x^3 - x), x) + a^2*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b \operatorname{asin}\left(\frac{1}{c\,x}\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))^2/x,x)
```

```
[Out] int((a + b*asin(1/(c*x)))^2/x, x)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{acsc}\,(cx))^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))**2/x,x)
```

```
[Out] Integral((a + b*acsc(c*x))**2/x, x)
```

3.20 $\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x^2}\,dx$

Optimal. Leaf size=50

$$-2bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)-\frac{\left(a+b\csc^{-1}(cx)\right)^2}{x}+\frac{2b^2}{x}$$

```
[Out] 2*b^2/x-(a+b*arccsc(c*x))^2/x-2*b*c*(a+b*arccsc(c*x))*(1-1/c^2/x^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5223, 3296, 2637}

$$-2bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)-\frac{\left(a+b\csc^{-1}(cx)\right)^2}{x}+\frac{2b^2}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])^2/x^2,x]
```

```
[Out] (2*b^2)/x - 2*b*c*Sqrt[1 - 1/(c^2*x^2)]*(a + b*ArcCsc[c*x]) - (a + b*ArcCsc
[c*x])^2/x
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5223

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x^2}\,dx &= -\left(c\,\text{Subst}\left(\int (a+bx)^2\cos(x)\,dx,x,\csc^{-1}(cx)\right)\right)\\
&= -\frac{\left(a+b\csc^{-1}(cx)\right)^2}{x}+(2bc)\,\text{Subst}\left(\int (a+bx)\sin(x)\,dx,x,\csc^{-1}(cx)\right)\\
&= -2bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)-\frac{\left(a+b\csc^{-1}(cx)\right)^2}{x}+\left(2b^2c\right)\text{Subst}\left(\int \cos(x)\,dx,x\right.\\
&= \frac{2b^2}{x}-2bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)-\frac{\left(a+b\csc^{-1}(cx)\right)^2}{x}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 71, normalized size = 1.42

$$-\frac{a^2+2abcx\sqrt{1-\frac{1}{c^2x^2}}+2b\csc^{-1}(cx)\left(a+bcx\sqrt{1-\frac{1}{c^2x^2}}\right)+b^2\csc^{-1}(cx)^2-2b^2}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])^2/x^2,x]
```

```
[Out] -((a^2 - 2*b^2 + 2*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x + 2*b*(a + b*c*Sqrt[1 - 1/
(c^2*x^2)]*x)*ArcCsc[c*x] + b^2*ArcCsc[c*x]^2)/x)
```

fricas [A] time = 2.27, size = 57, normalized size = 1.14

$$-\frac{b^2\operatorname{arccsc}(cx)^2+2\,ab\operatorname{arccsc}(cx)+a^2-2\,b^2+2\,\sqrt{c^2x^2-1}\left(b^2\operatorname{arccsc}(cx)+ab\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] -(b^2*arccsc(c*x)^2 + 2*a*b*arccsc(c*x) + a^2 - 2*b^2 + 2*sqrt(c^2*x^2 - 1)
*(b^2*arccsc(c*x) + a*b))/x
```

giac [B] time = 0.16, size = 104, normalized size = 2.08

$$-\left(2\,b^2\sqrt{-\frac{1}{c^2x^2}+1}\,\arcsin\left(\frac{1}{cx}\right)+2\,ab\sqrt{-\frac{1}{c^2x^2}+1}+\frac{b^2\arcsin\left(\frac{1}{cx}\right)^2}{cx}+\frac{2\,ab\arcsin\left(\frac{1}{cx}\right)}{cx}+\frac{a^2}{cx}-\frac{2\,b^2}{cx}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] -(2*b^2*sqrt(-1/(c^2*x^2) + 1)*arcsin(1/(c*x)) + 2*a*b*sqrt(-1/(c^2*x^2) +
1) + b^2*arcsin(1/(c*x))^2/(c*x) + 2*a*b*arcsin(1/(c*x))/(c*x) + a^2/(c*x)
- 2*b^2/(c*x))*c
```

maple [B] time = 0.18, size = 118, normalized size = 2.36

$$c\left(-\frac{a^2}{cx}+b^2\left(-\frac{\operatorname{arccsc}(cx)^2}{cx}+\frac{2}{cx}-2\operatorname{arccsc}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)+2ab\left(-\frac{\operatorname{arccsc}(cx)}{cx}-\frac{c^2x^2-1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2x^2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))^2/x^2,x)
```

```
[Out] c*(-a^2/c/x+b^2*(-1/c/x*arccsc(c*x)^2+2/c/x-2*arccsc(c*x)*((c^2*x^2-1)/c^2/
x^2)^(1/2))+2*a*b*(-1/c/x*arccsc(c*x)-1/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^2/x^2
*(c^2*x^2-1)))
```

maxima [A] time = 0.36, size = 79, normalized size = 1.58

$$-2\left(c\sqrt{-\frac{1}{c^2x^2}+1}+\frac{\operatorname{arccsc}(cx)}{x}\right)ab-2\left(c\sqrt{-\frac{1}{c^2x^2}+1}\,\operatorname{arccsc}(cx)-\frac{1}{x}\right)b^2-\frac{b^2\operatorname{arccsc}(cx)^2}{x}-\frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] -2*(c*sqrt(-1/(c^2*x^2) + 1) + arccsc(c*x)/x)*a*b - 2*(c*sqrt(-1/(c^2*x^2)
+ 1)*arccsc(c*x) - 1/x)*b^2 - b^2*arccsc(c*x)^2/x - a^2/x
```

mupad [B] time = 0.81, size = 88, normalized size = 1.76

$$-\frac{a^2}{x}-\frac{b^2\left(\mathrm{asin}\left(\frac{1}{c\,x}\right)^2-2\right)}{x}-2\,b^2\,c\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\,\sqrt{1-\frac{1}{c^2\,x^2}}-2\,a\,b\,c\,\left(\sqrt{1-\frac{1}{c^2\,x^2}}+\frac{\mathrm{asin}\left(\frac{1}{c\,x}\right)}{c\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))^2/x^2,x)
```

```
[Out] - a^2/x - (b^2*(asin(1/(c*x))^2 - 2))/x - 2*b^2*c*asin(1/(c*x))*(1 - 1/(c^2
*x^2))^(1/2) - 2*a*b*c*((1 - 1/(c^2*x^2))^(1/2) + asin(1/(c*x))/(c*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{acsc}\,(cx))^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))**2/x**2,x)
```

```
[Out] Integral((a + b*acsc(c*x))**2/x**2, x)
```

3.21 $\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x^3}\,dx$

Optimal. Leaf size=88

$$-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{2x}+\frac{1}{2}abc^2\csc^{-1}(cx)-\frac{\left(a+b\csc^{-1}(cx)\right)^2}{2x^2}+\frac{1}{4}b^2c^2\csc^{-1}(cx)^2+\frac{b^2}{4x^2}$$

```
[Out] 1/4*b^2/x^2+1/2*a*b*c^2*arccsc(c*x)+1/4*b^2*c^2*arccsc(c*x)^2-1/2*(a+b*arcc
sc(c*x))^2/x^2-1/2*b*c*(a+b*arccsc(c*x))*(1-1/c^2/x^2)^(1/2)/x
```

Rubi [A] time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5223, 4404, 3310}

$$-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{2x}+\frac{1}{2}abc^2\csc^{-1}(cx)-\frac{\left(a+b\csc^{-1}(cx)\right)^2}{2x^2}+\frac{1}{4}b^2c^2\csc^{-1}(cx)^2+\frac{b^2}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])^2/x^3,x]
```

```
[Out] b^2/(4*x^2) + (a*b*c^2*ArcCsc[c*x])/2 + (b^2*c^2*ArcCsc[c*x]^2)/4 - (b*c*Sq
rt[1 - 1/(c^2*x^2)]*(a + b*ArcCsc[c*x]))/(2*x) - (a + b*ArcCsc[c*x])^2/(2*x
^2)
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5223

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x^3}\,dx = -\left(c^2\,\text{Subst}\left(\int (a+bx)^2\cos(x)\sin(x)\,dx,x,\csc^{-1}(cx)\right)\right)$$
$$= -\frac{\left(a+b\csc^{-1}(cx)\right)^2}{2x^2} + \left(bc^2\right)\text{Subst}\left(\int (a+bx)\sin^2(x)\,dx,x,\csc^{-1}(cx)\right)$$
$$= \frac{b^2}{4x^2} - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{2x} - \frac{\left(a+b\csc^{-1}(cx)\right)^2}{2x^2} + \frac{1}{2}\left(bc^2\right)\text{Subst}\left(\int (a+bx\right.$$
$$= \frac{b^2}{4x^2} + \frac{1}{2}abc^2\csc^{-1}(cx) + \frac{1}{4}b^2c^2\csc^{-1}(cx)^2 - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{2x} - \frac{\left(a+b\,\mathrm{cs}\right.}{2}$$

Mathematica [A] time = 0.12, size = 102, normalized size = 1.16

$$\frac{-2a^2-2abcx\sqrt{1-\frac{1}{c^2x^2}}+2abc^2x^2\sin^{-1}\left(\frac{1}{cx}\right)-2b\csc^{-1}(cx)\left(2a+bcx\sqrt{1-\frac{1}{c^2x^2}}\right)+b^2\left(c^2x^2-2\right)\csc^{-1}(cx)^2+b^2}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])^2/x^3,x]
```

```
[Out] (-2*a^2 + b^2 - 2*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x - 2*b*(2*a + b*c*Sqrt[1 - 1
/(c^2*x^2)]*x)*ArcCsc[c*x] + b^2*(-2 + c^2*x^2)*ArcCsc[c*x]^2 + 2*a*b*c^2*x
^2*ArcSin[1/(c*x)])/(4*x^2)
```

fricas [A] time = 1.11, size = 82, normalized size = 0.93

$$\frac{\left(b^2c^2x^2-2\,b^2\right)\mathrm{arccsc}\,(cx)^2-2\,a^2+b^2+2\left(abc^2x^2-2\,ab\right)\mathrm{arccsc}\,(cx)-2\,\sqrt{c^2x^2-1}\left(b^2\,\mathrm{arccsc}\,(cx)+ab\right)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*((b^2*c^2*x^2 - 2*b^2)*arccsc(c*x)^2 - 2*a^2 + b^2 + 2*(a*b*c^2*x^2 - 2
*a*b)*arccsc(c*x) - 2*sqrt(c^2*x^2 - 1)*(b^2*arccsc(c*x) + a*b))/x^2
```

giac [B] time = 0.18, size = 163, normalized size = 1.85

$$-\frac{1}{8}\left(4\,b^2c\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)^2+8\,abc\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)+2\,b^2c\arcsin\left(\frac{1}{cx}\right)^2+4\,a^2c\left(\frac{1}{c^2x^2}-1\right)-2\,b^2c\right(\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] -1/8*(4*b^2*c*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))^2 + 8*a*b*c*(1/(c^2*x^2) -
1)*arcsin(1/(c*x)) + 2*b^2*c*arcsin(1/(c*x))^2 + 4*a^2*c*(1/(c^2*x^2) - 1)
- 2*b^2*c*(1/(c^2*x^2) - 1) + 4*a*b*c*arcsin(1/(c*x)) - b^2*c + 4*b^2*sqrt(
-1/(c^2*x^2) + 1)*arcsin(1/(c*x))/x + 4*a*b*sqrt(-1/(c^2*x^2) + 1)/x)*c
```

maple [B] time = 0.15, size = 191, normalized size = 2.17

$$-\frac{a^2}{2x^2}+\frac{b^2c^2\mathrm{arccsc}\,(cx)^2}{4}-\frac{b^2\mathrm{arccsc}\,(cx)^2}{2x^2}-\frac{c\,b^2\mathrm{arccsc}\,(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{2x}+\frac{b^2}{4x^2}-\frac{ab\,\mathrm{arccsc}\,(cx)}{x^2}+\frac{cab\sqrt{c^2x^2-1}\;\arctan\left(\cdots\right.}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\;x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))^2/x^3,x)
```

```
[Out] -1/2*a^2/x^2+1/4*b^2*c^2*arccsc(c*x)^2-1/2*b^2*arccsc(c*x)^2/x^2-1/2*c*b^2*
arccsc(c*x)/x*((c^2*x^2-1)/c^2/x^2)^(1/2)+1/4*b^2/x^2-a*b/x^2*arccsc(c*x)+1
/2*c*a*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*arctan(1/(c^2*x^2-
1)^(1/2))-1/2*c*a*b/((c^2*x^2-1)/c^2/x^2)^(1/2)/x+1/2/c*a*b/((c^2*x^2-1)/c^
2/x^2)^(1/2)/x^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*b*((c^4*x*sqrt(-1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) - c^
3*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)))/c - 2*arccsc(c*x)/x^2) - 1/8*(4*(c^2*
(log(c*x + 1) + log(c*x - 1) - 2*log(x))*log(c)^2 - 4*c^2*integrate(1/2*x^2
*log(c^2*x^2)/(c^2*x^5 - x^3), x)*log(c) + 8*c^2*integrate(1/2*x^2*log(x)/(
c^2*x^5 - x^3), x)*log(c) - 4*c^2*integrate(1/2*x^2*log(c^2*x^2)*log(x)/(c^
2*x^5 - x^3), x) + 4*c^2*integrate(1/2*x^2*log(x)^2/(c^2*x^5 - x^3), x) + 2
*c^2*integrate(1/2*x^2*log(c^2*x^2)/(c^2*x^5 - x^3), x) - (c^2*log(c*x + 1)
 + c^2*log(c*x - 1) - 2*c^2*log(x) + 1/x^2)*log(c)^2 + 4*integrate(1/2*log(
c^2*x^2)/(c^2*x^5 - x^3), x)*log(c) - 8*integrate(1/2*log(x)/(c^2*x^5 - x^3
), x)*log(c) + 4*integrate(1/2*sqrt(c*x + 1)*sqrt(c*x - 1)*arctan(1/(sqrt(c
*x + 1)*sqrt(c*x - 1)))/(c^2*x^5 - x^3), x) + 4*integrate(1/2*log(c^2*x^2)*
log(x)/(c^2*x^5 - x^3), x) - 4*integrate(1/2*log(x)^2/(c^2*x^5 - x^3), x) -
 2*integrate(1/2*log(c^2*x^2)/(c^2*x^5 - x^3), x))*x^2 + 4*arctan2(1, sqrt(
c*x + 1)*sqrt(c*x - 1))^2 - log(c^2*x^2)^2)*b^2/x^2 - 1/2*a^2/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^2}{x^3} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))^2/x^3,x)
```

```
[Out] int((a + b*asin(1/(c*x)))^2/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{acsc}(cx))^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))**2/x**3,x)
```

```
[Out] Integral((a + b*acsc(c*x))**2/x**3, x)
```

3.22 $\int \frac{\left(a+b \csc^{-1}(cx)\right)^2}{x^4} \, dx$

Optimal. Leaf size=102

$$-\frac{2bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{9x^2}-\frac{4}{9}bc^3\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)-\frac{\left(a+b\csc^{-1}(cx)\right)^2}{3x^3}+\frac{4b^2c^2}{9x}+\frac{2b^2}{27x^3}$$

```
[Out] 2/27*b^2/x^3+4/9*b^2*c^2/x-1/3*(a+b*arccsc(c*x))^2/x^3-4/9*b*c^3*(a+b*arccs
c(c*x))*(1-1/c^2/x^2)^(1/2)-2/9*b*c*(a+b*arccsc(c*x))*(1-1/c^2/x^2)^(1/2)/x
^2
```

Rubi [A] time = 0.10, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5223, 4404, 3310, 3296, 2637}

$$-\frac{4}{9}bc^3\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)-\frac{2bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{9x^2}-\frac{\left(a+b\csc^{-1}(cx)\right)^2}{3x^3}+\frac{4b^2c^2}{9x}+\frac{2b^2}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])^2/x^4,x]
```

```
[Out] (2*b^2)/(27*x^3) + (4*b^2*c^2)/(9*x) - (4*b*c^3*Sqrt[1 - 1/(c^2*x^2)]*(a +
b*ArcCsc[c*x]))/9 - (2*b*c*Sqrt[1 - 1/(c^2*x^2)]*(a + b*ArcCsc[c*x]))/(9*x^
2) - (a + b*ArcCsc[c*x])^2/(3*x^3)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5223

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x^4}\,dx = -\left(c^3\,\text{Subst}\left(\int (a+bx)^2\cos(x)\sin^2(x)\,dx, x, \csc^{-1}(cx)\right)\right)$$
$$= -\frac{\left(a+b\csc^{-1}(cx)\right)^2}{3x^3} + \frac{1}{3}\left(2bc^3\right)\text{Subst}\left(\int (a+bx)\sin^3(x)\,dx, x, \csc^{-1}(cx)\right)$$
$$= \frac{2b^2}{27x^3} - \frac{2bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{9x^2} - \frac{\left(a+b\csc^{-1}(cx)\right)^2}{3x^3} + \frac{1}{9}\left(4bc^3\right)\text{Subst}\left(\int (\right.$$
$$= \frac{2b^2}{27x^3} - \frac{4}{9}bc^3\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right) - \frac{2bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{9x^2} - \frac{(a+}{}$$
$$= \frac{2b^2}{27x^3} + \frac{4b^2c^2}{9x} - \frac{4}{9}bc^3\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right) - \frac{2bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx\right.}{9x^2}$$

Mathematica [A] time = 0.24, size = 108, normalized size = 1.06

$$-\frac{9a^2+6abcx\sqrt{1-\frac{1}{c^2x^2}}\left(2c^2x^2+1\right)+6b\csc^{-1}(cx)\left(3a+bcx\sqrt{1-\frac{1}{c^2x^2}}\left(2c^2x^2+1\right)\right)-2b^2\left(6c^2x^2+1\right)+9b^2\,\text{cs}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])^2/x^4,x]
```

```
[Out] -1/27*(9*a^2 + 6*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2) - 2*b^2*(1 +
 6*c^2*x^2) + 6*b*(3*a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2))*ArcCs
c[c*x] + 9*b^2*ArcCsc[c*x]^2)/x^3
```

fricas [A] time = 0.97, size = 93, normalized size = 0.91

$$\frac{12\,b^2c^2x^2-9\,b^2\,\text{arccsc}\,(cx)^2-18\,ab\,\text{arccsc}\,(cx)-9\,a^2+2\,b^2-6\left(2\,abc^2x^2+ab+\left(2\,b^2c^2x^2+b^2\right)\text{arccsc}\,(cx)\right.}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/27*(12*b^2*c^2*x^2 - 9*b^2*arccsc(c*x)^2 - 18*a*b*arccsc(c*x) - 9*a^2 + 2
*b^2 - 6*(2*a*b*c^2*x^2 + a*b + (2*b^2*c^2*x^2 + b^2)*arccsc(c*x))*sqrt(c^2
*x^2 - 1))/x^3
```

giac [B] time = 0.17, size = 224, normalized size = 2.20

$$\frac{1}{27}\left(6\,b^2c^2\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}\arcsin\left(\frac{1}{cx}\right)+6\,abc^2\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-18\,b^2c^2\sqrt{-\frac{1}{c^2x^2}+1}\,\arcsin\left(\frac{1}{cx}\right)-\frac{9\,b^2c\left(\frac{1}{c^2x^2}-1\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] 1/27*(6*b^2*c^2*(-1/(c^2*x^2) + 1)^(3/2)*arcsin(1/(c*x)) + 6*a*b*c^2*(-1/(c
^2*x^2) + 1)^(3/2) - 18*b^2*c^2*sqrt(-1/(c^2*x^2) + 1)*arcsin(1/(c*x)) - 9*
b^2*c*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))^2/x - 18*a*b*c^2*sqrt(-1/(c^2*x^2)
+ 1) - 18*a*b*c*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))/x - 9*b^2*c*arcsin(1/(c*x
```

```
))^2/x + 2*b^2*c*(1/(c^2*x^2) - 1)/x - 18*a*b*c*arcsin(1/(c*x))/x + 14*b^2*
c/x - 9*a^2/(c*x^3))*c
```

maple [A] time = 0.57, size = 154, normalized size = 1.51

$$c^3\left(-\frac{a^2}{3c^3x^3}+b^2\left(-\frac{\operatorname{arccsc}(cx)^2}{3c^3x^3}-\frac{2\operatorname{arccsc}(cx)\left(2c^2x^2+1\right)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{9c^2x^2}+\frac{2}{27c^3x^3}+\frac{4}{9cx}\right)+2ab\left(-\frac{\operatorname{arccsc}(cx)}{3c^3x^3}-\frac{(c^2\cdots}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))^2/x^4,x)
```

```
[Out] c^3*(-1/3*a^2/c^3/x^3+b^2*(-1/3/c^3/x^3*arccsc(c*x)^2-2/9*arccsc(c*x)*(2*c^
2*x^2+1)/c^2/x^2*((c^2*x^2-1)/c^2/x^2)^(1/2)+2/27/c^3/x^3+4/9/c/x)+2*a*b*(-
1/3/c^3/x^3*arccsc(c*x)-1/9*(c^2*x^2-1)*(2*c^2*x^2+1)/((c^2*x^2-1)/c^2/x^2)
^(1/2)/c^4/x^4))
```

maxima [B] time = 1.95, size = 197, normalized size = 1.93

$$\frac{2}{9}\,ab\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c}-\frac{3\,\operatorname{arccsc}(cx)}{x^3}\right)-\frac{b^2\operatorname{arccsc}(cx)^2}{3\,x^3}-\frac{a^2}{3\,x^3}-\frac{2\left(6\,c^5x^4\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx\cdots}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] 2/9*a*b*((c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c -
3*arccsc(c*x)/x^3) - 1/3*b^2*arccsc(c*x)^2/x^3 - 1/3*a^2/x^3 - 2/27*(6*c^5*
x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) - 3*c^3*x^2*arctan2(1, sqrt(c*x
 + 1)*sqrt(c*x - 1)) - (6*c^3*x^2 + c)*sqrt(c*x + 1)*sqrt(c*x - 1) - 3*c*ar
ctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*b^2/(sqrt(c*x + 1)*sqrt(c*x - 1)*c*x
^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))^2/x^4,x)
```

```
[Out] int((a + b*asin(1/(c*x)))^2/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{acsc}(cx))^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))**2/x**4,x)
```

```
[Out] Integral((a + b*acsc(c*x))**2/x**4, x)
```

3.23 $\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x^5}\,dx$

Optimal. Leaf size=134

$$\frac{3}{16}abc^4\csc^{-1}(cx)-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{8x^3}-\frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{16x}-\frac{\left(a+b\csc^{-1}(cx)\right)^2}{4x^4}+\frac{3}{32}b^2c$$

```
[Out] 1/32*b^2/x^4+3/32*b^2*c^2/x^2+3/16*a*b*c^4*arccsc(c*x)+3/32*b^2*c^4*arccsc(
c*x)^2-1/4*(a+b*arccsc(c*x))^2/x^4-1/8*b*c*(a+b*arccsc(c*x))*(1-1/c^2/x^2)^
(1/2)/x^3-3/16*b*c^3*(a+b*arccsc(c*x))*(1-1/c^2/x^2)^(1/2)/x
```

Rubi [A] time = 0.11, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5223, 4404, 3310}

$$-\frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{16x}-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{8x^3}+\frac{3}{16}abc^4\csc^{-1}(cx)-\frac{\left(a+b\csc^{-1}(cx)\right)^2}{4x^4}+\frac{3b^2c}{32x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])^2/x^5,x]
```

```
[Out] b^2/(32*x^4) + (3*b^2*c^2)/(32*x^2) + (3*a*b*c^4*ArcCsc[c*x])/16 + (3*b^2*c
^4*ArcCsc[c*x]^2)/32 - (b*c*Sqrt[1 - 1/(c^2*x^2)]*(a + b*ArcCsc[c*x]))/(8*x
^3) - (3*b*c^3*Sqrt[1 - 1/(c^2*x^2)]*(a + b*ArcCsc[c*x]))/(16*x) - (a + b*A
rcCsc[c*x])^2/(4*x^4)
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5223

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x^5}\,dx = -\left(c^4\,\text{Subst}\left(\int (a+bx)^2\cos(x)\sin^3(x)\,dx, x, \csc^{-1}(cx)\right)\right)$$
$$= -\frac{\left(a+b\csc^{-1}(cx)\right)^2}{4x^4} + \frac{1}{2}\left(bc^4\right)\text{Subst}\left(\int (a+bx)\sin^4(x)\,dx, x, \csc^{-1}(cx)\right)$$
$$= \frac{b^2}{32x^4} - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{8x^3} - \frac{\left(a+b\csc^{-1}(cx)\right)^2}{4x^4} + \frac{1}{8}\left(3bc^4\right)\text{Subst}\left(\int (a+$$
$$= \frac{b^2}{32x^4} + \frac{3b^2c^2}{32x^2} - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{8x^3} - \frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{16x} - $$
$$= \frac{b^2}{32x^4} + \frac{3b^2c^2}{32x^2} + \frac{3}{16}abc^4\csc^{-1}(cx) + \frac{3}{32}b^2c^4\csc^{-1}(cx)^2 - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right.}{8x^3}$$

Mathematica [A] time = 0.18, size = 148, normalized size = 1.10

$$\frac{-8a^2+6abc^4x^4\sin^{-1}\left(\frac{1}{cx}\right)-4abcx\sqrt{1-\frac{1}{c^2x^2}}-2b\csc^{-1}(cx)\left(8a+bcx\sqrt{1-\frac{1}{c^2x^2}}\left(3c^2x^2+2\right)\right)-6abc^3x^3\sqrt{1-\frac{1}{c^2x^2}}}{32x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])^2/x^5,x]
```

```
[Out] (-8*a^2 + b^2 - 4*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x + 3*b^2*c^2*x^2 - 6*a*b*c^3
*Sqrt[1 - 1/(c^2*x^2)]*x^3 - 2*b*(8*a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(2 + 3*
c^2*x^2))*ArcCsc[c*x] + b^2*(-8 + 3*c^4*x^4)*ArcCsc[c*x]^2 + 6*a*b*c^4*x^4*
ArcSin[1/(c*x)])/(32*x^4)
```

fricas [A] time = 0.96, size = 120, normalized size = 0.90

$$\frac{3\,b^2c^2x^2+\left(3\,b^2c^4x^4-8\,b^2\right)\text{arccsc}\left(cx\right)^2-8\,a^2+b^2+2\left(3\,abc^4x^4-8\,ab\right)\text{arccsc}\left(cx\right)-2\left(3\,abc^2x^2+2\,ab+\left(3\,b^2c\right.\right.}{32\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/32*(3*b^2*c^2*x^2 + (3*b^2*c^4*x^4 - 8*b^2)*arccsc(c*x)^2 - 8*a^2 + b^2 +
 2*(3*a*b*c^4*x^4 - 8*a*b)*arccsc(c*x) - 2*(3*a*b*c^2*x^2 + 2*a*b + (3*b^2*
c^2*x^2 + 2*b^2)*arccsc(c*x))*sqrt(c^2*x^2 - 1))/x^4
```

giac [B] time = 0.17, size = 304, normalized size = 2.27

$$-\frac{1}{256}\left(64\,b^2c^3\left(\frac{1}{c^2x^2}-1\right)^2\arcsin\left(\frac{1}{cx}\right)^2+128\,abc^3\left(\frac{1}{c^2x^2}-1\right)^2\arcsin\left(\frac{1}{cx}\right)+128\,b^2c^3\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^2/x^5,x, algorithm="giac")
```

```
[Out] -1/256*(64*b^2*c^3*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*x))^2 + 128*a*b*c^3*(1/(
c^2*x^2) - 1)^2*arcsin(1/(c*x)) + 128*b^2*c^3*(1/(c^2*x^2) - 1)*arcsin(1/(c
*x))^2 - 8*b^2*c^3*(1/(c^2*x^2) - 1)^2 + 256*a*b*c^3*(1/(c^2*x^2) - 1)*arcs
in(1/(c*x)) + 40*b^2*c^3*arcsin(1/(c*x))^2 - 40*b^2*c^3*(1/(c^2*x^2) - 1) +
```

```
80*a*b*c^3*arcsin(1/(c*x)) - 32*b^2*c^2*(-1/(c^2*x^2) + 1)^(3/2)*arcsin(1/
(c*x))/x - 17*b^2*c^3 - 32*a*b*c^2*(-1/(c^2*x^2) + 1)^(3/2)/x + 80*b^2*c^2*
sqrt(-1/(c^2*x^2) + 1)*arcsin(1/(c*x))/x + 80*a*b*c^2*sqrt(-1/(c^2*x^2) + 1
)/x + 64*a^2/(c*x^4))*c
```

maple [B] time = 0.46, size = 265, normalized size = 1.98

$$-\frac{a^2}{4x^4}-\frac{b^2\text{arccsc}\,(cx)^2}{4x^4}+\frac{3b^2c^4\text{arccsc}\,(cx)^2}{32}-\frac{3c^3b^2\text{arccsc}\,(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{16x}-\frac{c\,b^2\text{arccsc}\,(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{8x^3}+\frac{b^2}{32x^4}+\frac{3b^2c^2}{32x^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))^2/x^5,x)
```

```
[Out] -1/4*a^2/x^4-1/4*b^2/x^4*arccsc(c*x)^2+3/32*b^2*c^4*arccsc(c*x)^2-3/16*c^3*
b^2*arccsc(c*x)/x*((c^2*x^2-1)/c^2/x^2)^(1/2)-1/8*c*b^2*arccsc(c*x)/x^3*((c
^2*x^2-1)/c^2/x^2)^(1/2)+1/32*b^2/x^4+3/32*b^2*c^2/x^2-1/2*a*b/x^4*arccsc(c
*x)+3/16*c^3*a*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*arctan(1/(
c^2*x^2-1)^(1/2))-3/16*c^3*a*b/((c^2*x^2-1)/c^2/x^2)^(1/2)/x+1/16*c*a*b/((c
^2*x^2-1)/c^2/x^2)^(1/2)/x^3+1/8/c*a*b/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/16*a*b*((3*c^5*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)) + (3*c^8*x^3*(-1/(c^2*
x^2) + 1)^(3/2) + 5*c^6*x*sqrt(-1/(c^2*x^2) + 1))/(c^4*x^4*(1/(c^2*x^2) - 1
)^2 - 2*c^2*x^2*(1/(c^2*x^2) - 1) + 1))/c + 8*arccsc(c*x)/x^4) - 1/16*(4*(2
*(c^2*log(c*x + 1) + c^2*log(c*x - 1) - 2*c^2*log(x) + 1/x^2)*c^2*log(c)^2
- 16*c^2*integrate(1/4*x^2*log(c^2*x^2)/(c^2*x^7 - x^5), x)*log(c) + 32*c^2
*integrate(1/4*x^2*log(x)/(c^2*x^7 - x^5), x)*log(c) - 16*c^2*integrate(1/4
*x^2*log(c^2*x^2)*log(x)/(c^2*x^7 - x^5), x) + 16*c^2*integrate(1/4*x^2*log
(x)^2/(c^2*x^7 - x^5), x) + 4*c^2*integrate(1/4*x^2*log(c^2*x^2)/(c^2*x^7 -
 x^5), x) - (2*c^4*log(c*x + 1) + 2*c^4*log(c*x - 1) - 4*c^4*log(x) + (2*c^
2*x^2 + 1)/x^4)*log(c)^2 + 16*integrate(1/4*log(c^2*x^2)/(c^2*x^7 - x^5), x
)*log(c) - 32*integrate(1/4*log(x)/(c^2*x^7 - x^5), x)*log(c) + 8*integrate
(1/4*sqrt(c*x + 1)*sqrt(c*x - 1)*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c
^2*x^7 - x^5), x) + 16*integrate(1/4*log(c^2*x^2)*log(x)/(c^2*x^7 - x^5), x
) - 16*integrate(1/4*log(x)^2/(c^2*x^7 - x^5), x) - 4*integrate(1/4*log(c^2
*x^2)/(c^2*x^7 - x^5), x))*x^4 + 4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^
2 - log(c^2*x^2)^2)*b^2/x^4 - 1/4*a^2/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))^2/x^5,x)
```

```
[Out] int((a + b*asin(1/(c*x)))^2/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\text{acsc}\,(cx))^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))**2/x**5,x)
```

```
[Out] Integral((a + b*acsc(c*x))**2/x**5, x)
```

3.24 $\int x^3 \left(a+b\csc^{-1}(cx)\right)^3 \, dx$

Optimal. Leaf size=207

$$-\frac{b^2\log\left(1-e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)}{c^4}+\frac{b^2x^2\left(a+b\csc^{-1}(cx)\right)}{4c^2}+\frac{ib\left(a+b\csc^{-1}(cx)\right)^2}{2c^4}+\frac{bx^3\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\right.}{4c}$$

```
[Out] 1/4*b^2*x^2*(a+b*arccsc(c*x))/c^2+1/2*I*b*(a+b*arccsc(c*x))^2/c^4+1/4*x^4*(
a+b*arccsc(c*x))^3-b^2*(a+b*arccsc(c*x))*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2
)/c^4+1/2*I*b^3*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)/c^4+1/4*b^3*x*(1-1
/c^2/x^2)^(1/2)/c^3+1/2*b*x*(a+b*arccsc(c*x))^2*(1-1/c^2/x^2)^(1/2)/c^3+1/4
*b*x^3*(a+b*arccsc(c*x))^2*(1-1/c^2/x^2)^(1/2)/c
```

Rubi [A] time = 0.22, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5223, 4410, 4186, 3767, 8, 4184, 3717, 2190, 2279, 2391}

$$\frac{ib^3\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2c^4}+\frac{b^2x^2\left(a+b\csc^{-1}(cx)\right)}{4c^2}-\frac{b^2\log\left(1-e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)}{c^4}+\frac{bx^3\sqrt{1-\frac{1}{c^2x^2}}\left(a\right.}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcCsc[c*x])^3,x]
```

```
[Out] (b^3*Sqrt[1 - 1/(c^2*x^2)]*x)/(4*c^3) + (b^2*x^2*(a + b*ArcCsc[c*x]))/(4*c^
2) + ((I/2)*b*(a + b*ArcCsc[c*x])^2)/c^4 + (b*Sqrt[1 - 1/(c^2*x^2)]*x*(a +
b*ArcCsc[c*x])^2)/(2*c^3) + (b*Sqrt[1 - 1/(c^2*x^2)]*x^3*(a + b*ArcCsc[c*x]
)^2)/(4*c) + (x^4*(a + b*ArcCsc[c*x])^3)/4 - (b^2*(a + b*ArcCsc[c*x])*Log[1
 - E^((2*I)*ArcCsc[c*x])])/c^4 + ((I/2)*b^3*PolyLog[2, E^((2*I)*ArcCsc[c*x]
)])/c^4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
```

```
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5223

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\int x^3\left(a+b\csc^{-1}(cx)\right)^3dx=-\frac{\text{Subst}\left(\int(a+bx)^3\cot(x)\csc^4(x)\,dx,x,\csc^{-1}(cx)\right)}{c^4}$$
$$=\frac{1}{4}x^4\left(a+b\csc^{-1}(cx)\right)^3-\frac{(3b)\,\text{Subst}\left(\int(a+bx)^2\csc^4(x)\,dx,x,\csc^{-1}(cx)\right)}{4c^4}$$
$$=\frac{b^2x^2\left(a+b\csc^{-1}(cx)\right)}{4c^2}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^3\left(a+b\csc^{-1}(cx)\right)^2}{4c}+\frac{1}{4}x^4\left(a+b\csc^{-1}(cx)\right.$$
$$=\frac{b^2x^2\left(a+b\csc^{-1}(cx)\right)}{4c^2}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(a+b\csc^{-1}(cx)\right)^2}{2c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^3\left(a+b\right.}{4c}$$
$$=\frac{b^3\sqrt{1-\frac{1}{c^2x^2}}\,x}{4c^3}+\frac{b^2x^2\left(a+b\csc^{-1}(cx)\right)}{4c^2}+\frac{ib\left(a+b\csc^{-1}(cx)\right)^2}{2c^4}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(a\right.}{2}$$
$$=\frac{b^3\sqrt{1-\frac{1}{c^2x^2}}\,x}{4c^3}+\frac{b^2x^2\left(a+b\csc^{-1}(cx)\right)}{4c^2}+\frac{ib\left(a+b\csc^{-1}(cx)\right)^2}{2c^4}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(a\right.}{2}$$
$$=\frac{b^3\sqrt{1-\frac{1}{c^2x^2}}\,x}{4c^3}+\frac{b^2x^2\left(a+b\csc^{-1}(cx)\right)}{4c^2}+\frac{ib\left(a+b\csc^{-1}(cx)\right)^2}{2c^4}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(a\right.}{2}$$
$$=\frac{b^3\sqrt{1-\frac{1}{c^2x^2}}\,x}{4c^3}+\frac{b^2x^2\left(a+b\csc^{-1}(cx)\right)}{4c^2}+\frac{ib\left(a+b\csc^{-1}(cx)\right)^2}{2c^4}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(a\right.}{2}$$

Mathematica [A] time = 0.91, size = 285, normalized size = 1.38

$$\frac{a^3c^4x^4+b\csc^{-1}(cx)\left(cx\left(3a^2c^3x^3+2ab\sqrt{1-\frac{1}{c^2x^2}}\left(c^2x^2+2\right)+b^2cx\right)-4b^2\log\left(1-e^{2i\csc^{-1}(cx)}\right)\right)+2a^2bcx\sqrt{1-}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(a + b*ArcCsc[c*x])^3,x]
```

```
[Out] (2*a^2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x + b^3*c*Sqrt[1 - 1/(c^2*x^2)]*x + a*b^2*
c^2*x^2 + a^2*b*c^3*Sqrt[1 - 1/(c^2*x^2)]*x^3 + a^3*c^4*x^4 + b^2*(3*a*c^4*
x^4 + b*(2*I + 2*c*Sqrt[1 - 1/(c^2*x^2)]*x + c^3*Sqrt[1 - 1/(c^2*x^2)]*x^3)
)*ArcCsc[c*x]^2 + b^3*c^4*x^4*ArcCsc[c*x]^3 + b*ArcCsc[c*x]*(c*x*(b^2*c*x +
 3*a^2*c^3*x^3 + 2*a*b*Sqrt[1 - 1/(c^2*x^2)]*(2 + c^2*x^2)) - 4*b^2*Log[1 -
 E^((2*I)*ArcCsc[c*x])]) - 4*a*b^2*Log[1/(c*x)] + (2*I)*b^3*PolyLog[2, E^((
2*I)*ArcCsc[c*x])])/(4*c^4)
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3x^3\operatorname{arccsc}(cx)^3+3\,ab^2x^3\operatorname{arccsc}(cx)^2+3\,a^2bx^3\operatorname{arccsc}(cx)+a^3x^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^3*arccsc(c*x)^3 + 3*a*b^2*x^3*arccsc(c*x)^2 + 3*a^2*b*x^3*ar
ccsc(c*x) + a^3*x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(b\operatorname{arccsc}(cx)+a)^3x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)^3*x^3, x)
```

maple [B] time = 1.06, size = 510, normalized size = 2.46

$$\frac{x^4a^3}{4}+\frac{b^3\mathrm{arccsc}\,(cx)^3\,x^4}{4}+\frac{b^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}\ \mathrm{arccsc}\,(cx)^2\,x^3}{4c}+\frac{b^3\mathrm{arccsc}\,(cx)^2\,\sqrt{\frac{c^2x^2-1}{c^2x^2}}\ x}{2c^3}+\frac{ib^3\mathrm{arccsc}\,(cx)^2}{2c^4}+\frac{b^3\mathrm{arccsc}\,(cx)\,x^2}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsc(c*x))^3,x)
```

```
[Out] 1/4*x^4*a^3+1/4*b^3*arccsc(c*x)^3*x^4+1/4/c*b^3*((c^2*x^2-1)/c^2/x^2)^(1/2)
*arccsc(c*x)^2*x^3+1/2/c^3*b^3*arccsc(c*x)^2*((c^2*x^2-1)/c^2/x^2)^(1/2)*x+
1/2*I/c^4*b^3*arccsc(c*x)^2+1/4/c^2*b^3*arccsc(c*x)*x^2+1/4/c^3*b^3*((c^2*x
^2-1)/c^2/x^2)^(1/2)*x+I/c^4*b^3*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))-1/c^4
*b^3*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))-1/c^4*b^3*arccsc(c*x)*ln(1
-I/c/x-(1-1/c^2/x^2)^(1/2))+I/c^4*b^3*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))
-1/4*I/c^4*b^3+3/4*a^2*b*x^4*arccsc(c*x)+1/4/c*a^2*b/((c^2*x^2-1)/c^2/x^2)^
(1/2)*x^3+1/4/c^3*a^2*b/((c^2*x^2-1)/c^2/x^2)^(1/2)*x-1/2/c^5*a^2*b/((c^2*x
^2-1)/c^2/x^2)^(1/2)/x+3/4*a*b^2*arccsc(c*x)^2*x^4+1/2/c*a*b^2*((c^2*x^2-1)
/c^2/x^2)^(1/2)*arccsc(c*x)*x^3+1/4/c^2*x^2*a*b^2+1/c^3*a*b^2*((c^2*x^2-1)/
c^2/x^2)^(1/2)*arccsc(c*x)*x-1/c^4*a*b^2*ln(1/c/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3}{4}\,ab^2x^4\,\mathrm{arccsc}\,(cx)^2+\frac{1}{4}\,a^3x^4+\frac{1}{4}\left(3\,x^4\,\mathrm{arccsc}\,(cx)+\frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+3\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)a^2b+\frac{1}{16}\left(4\,x^4\,\mathrm{arctan}\left(1,\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))^3,x, algorithm="maxima")
```

```
[Out] 3/4*a*b^2*x^4*arccsc(c*x)^2 + 1/4*a^3*x^4 + 1/4*(3*x^4*arccsc(c*x) + (c^2*x
^3*(-1/(c^2*x^2) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*a^2*b + 1/16
*(4*x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 3*x^4*arctan2(1, sqrt(c
*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 - 16*integrate(3/16*(16*c^2*x^5*arcta
n2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c)^2 - 16*x^3*arctan2(1, sqrt(c*x +
1)*sqrt(c*x - 1))*log(c)^2 + 16*(c^2*x^5*arctan2(1, sqrt(c*x + 1)*sqrt(c*x
- 1)) - x^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)^2 - (4*x^3*arct
an2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2 - x^3*log(c^2*x^2)^2)*sqrt(c*x + 1)*s
qrt(c*x - 1) - 4*((4*c^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c) + c
^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*x^5 - (4*arctan2(1, sqrt(c*x +
1)*sqrt(c*x - 1))*log(c) + arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*x^3 + 4
*(c^2*x^5*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) - x^3*arctan2(1, sqrt(c*x
 + 1)*sqrt(c*x - 1)))*log(x))*log(c^2*x^2) + 32*(c^2*x^5*arctan2(1, sqrt(c*
x + 1)*sqrt(c*x - 1))*log(c) - x^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*
log(c))*log(x))/(c^2*x^2 - 1), x))*b^3 + 1/4*(2*c^4*x^4*arctan2(1, sqrt(c*x
 + 1)*sqrt(c*x - 1)) + 2*c^2*x^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) +
(c^2*x^2 + 2*log(x^2))*sqrt(c*x + 1)*sqrt(c*x - 1) - 4*arctan2(1, sqrt(c*x
+ 1)*sqrt(c*x - 1)))*a*b^2/(sqrt(c*x + 1)*sqrt(c*x - 1)*c^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asin(1/(c*x)))^3,x)
```

```
[Out] int(x^3*(a + b*asin(1/(c*x)))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 (a + b \operatorname{acsc}(cx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsc(c*x))**3,x)
```

```
[Out] Integral(x**3*(a + b*acsc(c*x))**3, x)
```

3.25 $\int x^2 \left(a+b\csc^{-1}(cx)\right)^3 dx$

Optimal. Leaf size=220

$$-\frac{ib^2\text{Li}_2\left(-e^{i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)}{c^3}+\frac{ib^2\text{Li}_2\left(e^{i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)}{c^3}+\frac{b^2x\left(a+b\csc^{-1}(cx)\right)}{c^2}+\frac{b\tanh^{-1}\left(e^{i\csc^{-1}(cx)}\right.}{}$$

```
[Out] b^2*x*(a+b*arccsc(c*x))/c^2+1/3*x^3*(a+b*arccsc(c*x))^3+b*(a+b*arccsc(c*x))
^2*arctanh(I/c/x+(1-1/c^2/x^2)^(1/2))/c^3+b^3*arctanh((1-1/c^2/x^2)^(1/2))/
c^3-I*b^2*(a+b*arccsc(c*x))*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))/c^3+I*b^2
*(a+b*arccsc(c*x))*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))/c^3+b^3*polylog(3,-
I/c/x-(1-1/c^2/x^2)^(1/2))/c^3-b^3*polylog(3,I/c/x+(1-1/c^2/x^2)^(1/2))/c^3
+1/2*b*x^2*(a+b*arccsc(c*x))^2*(1-1/c^2/x^2)^(1/2)/c
```

Rubi [A] time = 0.19, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5223, 4410, 4186, 3770, 4183, 2531, 2282, 6589}

$$-\frac{ib^2\text{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)}{c^3}+\frac{ib^2\text{PolyLog}\left(2,e^{i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)}{c^3}+\frac{b^3\text{PolyLog}\left(3,-e^{i\csc^{-1}(cx)}\right.}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcCsc[c*x])^3,x]
```

```
[Out] (b^2*x*(a + b*ArcCsc[c*x]))/c^2 + (b*Sqrt[1 - 1/(c^2*x^2)]*x^2*(a + b*ArcCs
c[c*x])^2)/(2*c) + (x^3*(a + b*ArcCsc[c*x])^3)/3 + (b*(a + b*ArcCsc[c*x])^2
*ArcTanh[E^(I*ArcCsc[c*x])])/c^3 + (b^3*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/c^3
 - (I*b^2*(a + b*ArcCsc[c*x])*PolyLog[2, -E^(I*ArcCsc[c*x])])/c^3 + (I*b^2*
(a + b*ArcCsc[c*x])*PolyLog[2, E^(I*ArcCsc[c*x])])/c^3 + (b^3*PolyLog[3, -E
^(I*ArcCsc[c*x])])/c^3 - (b^3*PolyLog[3, E^(I*ArcCsc[c*x])])/c^3
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
```

```
[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5223

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\left(a+b\csc^{-1}(cx)\right)^3\,dx=-\frac{\text{Subst}\left(\int(a+bx)^3\cot(x)\csc^3(x)\,dx,x,\csc^{-1}(cx)\right)}{c^3}$$

$$=\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)^3-\frac{b\,\text{Subst}\left(\int(a+bx)^2\csc^3(x)\,dx,x,\csc^{-1}(cx)\right)}{c^3}$$

$$=\frac{b^2x\left(a+b\csc^{-1}(cx)\right)}{c^2}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^2\left(a+b\csc^{-1}(cx)\right)^2}{2c}+\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)$$

$$=\frac{b^2x\left(a+b\csc^{-1}(cx)\right)}{c^2}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^2\left(a+b\csc^{-1}(cx)\right)^2}{2c}+\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)$$

$$=\frac{b^2x\left(a+b\csc^{-1}(cx)\right)}{c^2}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^2\left(a+b\csc^{-1}(cx)\right)^2}{2c}+\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)$$

$$=\frac{b^2x\left(a+b\csc^{-1}(cx)\right)}{c^2}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^2\left(a+b\csc^{-1}(cx)\right)^2}{2c}+\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)$$

$$=\frac{b^2x\left(a+b\csc^{-1}(cx)\right)}{c^2}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^2\left(a+b\csc^{-1}(cx)\right)^2}{2c}+\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)$$

Mathematica [B] time = 7.82, size = 580, normalized size = 2.64

$$\frac{a^3x^3}{3}+\frac{a^2bx^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{2c}+\frac{a^2b\log\left(x\left(\sqrt{\frac{c^2x^2-1}{c^2x^2}}+1\right)\right)}{2c^3}+a^2bx^3\csc^{-1}(cx)+\frac{ab^2\left(2c^3x^3\left(\frac{4i\text{Li}_2\left(e^{i\csc^{-1}(cx)}\right)}{c^3x^3}+4\csc^{-1}(cx)^2-2\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcCsc[c*x])^3,x]
```

```
[Out] (a^3*x^3)/3 + (a^2*b*x^2*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(2*c) + a^2*b*x^3*
ArcCsc[c*x] + (a^2*b*Log[x*(1 + Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])])/(2*c^3) +
 (a*b^2*((-8*I)*PolyLog[2, -E^(I*ArcCsc[c*x])] + 2*c^3*x^3*(2 + 4*ArcCsc[c*
x]^2 - 2*Cos[2*ArcCsc[c*x]] - (3*ArcCsc[c*x]*Log[1 - E^(I*ArcCsc[c*x])])/(c
*x) + (3*ArcCsc[c*x]*Log[1 + E^(I*ArcCsc[c*x])])/(c*x) + ((4*I)*PolyLog[2,
E^(I*ArcCsc[c*x])])/(c^3*x^3) + 2*ArcCsc[c*x]*Sin[2*ArcCsc[c*x]] + ArcCsc[c
*x]*Log[1 - E^(I*ArcCsc[c*x])]*Sin[3*ArcCsc[c*x]] - ArcCsc[c*x]*Log[1 + E^(
I*ArcCsc[c*x])]*Sin[3*ArcCsc[c*x]])))/(8*c^3) + (b^3*(24*ArcCsc[c*x]*Cot[Ar
cCsc[c*x]/2] + 4*ArcCsc[c*x]^3*Cot[ArcCsc[c*x]/2] + 6*ArcCsc[c*x]^2*Csc[Arc
Csc[c*x]/2]^2 + (ArcCsc[c*x]^3*Csc[ArcCsc[c*x]/2]^4)/(c*x) - 24*ArcCsc[c*x]
^2*Log[1 - E^(I*ArcCsc[c*x])] + 24*ArcCsc[c*x]^2*Log[1 + E^(I*ArcCsc[c*x])]
 - 48*Log[Tan[ArcCsc[c*x]/2]] - (48*I)*ArcCsc[c*x]*PolyLog[2, -E^(I*ArcCsc[
c*x])] + (48*I)*ArcCsc[c*x]*PolyLog[2, E^(I*ArcCsc[c*x])] + 48*PolyLog[3, -
E^(I*ArcCsc[c*x])] - 48*PolyLog[3, E^(I*ArcCsc[c*x])] - 6*ArcCsc[c*x]^2*Sec
[ArcCsc[c*x]/2]^2 + 16*c^3*x^3*ArcCsc[c*x]^3*Sin[ArcCsc[c*x]/2]^4 + 24*ArcC
sc[c*x]*Tan[ArcCsc[c*x]/2] + 4*ArcCsc[c*x]^3*Tan[ArcCsc[c*x]/2]))/(48*c^3)
```

fricas [F] time = 1.92, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3x^2\,\text{arccsc}\,(cx)^3+3\,ab^2x^2\,\text{arccsc}\,(cx)^2+3\,a^2bx^2\,\text{arccsc}\,(cx)+a^3x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^2*arccsc(c*x)^3 + 3*a*b^2*x^2*arccsc(c*x)^2 + 3*a^2*b*x^2*ar
ccsc(c*x) + a^3*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(b\,\text{arccsc}\,(cx)+a)^3x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)^3*x^2, x)
```

maple [B] time = 1.55, size = 648, normalized size = 2.95

$$\frac{a^3x^3}{3}+\frac{x^3b^3\text{arccsc}\,(cx)^3}{3}+\frac{b^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,\text{arccsc}\,(cx)^2\,x^2}{2c}+\frac{b^3\text{arccsc}\,(cx)\,x}{c^2}-\frac{b^3\text{arccsc}\,(cx)^2\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)}{2c^3}+\frac{ib^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsc(c*x))^3,x)
```

```
[Out] 1/3*a^3*x^3+1/3*x^3*b^3*arccsc(c*x)^3+1/2/c*b^3*((c^2*x^2-1)/c^2/x^2)^(1/2)
*arccsc(c*x)^2*x^2+1/c^2*b^3*arccsc(c*x)*x-1/2/c^3*b^3*arccsc(c*x)^2*ln(1-I
/c/x-(1-1/c^2/x^2)^(1/2))+I/c^3*b^3*arccsc(c*x)*polylog(2,I/c/x+(1-1/c^2/x^
```

```
2)^(1/2))-b^3*polylog(3,I/c/x+(1-1/c^2/x^2)^(1/2))/c^3+1/2/c^3*b^3*arccsc(c
*x)^2*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))+I/c^3*a*b^2*polylog(2,I/c/x+(1-1/c^2/
x^2)^(1/2))+b^3*polylog(3,-I/c/x-(1-1/c^2/x^2)^(1/2))/c^3+2/c^3*b^3*arctanh
(I/c/x+(1-1/c^2/x^2)^(1/2))+b^2*x^3*a*arccsc(c*x)^2+1/c*a*b^2*((c^2*x^2-1)/
c^2/x^2)^(1/2)*arccsc(c*x)*x^2-I/c^3*b^3*arccsc(c*x)*polylog(2,-I/c/x-(1-1/
c^2/x^2)^(1/2))-I/c^3*a*b^2*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))-1/c^3*a*b
^2*arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))+1/c^3*a*b^2*arccsc(c*x)*ln(1
+I/c/x+(1-1/c^2/x^2)^(1/2))+a*b^2*x/c^2+x^3*a^2*b*arccsc(c*x)+1/2/c*a^2*b/(
(c^2*x^2-1)/c^2/x^2)^(1/2)*x^2-1/2/c^3*a^2*b/((c^2*x^2-1)/c^2/x^2)^(1/2)+1/
2/c^4*a^2*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*ln(c*x+(c^2*x^2
-1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*x^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 1/4*b^3*x^3*arctan2
(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 - 1/2*a*b^2*c^2*(2*(c^2*x^3
 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5)*log(c)^2 - 12*b^3*c^
2*integrate(1/4*x^4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^2 - 1),
x)*log(c)^2 + 12*b^3*c^2*integrate(1/4*x^4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x
 - 1)))*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) - 24*b^3*c^2*integrate(1/4*x^
4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)/(c^2*x^2 - 1), x)*log(c) +
 12*a*b^2*c^2*integrate(1/4*x^4*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) - 24*
a*b^2*c^2*integrate(1/4*x^4*log(x)/(c^2*x^2 - 1), x)*log(c) + 1/3*a^3*x^3 +
 12*b^3*c^2*integrate(1/4*x^4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c
^2*x^2)*log(x)/(c^2*x^2 - 1), x) - 12*b^3*c^2*integrate(1/4*x^4*arctan(1/(s
qrt(c*x + 1)*sqrt(c*x - 1)))*log(x)^2/(c^2*x^2 - 1), x) + 12*a*b^2*c^2*inte
grate(1/4*x^4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))^2/(c^2*x^2 - 1), x) +
 4*b^3*c^2*integrate(1/4*x^4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^
2*x^2)/(c^2*x^2 - 1), x) - 3*a*b^2*c^2*integrate(1/4*x^4*log(c^2*x^2)^2/(c^
2*x^2 - 1), x) + 12*a*b^2*c^2*integrate(1/4*x^4*log(c^2*x^2)*log(x)/(c^2*x^
2 - 1), x) - 12*a*b^2*c^2*integrate(1/4*x^4*log(x)^2/(c^2*x^2 - 1), x) + 3/
2*a*b^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3)*log(c)^2 + 12*b^3*i
ntegrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^2 - 1), x)*
log(c)^2 - 12*b^3*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))
*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) + 24*b^3*integrate(1/4*x^2*arctan(1/
(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)/(c^2*x^2 - 1), x)*log(c) - 12*a*b^2*i
ntegrate(1/4*x^2*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) + 24*a*b^2*integrate
(1/4*x^2*log(x)/(c^2*x^2 - 1), x)*log(c) + 1/4*(4*x^3*arccsc(c*x) + (2*sqrt
(-1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) +
1) + 1)/c^2 - log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^2)/c)*a^2*b + 4*b^3*integra
te(1/4*sqrt(c*x + 1)*sqrt(c*x - 1)*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1
)))^2/(c^2*x^2 - 1), x) - b^3*integrate(1/4*sqrt(c*x + 1)*sqrt(c*x - 1)*x^2
*log(c^2*x^2)^2/(c^2*x^2 - 1), x) - 12*b^3*integrate(1/4*x^2*arctan(1/(sqrt
(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) + 12*b^3*in
tegrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)^2/(c^2*x^2 -
 1), x) - 12*a*b^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1))
)^2/(c^2*x^2 - 1), x) - 4*b^3*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqr
t(c*x - 1)))*log(c^2*x^2)/(c^2*x^2 - 1), x) + 3*a*b^2*integrate(1/4*x^2*log
(c^2*x^2)^2/(c^2*x^2 - 1), x) - 12*a*b^2*integrate(1/4*x^2*log(c^2*x^2)*log
(x)/(c^2*x^2 - 1), x) + 12*a*b^2*integrate(1/4*x^2*log(x)^2/(c^2*x^2 - 1),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asin(1/(c*x)))^3,x)
```

```
[Out] int(x^2*(a + b*asin(1/(c*x)))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + b \operatorname{acsc}(cx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsc(c*x))**3,x)
```

```
[Out] Integral(x**2*(a + b*acsc(c*x))**3, x)
```

3.26 $\int x \left(a+b \csc^{-1}(cx)\right)^3 \, dx$

Optimal. Leaf size=126

$$-\frac{3b^2 \log\left(1-e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)}{c^2}+\frac{3bx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2}{2c}+\frac{3ib\left(a+b\csc^{-1}(cx)\right)^2}{2c^2}+\frac{1}{2}x^2\left(a-\right.$$

```
[Out] 3/2*I*b*(a+b*arccsc(c*x))^2/c^2+1/2*x^2*(a+b*arccsc(c*x))^3-3*b^2*(a+b*arcc
sc(c*x))*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)/c^2+3/2*I*b^3*polylog(2,(I/c/x
+(1-1/c^2/x^2)^(1/2))^2)/c^2+3/2*b*x*(a+b*arccsc(c*x))^2*(1-1/c^2/x^2)^(1/2
)/c
```

Rubi [A] time = 0.15, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5223, 4410, 4184, 3717, 2190, 2279, 2391}

$$\frac{3ib^3\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2c^2}-\frac{3b^2\log\left(1-e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)}{c^2}+\frac{3bx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2}{2c}+\frac{3ib}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcCsc[c*x])^3,x]
```

```
[Out] (((3*I)/2)*b*(a + b*ArcCsc[c*x])^2)/c^2 + (3*b*Sqrt[1 - 1/(c^2*x^2)]*x*(a +
 b*ArcCsc[c*x])^2)/(2*c) + (x^2*(a + b*ArcCsc[c*x])^3)/2 - (3*b^2*(a + b*Ar
cCsc[c*x])*Log[1 - E^((2*I)*ArcCsc[c*x])])/c^2 + (((3*I)/2)*b^3*PolyLog[2,
E^((2*I)*ArcCsc[c*x])])/c^2
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
```

```
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5223

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\int x\left(a+b\csc^{-1}(cx)\right)^3\,dx=-\frac{\text{Subst}\left(\int(a+bx)^3\cot(x)\csc^2(x)\,dx,x,\csc^{-1}(cx)\right)}{c^2}$$
$$=\frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)^3-\frac{(3b)\,\text{Subst}\left(\int(a+bx)^2\csc^2(x)\,dx,x,\csc^{-1}(cx)\right)}{2c^2}$$
$$=\frac{3b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(a+b\csc^{-1}(cx)\right)^2}{2c}+\frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)^3-\frac{\left(3b^2\right)\text{Subst}\left(\int(a+bx)\cdots\right.}{c^2\cdots}$$
$$=\frac{3ib\left(a+b\csc^{-1}(cx)\right)^2}{2c^2}+\frac{3b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(a+b\csc^{-1}(cx)\right)^2}{2c}+\frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)^3+\cdots$$
$$=\frac{3ib\left(a+b\csc^{-1}(cx)\right)^2}{2c^2}+\frac{3b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(a+b\csc^{-1}(cx)\right)^2}{2c}+\frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)^3-\cdots$$
$$=\frac{3ib\left(a+b\csc^{-1}(cx)\right)^2}{2c^2}+\frac{3b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(a+b\csc^{-1}(cx)\right)^2}{2c}+\frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)^3-\cdots$$
$$=\frac{3ib\left(a+b\csc^{-1}(cx)\right)^2}{2c^2}+\frac{3b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(a+b\csc^{-1}(cx)\right)^2}{2c}+\frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)^3-\cdots$$

Mathematica [**A**] time = 0.49, size = 182, normalized size = 1.44

$$\frac{a\left(acx\left(acx+3b\sqrt{1-\frac{1}{c^2x^2}}\right)-6b^2\log\left(\frac{1}{cx}\right)\right)+3b^2\csc^{-1}(cx)^2\left(ac^2x^2+b\left(cx\sqrt{1-\frac{1}{c^2x^2}}+i\right)\right)+3b\csc^{-1}(cx)\left(acx\left(a\cdots\right.\right.}{2c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcCsc[c*x])^3,x]
```

```
[Out] (3*b^2*(a*c^2*x^2 + b*(I + c*Sqrt[1 - 1/(c^2*x^2)]*x))*ArcCsc[c*x]^2 + b^3*
c^2*x^2*ArcCsc[c*x]^3 + 3*b*ArcCsc[c*x]*(a*c*x*(2*b*Sqrt[1 - 1/(c^2*x^2)] +
 a*c*x) - 2*b^2*Log[1 - E^((2*I)*ArcCsc[c*x])]) + a*(a*c*x*(3*b*Sqrt[1 - 1/
(c^2*x^2)] + a*c*x) - 6*b^2*Log[1/(c*x)]) + (3*I)*b^3*PolyLog[2, E^((2*I)*A
rcCsc[c*x])])/(2*c^2)
```

fricas [**F**] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3x\,\text{arccsc}\,(cx)^3+3\,ab^2x\,\text{arccsc}\,(cx)^2+3\,a^2bx\,\text{arccsc}\,(cx)+a^3x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x*arccsc(c*x)^3 + 3*a*b^2*x*arccsc(c*x)^2 + 3*a^2*b*x*arccsc(c
*x) + a^3*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arccsc}(cx) + a)^3 x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)^3*x, x)
```

maple [B] time = 0.88, size = 349, normalized size = 2.77

$$\frac{x^2a^3}{2}+\frac{x^2b^3\operatorname{arccsc}(cx)^3}{2}+\frac{3b^3\operatorname{arccsc}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}{2c}+\frac{3ib^3\operatorname{arccsc}(cx)^2}{2c^2}-\frac{3b^3\operatorname{arccsc}(cx)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsc(c*x))^3,x)
```

```
[Out] 1/2*x^2*a^3+1/2*x^2*b^3*arccsc(c*x)^3+3/2/c*b^3*arccsc(c*x)^2*((c^2*x^2-1)/
c^2/x^2)^(1/2)*x+3/2*I/c^2*b^3*arccsc(c*x)^2-3/c^2*b^3*arccsc(c*x)*ln(1-I/c
/x-(1-1/c^2/x^2)^(1/2))-3/c^2*b^3*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2
))+3*I/c^2*b^3*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))+3*I/c^2*b^3*polylog(2,
I/c/x+(1-1/c^2/x^2)^(1/2))+3/2*x^2*a*b^2*arccsc(c*x)^2+3/c*a*b^2*((c^2*x^2-
1)/c^2/x^2)^(1/2)*arccsc(c*x)*x-3/c^2*a*b^2*ln(1/c/x)+3/2*a^2*b*x^2*arccsc(
c*x)+3/2/c*a^2*b/((c^2*x^2-1)/c^2/x^2)^(1/2)*x-3/2/c^3*a^2*b/((c^2*x^2-1)/c
^2/x^2)^(1/2)/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3}{2}ab^2x^2\operatorname{arccsc}(cx)^2+\frac{1}{2}a^3x^2+\frac{3}{2}\left(x^2\operatorname{arccsc}(cx)+\frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)a^2b+3\left(\frac{x\sqrt{-\frac{1}{c^2x^2}+1}\operatorname{arccsc}(cx)}{c}+\frac{\log(x)}{c^2}\right)ab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))^3,x, algorithm="maxima")
```

```
[Out] 3/2*a*b^2*x^2*arccsc(c*x)^2 + 1/2*a^3*x^2 + 3/2*(x^2*arccsc(c*x) + x*sqrt(-
1/(c^2*x^2) + 1)/c)*a^2*b + 3*(x*sqrt(-1/(c^2*x^2) + 1)*arccsc(c*x)/c + log
(x)/c^2)*a*b^2 + 1/8*(4*x^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 3*x
^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 - 8*integrate(3/8
*(8*c^2*x^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c)^2 - 8*x*arctan2(
1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c)^2 + 8*(c^2*x^3*arctan2(1, sqrt(c*x +
 1)*sqrt(c*x - 1)) - x*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)^2 -
(4*x*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2 - x*log(c^2*x^2)^2)*sqrt(c*x
 + 1)*sqrt(c*x - 1) - 4*((2*c^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log
(c) + c^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*x^3 - (2*arctan2(1, sqrt
(c*x + 1)*sqrt(c*x - 1))*log(c) + arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*
x + 2*(c^2*x^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) - x*arctan2(1, sqrt(
c*x + 1)*sqrt(c*x - 1)))*log(x))*log(c^2*x^2) + 16*(c^2*x^3*arctan2(1, sqrt
(c*x + 1)*sqrt(c*x - 1))*log(c) - x*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))
*log(c))*log(x))/(c^2*x^2 - 1), x))*b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asin(1/(c*x)))^3,x)
[Out] int(x*(a + b*asin(1/(c*x)))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\,\mathrm{acsc}\left(cx\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsc(c*x))**3,x)
[Out] Integral(x*(a + b*acsc(c*x))**3, x)
```

3.27 $\int \left(a+b\csc^{-1}(cx)\right)^3\,dx$

Optimal. Leaf size=144

$$-\frac{6ib^2\text{Li}_2\left(-e^{i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)}{c}+\frac{6ib^2\text{Li}_2\left(e^{i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)}{c}+x\left(a+b\csc^{-1}(cx)\right)^3+\frac{6b\tanh^{-}}{}$$

```
[Out] x*(a+b*arccsc(c*x))^3+6*b*(a+b*arccsc(c*x))^2*arctanh(I/c/x+(1-1/c^2/x^2)^(
1/2))/c-6*I*b^2*(a+b*arccsc(c*x))*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))/c+6
*I*b^2*(a+b*arccsc(c*x))*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))/c+6*b^3*polyl
og(3,-I/c/x-(1-1/c^2/x^2)^(1/2))/c-6*b^3*polylog(3,I/c/x+(1-1/c^2/x^2)^(1/2
))/c
```

Rubi [A] time = 0.11, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5217, 4410, 4183, 2531, 2282, 6589}

$$-\frac{6ib^2\text{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)}{c}+\frac{6ib^2\text{PolyLog}\left(2,e^{i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)}{c}+\frac{6b^3\text{PolyLog}(}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])^3,x]
```

```
[Out] x*(a + b*ArcCsc[c*x])^3 + (6*b*(a + b*ArcCsc[c*x])^2*ArcTanh[E^(I*ArcCsc[c*
x])])/c - ((6*I)*b^2*(a + b*ArcCsc[c*x])*PolyLog[2, -E^(I*ArcCsc[c*x])])/c
+ ((6*I)*b^2*(a + b*ArcCsc[c*x])*PolyLog[2, E^(I*ArcCsc[c*x])])/c + (6*b^3*
PolyLog[3, -E^(I*ArcCsc[c*x])])/c - (6*b^3*PolyLog[3, E^(I*ArcCsc[c*x])])/c
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5217

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[c^(-1), Sub
st[Int[(a + b*x)^n*Csc[x]*Cot[x], x], x, ArcCsc[c*x]], x] /; FreeQ[{a, b, c
, n}, x] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \left(a+b\csc^{-1}(cx)\right)^3 dx = -\frac{\text{Subst}\left(\int (a+bx)^3\cot(x)\csc(x)\,dx, x, \csc^{-1}(cx)\right)}{c}$$

$$= x\left(a+b\csc^{-1}(cx)\right)^3 - \frac{(3b)\,\text{Subst}\left(\int (a+bx)^2\csc(x)\,dx, x, \csc^{-1}(cx)\right)}{c}$$

$$= x\left(a+b\csc^{-1}(cx)\right)^3 + \frac{6b\left(a+b\csc^{-1}(cx)\right)^2\tanh^{-1}\left(e^{i\csc^{-1}(cx)}\right)}{c} + \frac{\left(6b^2\right)\text{Subst}\left(\int (a+b\right.}{}$$

$$= x\left(a+b\csc^{-1}(cx)\right)^3 + \frac{6b\left(a+b\csc^{-1}(cx)\right)^2\tanh^{-1}\left(e^{i\csc^{-1}(cx)}\right)}{c} - \frac{6ib^2\left(a+b\csc^{-1}(cx)\right.}{c}$$

$$= x\left(a+b\csc^{-1}(cx)\right)^3 + \frac{6b\left(a+b\csc^{-1}(cx)\right)^2\tanh^{-1}\left(e^{i\csc^{-1}(cx)}\right)}{c} - \frac{6ib^2\left(a+b\csc^{-1}(cx)\right.}{c}$$

$$= x\left(a+b\csc^{-1}(cx)\right)^3 + \frac{6b\left(a+b\csc^{-1}(cx)\right)^2\tanh^{-1}\left(e^{i\csc^{-1}(cx)}\right)}{c} - \frac{6ib^2\left(a+b\csc^{-1}(cx)\right.}{c}$$

Mathematica [A] time = 0.30, size = 265, normalized size = 1.84

$$\frac{a^3cx + 3a^2b\log\left(cx\left(\sqrt{1-\frac{1}{c^2x^2}}+1\right)\right) + 3a^2bcx\csc^{-1}(cx) - 6ib^2\text{Li}_2\left(-e^{i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right) + 6ib^2\text{Li}_2\left(e^{i\csc\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsc[c*x])^3,x]
```

```
[Out] (a^3*c*x + 3*a^2*b*c*x*ArcCsc[c*x] + 3*a*b^2*c*x*ArcCsc[c*x]^2 + b^3*c*x*Ar
cCsc[c*x]^3 - 6*a*b^2*ArcCsc[c*x]*Log[1 - E^(I*ArcCsc[c*x])] - 3*b^3*ArcCsc
[c*x]^2*Log[1 - E^(I*ArcCsc[c*x])] + 6*a*b^2*ArcCsc[c*x]*Log[1 + E^(I*ArcCs
c[c*x])] + 3*b^3*ArcCsc[c*x]^2*Log[1 + E^(I*ArcCsc[c*x])] + 3*a^2*b*Log[c*(
1 + Sqrt[1 - 1/(c^2*x^2)])*x] - (6*I)*b^2*(a + b*ArcCsc[c*x])*PolyLog[2, -E
^(I*ArcCsc[c*x])] + (6*I)*b^2*(a + b*ArcCsc[c*x])*PolyLog[2, E^(I*ArcCsc[c*
x])] + 6*b^3*PolyLog[3, -E^(I*ArcCsc[c*x])] - 6*b^3*PolyLog[3, E^(I*ArcCsc[
c*x])])/c
```

fricas [F] time = 2.19, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3\,\text{arccsc}\,(cx)^3 + 3\,ab^2\,\text{arccsc}\,(cx)^2 + 3\,a^2b\,\text{arccsc}\,(cx) + a^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*arccsc(c*x)^3 + 3*a*b^2*arccsc(c*x)^2 + 3*a^2*b*arccsc(c*x) +
a^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arccsc}(cx) + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)^3, x)
```

maple [B] time = 0.76, size = 437, normalized size = 3.03

$$x\,b^3\operatorname{arccsc}(cx)^3+3xa\,b^2\operatorname{arccsc}(cx)^2+\frac{3b^3\operatorname{arccsc}(cx)^2\ln\left(1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}-\frac{3b^3\operatorname{arccsc}(cx)^2\ln\left(1-\frac{i}{cx}-\sqrt{1\right.}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))^3,x)
```

```
[Out] x*b^3*arccsc(c*x)^3+3*x*a*b^2*arccsc(c*x)^2+3/c*b^3*arccsc(c*x)^2*ln(1+I/c/
x+(1-1/c^2/x^2)^(1/2))-3/c*b^3*arccsc(c*x)^2*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2)
)-6*I/c*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))*arccsc(c*x)*b^3+6*I/c*b^3*arc
csc(c*x)*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))+3*x*a^2*b*arccsc(c*x)+6*I/c*d
ilog(1-I/c/x-(1-1/c^2/x^2)^(1/2))*a*b^2-6*I/c*dilog(1+I/c/x+(1-1/c^2/x^2)^(
1/2))*a*b^2+6/c*a*b^2*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))-6/c*a*b^2
*arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))+x*a^3+6*b^3*polylog(3,-I/c/x-(
1-1/c^2/x^2)^(1/2))/c-6*b^3*polylog(3,I/c/x+(1-1/c^2/x^2)^(1/2))/c+3/c*ln(c
*x+c*x*(1-1/c^2/x^2)^(1/2))*a^2*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3,x, algorithm="maxima")
```

```
[Out] -3/2*a*b^2*c^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3)*log(c)^2 - 1
2*b^3*c^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^
2 - 1), x)*log(c)^2 + b^3*x*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 3/4
*b^3*x*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 + 12*b^3*c^2*
integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)/(c^2
*x^2 - 1), x)*log(c) - 24*b^3*c^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)
*sqrt(c*x - 1)))*log(x)/(c^2*x^2 - 1), x)*log(c) + 12*a*b^2*c^2*integrate(1
/4*x^2*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) - 24*a*b^2*c^2*integrate(1/4*x
^2*log(x)/(c^2*x^2 - 1), x)*log(c) + 12*b^3*c^2*integrate(1/4*x^2*arctan(1/
(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) - 12*b
^3*c^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)^2/(
c^2*x^2 - 1), x) + 12*a*b^2*c^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*s
qrt(c*x - 1)))^2/(c^2*x^2 - 1), x) + 12*b^3*c^2*integrate(1/4*x^2*arctan(1/
(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)/(c^2*x^2 - 1), x) - 3*a*b^2*c^2
*integrate(1/4*x^2*log(c^2*x^2)^2/(c^2*x^2 - 1), x) + 12*a*b^2*c^2*integrat
e(1/4*x^2*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) - 12*a*b^2*c^2*integrate(1/
4*x^2*log(x)^2/(c^2*x^2 - 1), x) - 3/2*a*b^2*(log(c*x + 1)/c - log(c*x - 1)
/c)*log(c)^2 + 12*b^3*integrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))
/(c^2*x^2 - 1), x)*log(c)^2 - 12*b^3*integrate(1/4*arctan(1/(sqrt(c*x + 1)*
sqrt(c*x - 1)))*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) + 24*b^3*integrate(1/
4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)/(c^2*x^2 - 1), x)*log(c) -
 12*a*b^2*integrate(1/4*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) + 24*a*b^2*in
tegrate(1/4*log(x)/(c^2*x^2 - 1), x)*log(c) + a^3*x + 12*b^3*integrate(1/4*
```

```
sqrt(c*x + 1)*sqrt(c*x - 1)*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))^2/(c^2*
x^2 - 1), x) - 3*b^3*integrate(1/4*sqrt(c*x + 1)*sqrt(c*x - 1)*log(c^2*x^2)
^2/(c^2*x^2 - 1), x) - 12*b^3*integrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt(c*
x - 1)))*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) + 12*b^3*integrate(1/4*arcta
n(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)^2/(c^2*x^2 - 1), x) - 12*a*b^2*in
tegrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))^2/(c^2*x^2 - 1), x) - 1
2*b^3*integrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)/(c
^2*x^2 - 1), x) + 3*a*b^2*integrate(1/4*log(c^2*x^2)^2/(c^2*x^2 - 1), x) -
12*a*b^2*integrate(1/4*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) + 12*a*b^2*int
egrate(1/4*log(x)^2/(c^2*x^2 - 1), x) + 3/2*(2*c*x*arccsc(c*x) + log(sqrt(-
1/(c^2*x^2) + 1) + 1) - log(-sqrt(-1/(c^2*x^2) + 1) + 1))*a^2*b/c
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))^3,x)
```

```
[Out] int((a + b*asin(1/(c*x)))^3, x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{acsc}{\left(c x \right)}\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))**3,x)
```

```
[Out] Integral((a + b*acsc(c*x))**3, x)
```

3.28 $\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x}\,dx$

Optimal. Leaf size=124

$$-\frac{3}{2}b^2\text{Li}_3\left(e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)+\frac{3}{2}ib\text{Li}_2\left(e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)^2+\frac{i\left(a+b\csc^{-1}(cx)\right)^4}{4b}-\log\left(1-e^{2}\right.$$

```
[Out] 1/4*I*(a+b*arccsc(c*x))^4/b-(a+b*arccsc(c*x))^3*ln(1-(I/c/x+(1-1/c^2/x^2)^(
1/2))^2)+3/2*I*b*(a+b*arccsc(c*x))^2*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^
2)-3/2*b^2*(a+b*arccsc(c*x))*polylog(3,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)-3/4*I
*b^3*polylog(4,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)
```

Rubi [A] time = 0.14, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5223, 3717, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3}{2}b^2\text{PolyLog}\left(3,e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)+\frac{3}{2}ib\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)^2-\frac{3}{4}ib^3\text{PolyLog}\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])^3/x,x]
```

```
[Out] ((I/4)*(a + b*ArcCsc[c*x])^4)/b - (a + b*ArcCsc[c*x])^3*Log[1 - E^((2*I)*Ar
cCsc[c*x])] + ((3*I)/2)*b*(a + b*ArcCsc[c*x])^2*PolyLog[2, E^((2*I)*ArcCsc[
c*x])] - (3*b^2*(a + b*ArcCsc[c*x])*PolyLog[3, E^((2*I)*ArcCsc[c*x])])/2 -
((3*I)/4)*b^3*PolyLog[4, E^((2*I)*ArcCsc[c*x])]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 5223

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x}\,dx &= -\text{Subst}\left(\int (a+bx)^3\cot(x)\,dx, x, \csc^{-1}(cx)\right)\\
&= \frac{i\left(a+b\csc^{-1}(cx)\right)^4}{4b}+2i\,\text{Subst}\left(\int \frac{e^{2ix}(a+bx)^3}{1-e^{2ix}}\,dx, x, \csc^{-1}(cx)\right)\\
&= \frac{i\left(a+b\csc^{-1}(cx)\right)^4}{4b}-\left(a+b\csc^{-1}(cx)\right)^3\log\left(1-e^{2i\csc^{-1}(cx)}\right)+(3b)\,\text{Subst}\left(\int (a+bx)^2\right.\\
&= \frac{i\left(a+b\csc^{-1}(cx)\right)^4}{4b}-\left(a+b\csc^{-1}(cx)\right)^3\log\left(1-e^{2i\csc^{-1}(cx)}\right)+\frac{3}{2}ib\left(a+b\csc^{-1}(cx)\right)^2\,\text{L}\\
&= \frac{i\left(a+b\csc^{-1}(cx)\right)^4}{4b}-\left(a+b\csc^{-1}(cx)\right)^3\log\left(1-e^{2i\csc^{-1}(cx)}\right)+\frac{3}{2}ib\left(a+b\csc^{-1}(cx)\right)^2\,\text{L}\\
&= \frac{i\left(a+b\csc^{-1}(cx)\right)^4}{4b}-\left(a+b\csc^{-1}(cx)\right)^3\log\left(1-e^{2i\csc^{-1}(cx)}\right)+\frac{3}{2}ib\left(a+b\csc^{-1}(cx)\right)^2\,\text{L}\\
&= \frac{i\left(a+b\csc^{-1}(cx)\right)^4}{4b}-\left(a+b\csc^{-1}(cx)\right)^3\log\left(1-e^{2i\csc^{-1}(cx)}\right)+\frac{3}{2}ib\left(a+b\csc^{-1}(cx)\right)^2\,\text{L}
\end{aligned}$$

Mathematica [A] time = 0.23, size = 242, normalized size = 1.95

$$a^3\log(cx)+\frac{3}{2}ia^2b\left(\text{Li}_2\left(e^{2i\csc^{-1}(cx)}\right)+\csc^{-1}(cx)\left(\csc^{-1}(cx)+2i\log\left(1-e^{2i\csc^{-1}(cx)}\right)\right)\right)+\frac{1}{8}iab^2\left(-24\csc^{-1}(cx)\text{Li}_2\left(e^{-}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsc[c*x])^3/x,x]
```

```
[Out] a^3*Log[c*x] + ((3*I)/2)*a^2*b*(ArcCsc[c*x]*(ArcCsc[c*x] + (2*I)*Log[1 - E^
((2*I)*ArcCsc[c*x])]) + PolyLog[2, E^((2*I)*ArcCsc[c*x])]) + (I/8)*a*b^2*(P
i^3 - 8*ArcCsc[c*x]^3 + (24*I)*ArcCsc[c*x]^2*Log[1 - E^((-2*I)*ArcCsc[c*x])
] - 24*ArcCsc[c*x]*PolyLog[2, E^((-2*I)*ArcCsc[c*x])] + (12*I)*PolyLog[3, E
^((-2*I)*ArcCsc[c*x])]) + (I/64)*b^3*(Pi^4 - 16*ArcCsc[c*x]^4 + (64*I)*ArcC
sc[c*x]^3*Log[1 - E^((-2*I)*ArcCsc[c*x])] - 96*ArcCsc[c*x]^2*PolyLog[2, E^(
```

```
(-2*I)*ArcCsc[c*x])] + (96*I)*ArcCsc[c*x]*PolyLog[3, E^((-2*I)*ArcCsc[c*x])
] + 48*PolyLog[4, E^((-2*I)*ArcCsc[c*x])])
```

fricas [F] time = 2.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\,\mathrm{arccsc}\,(cx)^3 + 3\,ab^2\,\mathrm{arccsc}\,(cx)^2 + 3\,a^2b\,\mathrm{arccsc}\,(cx) + a^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arccsc(c*x)^3 + 3*a*b^2*arccsc(c*x)^2 + 3*a^2*b*arccsc(c*x) +
 a^3)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arccsc}\,(cx) + a)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)^3/x, x)
```

maple [B] time = 0.26, size = 666, normalized size = 5.37

$$a^3\ln(cx)+3ia^2b\,\mathrm{polylog}\left(2, \frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)-b^3\mathrm{arccsc}\,(cx)^3\ln\left(1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)+6ia\,b^2\mathrm{arccsc}\,(cx)\,\mathrm{po}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))^3/x,x)
```

```
[Out] a^3*ln(c*x)-6*I*b^3*polylog(4,-I/c/x-(1-1/c^2/x^2)^(1/2))-b^3*arccsc(c*x)^3
*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))+1/4*I*b^3*arccsc(c*x)^4-6*b^3*arccsc(c*x)*
polylog(3,-I/c/x-(1-1/c^2/x^2)^(1/2))+I*a*b^2*arccsc(c*x)^3-b^3*arccsc(c*x)
^3*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))+3*I*b^3*arccsc(c*x)^2*polylog(2,-I/c/x-(
1-1/c^2/x^2)^(1/2))-6*b^3*arccsc(c*x)*polylog(3,I/c/x+(1-1/c^2/x^2)^(1/2))+
6*I*a*b^2*arccsc(c*x)*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))+3*I*a^2*b*polyl
og(2,-I/c/x-(1-1/c^2/x^2)^(1/2))-3*a*b^2*arccsc(c*x)^2*ln(1-I/c/x-(1-1/c^2/
x^2)^(1/2))+3/2*I*a^2*b*arccsc(c*x)^2-3*a*b^2*arccsc(c*x)^2*ln(1+I/c/x+(1-1
/c^2/x^2)^(1/2))-6*I*b^3*polylog(4,I/c/x+(1-1/c^2/x^2)^(1/2))-6*a*b^2*polyl
og(3,-I/c/x-(1-1/c^2/x^2)^(1/2))-6*a*b^2*polylog(3,I/c/x+(1-1/c^2/x^2)^(1/2
))+3*I*b^3*arccsc(c*x)^2*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))-3*a^2*b*arccs
c(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))-3*a^2*b*arccsc(c*x)*ln(1+I/c/x+(1-1/
c^2/x^2)^(1/2))+3*I*a^2*b*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))+6*I*a*b^2*ar
ccsc(c*x)*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3/x,x, algorithm="maxima")
```

```
[Out] -3/2*a*b^2*c^2*(log(c*x + 1)/c^2 + log(c*x - 1)/c^2)*log(c)^2 - 12*b^3*c^2*
integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^3 - x), x)
*log(c)^2 + 12*b^3*c^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x -
 1)))*log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) - 24*b^3*c^2*integrate(1/4*x^2*
```

```
arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)/(c^2*x^3 - x), x)*log(c) + 1
2*a*b^2*c^2*integrate(1/4*x^2*log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) - 24*a*
b^2*c^2*integrate(1/4*x^2*log(x)/(c^2*x^3 - x), x)*log(c) + b^3*arctan2(1,
sqrt(c*x + 1)*sqrt(c*x - 1))^3*log(x) - 3/4*b^3*arctan2(1, sqrt(c*x + 1)*sq
rt(c*x - 1))*log(c^2*x^2)^2*log(x) + 24*b^3*c^2*integrate(1/4*x^2*arctan(1/
(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)*log(x)/(c^2*x^3 - x), x) - 12*b
^3*c^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)^2/(
c^2*x^3 - x), x) + 12*a*b^2*c^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*s
qrt(c*x - 1)))^2/(c^2*x^3 - x), x) - 3*a*b^2*c^2*integrate(1/4*x^2*log(c^2*
x^2)^2/(c^2*x^3 - x), x) + 12*a*b^2*c^2*integrate(1/4*x^2*log(c^2*x^2)*log(
x)/(c^2*x^3 - x), x) - 12*a*b^2*c^2*integrate(1/4*x^2*log(x)^2/(c^2*x^3 - x
), x) + 12*a^2*b*c^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1
)))/(c^2*x^3 - x), x) + 3/2*a*b^2*(log(c*x + 1) + log(c*x - 1) - 2*log(x))*
log(c)^2 + 12*b^3*integrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^
2*x^3 - x), x)*log(c)^2 - 12*b^3*integrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt
(c*x - 1)))*log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) + 24*b^3*integrate(1/4*ar
ctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)/(c^2*x^3 - x), x)*log(c) - 12*
a*b^2*integrate(1/4*log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) + 24*a*b^2*integr
ate(1/4*log(x)/(c^2*x^3 - x), x)*log(c) + 12*b^3*integrate(1/4*sqrt(c*x + 1
)*sqrt(c*x - 1)*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))^2*log(x)/(c^2*x^3 -
 x), x) - 3*b^3*integrate(1/4*sqrt(c*x + 1)*sqrt(c*x - 1)*log(c^2*x^2)^2*lo
g(x)/(c^2*x^3 - x), x) - 24*b^3*integrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt(
c*x - 1)))*log(c^2*x^2)*log(x)/(c^2*x^3 - x), x) + 12*b^3*integrate(1/4*arc
tan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)^2/(c^2*x^3 - x), x) - 12*a*b^2*
integrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))^2/(c^2*x^3 - x), x) +
 3*a*b^2*integrate(1/4*log(c^2*x^2)^2/(c^2*x^3 - x), x) - 12*a*b^2*integrat
e(1/4*log(c^2*x^2)*log(x)/(c^2*x^3 - x), x) + 12*a*b^2*integrate(1/4*log(x)
^2/(c^2*x^3 - x), x) - 12*a^2*b*integrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt(
c*x - 1)))/(c^2*x^3 - x), x) + a^3*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))^3/x,x)
[Out] int((a + b*asin(1/(c*x)))^3/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{acsc}{\left(cx \right)}\right)^{3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))**3/x,x)
[Out] Integral((a + b*acsc(c*x))**3/x, x)
```

3.29 $\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x^2}\,dx$

Optimal. Leaf size=80

$$\frac{6b^2\left(a+b\csc^{-1}(cx)\right)}{x}-3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2-\frac{\left(a+b\csc^{-1}(cx)\right)^3}{x}+6b^3c\sqrt{1-\frac{1}{c^2x^2}}$$

```
[Out] 6*b^2*(a+b*arccsc(c*x))/x-(a+b*arccsc(c*x))^3/x+6*b^3*c*(1-1/c^2/x^2)^(1/2)
-3*b*c*(a+b*arccsc(c*x))^2*(1-1/c^2/x^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5223, 3296, 2638}

$$\frac{6b^2\left(a+b\csc^{-1}(cx)\right)}{x}-3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2-\frac{\left(a+b\csc^{-1}(cx)\right)^3}{x}+6b^3c\sqrt{1-\frac{1}{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])^3/x^2,x]
```

```
[Out] 6*b^3*c*Sqrt[1 - 1/(c^2*x^2)] + (6*b^2*(a + b*ArcCsc[c*x]))/x - 3*b*c*Sqrt[
1 - 1/(c^2*x^2)]*(a + b*ArcCsc[c*x])^2 - (a + b*ArcCsc[c*x])^3/x
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5223

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x^2}\,dx &= -\left(c\,\text{Subst}\left(\int (a+bx)^3\cos(x)\,dx,x,\csc^{-1}(cx)\right)\right)\\
&= -\frac{\left(a+b\csc^{-1}(cx)\right)^3}{x}+(3bc)\,\text{Subst}\left(\int (a+bx)^2\sin(x)\,dx,x,\csc^{-1}(cx)\right)\\
&= -3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2-\frac{\left(a+b\csc^{-1}(cx)\right)^3}{x}+\left(6b^2c\right)\text{Subst}\left(\int (a+bx\right.\\
&= \frac{6b^2\left(a+b\csc^{-1}(cx)\right)}{x}-3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2-\frac{\left(a+b\csc^{-1}(cx)\right)^3}{x}-(6\\
&= 6b^3c\sqrt{1-\frac{1}{c^2x^2}}+\frac{6b^2\left(a+b\csc^{-1}(cx)\right)}{x}-3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2-\frac{(a+}{}
\end{aligned}$$

Mathematica [A] time = 0.21, size = 135, normalized size = 1.69

$$-\frac{a^3+3b\csc^{-1}(cx)\left(a^2+2abcx\sqrt{1-\frac{1}{c^2x^2}}-2b^2\right)+3a^2bcx\sqrt{1-\frac{1}{c^2x^2}}+3b^2\csc^{-1}(cx)^2\left(a+bcx\sqrt{1-\frac{1}{c^2x^2}}\right)-6ab^2}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])^3/x^2,x]
```

```
[Out] -((a^3 - 6*a*b^2 + 3*a^2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x - 6*b^3*c*Sqrt[1 - 1/(
c^2*x^2)]*x + 3*b*(a^2 - 2*b^2 + 2*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x)*ArcCsc[c*
x] + 3*b^2*(a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x)*ArcCsc[c*x]^2 + b^3*ArcCsc[c*x
]^3)/x)
```

fricas [A] time = 1.11, size = 98, normalized size = 1.22

$$-\frac{b^3\operatorname{arccsc}(cx)^3+3\,ab^2\operatorname{arccsc}(cx)^2+a^3-6\,ab^2+3\left(a^2b-2\,b^3\right)\operatorname{arccsc}(cx)+3\left(b^3\operatorname{arccsc}(cx)^2+2\,ab^2\operatorname{arccsc}(\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3/x^2,x, algorithm="fricas")
```

```
[Out] -(b^3*arccsc(c*x)^3 + 3*a*b^2*arccsc(c*x)^2 + a^3 - 6*a*b^2 + 3*(a^2*b - 2*
b^3)*arccsc(c*x) + 3*(b^3*arccsc(c*x)^2 + 2*a*b^2*arccsc(c*x) + a^2*b - 2*b
^3)*sqrt(c^2*x^2 - 1))/x
```

giac [B] time = 0.19, size = 195, normalized size = 2.44

$$-\left(3\,b^3\sqrt{-\frac{1}{c^2x^2}+1}\,\arcsin\left(\frac{1}{cx}\right)^2+6\,ab^2\sqrt{-\frac{1}{c^2x^2}+1}\,\arcsin\left(\frac{1}{cx}\right)+\frac{b^3\arcsin\left(\frac{1}{cx}\right)^3}{cx}+3\,a^2b\sqrt{-\frac{1}{c^2x^2}+1}-6\,b^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3/x^2,x, algorithm="giac")
```

```
[Out] -(3*b^3*sqrt(-1/(c^2*x^2) + 1)*arcsin(1/(c*x))^2 + 6*a*b^2*sqrt(-1/(c^2*x^2
) + 1)*arcsin(1/(c*x)) + b^3*arcsin(1/(c*x))^3/(c*x) + 3*a^2*b*sqrt(-1/(c^2
*x^2) + 1) - 6*b^3*sqrt(-1/(c^2*x^2) + 1) + 3*a*b^2*arcsin(1/(c*x))^2/(c*x)
 + 3*a^2*b*arcsin(1/(c*x))/(c*x) - 6*b^3*arcsin(1/(c*x))/(c*x) + a^3/(c*x)
- 6*a*b^2/(c*x))*c
```

maple [B] time = 0.22, size = 199, normalized size = 2.49

$$c\left(-\frac{a^3}{cx}+b^3\left(-\frac{\operatorname{arccsc}(cx)^3}{cx}-3\operatorname{arccsc}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}+6\sqrt{\frac{c^2x^2-1}{c^2x^2}}+\frac{6\operatorname{arccsc}(cx)}{cx}\right)+3a\,b^2\left(-\frac{\operatorname{arccsc}(cx)^2}{cx}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))^3/x^2,x)
```

```
[Out] c*(-a^3/c/x+b^3*(-1/c/x*arccsc(c*x)^3-3*arccsc(c*x)^2*((c^2*x^2-1)/c^2/x^2)
^(1/2)+6*((c^2*x^2-1)/c^2/x^2)^(1/2)+6/c/x*arccsc(c*x))+3*a*b^2*(-1/c/x*arc
csc(c*x)^2+2/c/x-2*arccsc(c*x)*((c^2*x^2-1)/c^2/x^2)^(1/2))+3*a^2*b*(-1/c/x
*arccsc(c*x)-1/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^2/x^2*(c^2*x^2-1)))
```

maxima [A] time = 0.69, size = 147, normalized size = 1.84

$$-\frac{b^3\operatorname{arccsc}(cx)^3}{x}-3\left(c\sqrt{-\frac{1}{c^2x^2}+1}+\frac{\operatorname{arccsc}(cx)}{x}\right)a^2b-6\left(c\sqrt{-\frac{1}{c^2x^2}+1}\,\operatorname{arccsc}(cx)-\frac{1}{x}\right)ab^2-3\left(c\sqrt{-\frac{1}{c^2x^2}+1}\,\operatorname{ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3/x^2,x, algorithm="maxima")
```

```
[Out] -b^3*arccsc(c*x)^3/x - 3*(c*sqrt(-1/(c^2*x^2) + 1) + arccsc(c*x)/x)*a^2*b -
 6*(c*sqrt(-1/(c^2*x^2) + 1)*arccsc(c*x) - 1/x)*a*b^2 - 3*(c*sqrt(-1/(c^2*x
^2) + 1)*arccsc(c*x)^2 - 2*c*sqrt(-1/(c^2*x^2) + 1) - 2*arccsc(c*x)/x)*b^3
- 3*a*b^2*arccsc(c*x)^2/x - a^3/x
```

mupad [B] time = 0.79, size = 155, normalized size = 1.94

$$\frac{b^3\left(6\,\mathrm{asin}\left(\frac{1}{c\,x}\right)-\mathrm{asin}\left(\frac{1}{c\,x}\right)^3\right)}{x}-\frac{a^3}{x}-3\,a^2\,b\,c\left(\sqrt{1-\frac{1}{c^2\,x^2}}+\frac{\mathrm{asin}\left(\frac{1}{c\,x}\right)}{c\,x}\right)-b^3\,c\,\sqrt{1-\frac{1}{c^2\,x^2}}\left(3\,\mathrm{asin}\left(\frac{1}{c\,x}\right)^2-6\right)-3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))^3/x^2,x)
```

```
[Out] (b^3*(6*asin(1/(c*x)) - asin(1/(c*x))^3))/x - a^3/x - 3*a^2*b*c*((1 - 1/(c^
2*x^2))^(1/2) + asin(1/(c*x))/(c*x)) - b^3*c*(1 - 1/(c^2*x^2))^(1/2)*(3*asi
n(1/(c*x))^2 - 6) - 3*a*b^2*c*(2*asin(1/(c*x))*(1 - 1/(c^2*x^2))^(1/2) + (a
sin(1/(c*x))^2 - 2)/(c*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{acsc}\,(cx)\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))**3/x**2,x)
```

```
[Out] Integral((a + b*acsc(c*x))**3/x**2, x)
```

3.30 $\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x^3}\,dx$

Optimal. Leaf size=125

$$\frac{3b^2\left(a+b\csc^{-1}(cx)\right)}{4x^2}-\frac{3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2}{4x}+\frac{1}{4}c^2\left(a+b\csc^{-1}(cx)\right)^3-\frac{\left(a+b\csc^{-1}(cx)\right)^3}{2x^2}+\frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{8x}$$

```
[Out] -3/8*b^3*c^2*arccsc(c*x)+3/4*b^2*(a+b*arccsc(c*x))/x^2+1/4*c^2*(a+b*arccsc(
c*x))^3-1/2*(a+b*arccsc(c*x))^3/x^2+3/8*b^3*c*(1-1/c^2/x^2)^(1/2)/x-3/4*b*c
*(a+b*arccsc(c*x))^2*(1-1/c^2/x^2)^(1/2)/x
```

Rubi [A] time = 0.10, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5223, 4404, 3311, 32, 2635, 8}

$$\frac{3b^2\left(a+b\csc^{-1}(cx)\right)}{4x^2}-\frac{3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2}{4x}+\frac{1}{4}c^2\left(a+b\csc^{-1}(cx)\right)^3-\frac{\left(a+b\csc^{-1}(cx)\right)^3}{2x^2}+\frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{8x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])^3/x^3,x]
```

```
[Out] (3*b^3*c*Sqrt[1 - 1/(c^2*x^2)])/(8*x) - (3*b^3*c^2*ArcCsc[c*x])/8 + (3*b^2*
(a + b*ArcCsc[c*x]))/(4*x^2) - (3*b*c*Sqrt[1 - 1/(c^2*x^2)]*(a + b*ArcCsc[c
*x])^2)/(4*x) + (c^2*(a + b*ArcCsc[c*x])^3)/4 - (a + b*ArcCsc[c*x])^3/(2*x^
2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5223

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x^3}\,dx = -\left(c^2\,\text{Subst}\left(\int (a+bx)^3\cos(x)\sin(x)\,dx, x, \csc^{-1}(cx)\right)\right)$$

$$= -\frac{\left(a+b\csc^{-1}(cx)\right)^3}{2x^2} + \frac{1}{2}\left(3bc^2\right)\text{Subst}\left(\int (a+bx)^2\sin^2(x)\,dx, x, \csc^{-1}(cx)\right)$$

$$= \frac{3b^2\left(a+b\csc^{-1}(cx)\right)}{4x^2} - \frac{3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2}{4x} - \frac{\left(a+b\csc^{-1}(cx)\right)^3}{2x^2} + \frac{1}{4}\left(\right.$$

$$= \frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{8x} + \frac{3b^2\left(a+b\csc^{-1}(cx)\right)}{4x^2} - \frac{3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2}{4x} + \frac{1}{4}c^2\left(\right.$$

$$= \frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{8x} - \frac{3}{8}b^3c^2\csc^{-1}(cx) + \frac{3b^2\left(a+b\csc^{-1}(cx)\right)}{4x^2} - \frac{3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\,\text{cs}\right.}{4x}$$

Mathematica [A] time = 0.22, size = 186, normalized size = 1.49

$$\frac{-4a^3-3bc^2x^2\left(b^2-2a^2\right)\sin^{-1}\left(\frac{1}{cx}\right)+6b\csc^{-1}(cx)\left(-2a^2-2abcx\sqrt{1-\frac{1}{c^2x^2}}+b^2\right)-6a^2bcx\sqrt{1-\frac{1}{c^2x^2}}-6b^2\,\text{cs}}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])^3/x^3,x]
```

```
[Out] (-4*a^3 + 6*a*b^2 - 6*a^2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x + 3*b^3*c*Sqrt[1 - 1/
(c^2*x^2)]*x + 6*b*(-2*a^2 + b^2 - 2*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x)*ArcCsc[
c*x] - 6*b^2*(b*c*Sqrt[1 - 1/(c^2*x^2)]*x + a*(2 - c^2*x^2))*ArcCsc[c*x]^2
+ 2*b^3*(-2 + c^2*x^2)*ArcCsc[c*x]^3 - 3*b*(-2*a^2 + b^2)*c^2*x^2*ArcSin[1/
(c*x)])/(8*x^2)
```

fricas [A] time = 1.56, size = 150, normalized size = 1.20

$$\frac{2\left(b^3c^2x^2-2\,b^3\right)\text{arccsc}\,(cx)^3-4\,a^3+6\,ab^2+6\left(ab^2c^2x^2-2\,ab^2\right)\text{arccsc}\,(cx)^2+3\left(\left(2\,a^2b-b^3\right)c^2x^2-4\,a^2b+\right.}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(b^3*c^2*x^2 - 2*b^3)*arccsc(c*x)^3 - 4*a^3 + 6*a*b^2 + 6*(a*b^2*c^2
*x^2 - 2*a*b^2)*arccsc(c*x)^2 + 3*((2*a^2*b - b^3)*c^2*x^2 - 4*a^2*b + 2*b^
3)*arccsc(c*x) - 3*(2*b^3*arccsc(c*x)^2 + 4*a*b^2*arccsc(c*x) + 2*a^2*b - b
^3)*sqrt(c^2*x^2 - 1))/x^2
```

giac [B] time = 0.18, size = 302, normalized size = 2.42

$$-\frac{1}{8}\left(4\,b^3c\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)^3+12\,ab^2c\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)^2+2\,b^3c\arcsin\left(\frac{1}{cx}\right)^3+12\,a^2bc\left(\frac{1}{c^2x^2}-1\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3/x^3,x, algorithm="giac")
```

```
[Out] -1/8*(4*b^3*c*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))^3 + 12*a*b^2*c*(1/(c^2*x^2)
 - 1)*arcsin(1/(c*x))^2 + 2*b^3*c*arcsin(1/(c*x))^3 + 12*a^2*b*c*(1/(c^2*x^
2) - 1)*arcsin(1/(c*x)) - 6*b^3*c*(1/(c^2*x^2) - 1)*arcsin(1/(c*x)) + 6*a*b
^2*c*arcsin(1/(c*x))^2 + 4*a^3*c*(1/(c^2*x^2) - 1) - 6*a*b^2*c*(1/(c^2*x^2)
 - 1) + 6*a^2*b*c*arcsin(1/(c*x)) - 3*b^3*c*arcsin(1/(c*x)) + 6*b^3*sqrt(-1
/(c^2*x^2) + 1)*arcsin(1/(c*x))^2/x - 3*a*b^2*c + 12*a*b^2*sqrt(-1/(c^2*x^2
) + 1)*arcsin(1/(c*x))/x + 6*a^2*b*sqrt(-1/(c^2*x^2) + 1)/x - 3*b^3*sqrt(-1
/(c^2*x^2) + 1)/x)*c
```

maple [B] time = 0.38, size = 315, normalized size = 2.52

$$-\frac{a^3}{2x^2}+\frac{c^2b^3\operatorname{arccsc}(cx)^3}{4}-\frac{b^3\operatorname{arccsc}(cx)^3}{2x^2}-\frac{3c\,b^3\operatorname{arccsc}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{4x}-\frac{3b^3c^2\operatorname{arccsc}(cx)}{8}+\frac{3b^3\operatorname{arccsc}(cx)}{4x^2}+\frac{3c\,b^3\sqrt{\frac{c^2}{}}}{8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))^3/x^3,x)
```

```
[Out] -1/2*a^3/x^2+1/4*c^2*b^3*arccsc(c*x)^3-1/2*b^3*arccsc(c*x)^3/x^2-3/4*c*b^3*
arccsc(c*x)^2/x*((c^2*x^2-1)/c^2/x^2)^(1/2)-3/8*b^3*c^2*arccsc(c*x)+3/4*b^3
/x^2*arccsc(c*x)+3/8*c*b^3*((c^2*x^2-1)/c^2/x^2)^(1/2)/x+3/4*c^2*a*b^2*arcc
sc(c*x)^2-3/2*a*b^2*arccsc(c*x)^2/x^2-3/2*c*a*b^2*arccsc(c*x)/x*((c^2*x^2-1
)/c^2/x^2)^(1/2)+3/4*a*b^2/x^2-3/2*a^2*b/x^2*arccsc(c*x)+3/4*c*a^2*b*(c^2*x
^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*arctan(1/(c^2*x^2-1)^(1/2))-3/4*c
*a^2*b/((c^2*x^2-1)/c^2/x^2)^(1/2)/x+3/4/c*a^2*b/((c^2*x^2-1)/c^2/x^2)^(1/2
)/x^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3/x^3,x, algorithm="maxima")
```

```
[Out] 3/4*a^2*b*((c^4*x*sqrt(-1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) -
c^3*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)))/c - 2*arccsc(c*x)/x^2) - 1/2*a^3/x^
2 - 1/8*(4*b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 3*b^3*arctan2(1,
 sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 + 12*(a*b^2*c^2*(log(c*x + 1)
+ log(c*x - 1) - 2*log(x))*log(c)^2 + 16*b^3*c^2*integrate(1/8*x^2*arctan(1
/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^5 - x^3), x)*log(c)^2 - 16*b^3*c^2*i
ntegrate(1/8*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)/(c^2*
x^5 - x^3), x)*log(c) + 32*b^3*c^2*integrate(1/8*x^2*arctan(1/(sqrt(c*x + 1
)*sqrt(c*x - 1)))*log(x)/(c^2*x^5 - x^3), x)*log(c) - 16*a*b^2*c^2*integrat
e(1/8*x^2*log(c^2*x^2)/(c^2*x^5 - x^3), x)*log(c) + 32*a*b^2*c^2*integrate(
1/8*x^2*log(x)/(c^2*x^5 - x^3), x)*log(c) - 16*b^3*c^2*integrate(1/8*x^2*ar
ctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)*log(x)/(c^2*x^5 - x^3),
x) + 16*b^3*c^2*integrate(1/8*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*l
og(x)^2/(c^2*x^5 - x^3), x) - 16*a*b^2*c^2*integrate(1/8*x^2*arctan(1/(sqrt
(c*x + 1)*sqrt(c*x - 1)))^2/(c^2*x^5 - x^3), x) + 8*b^3*c^2*integrate(1/8*x
^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)/(c^2*x^5 - x^3), x)
 + 4*a*b^2*c^2*integrate(1/8*x^2*log(c^2*x^2)^2/(c^2*x^5 - x^3), x) - 16*a*
b^2*c^2*integrate(1/8*x^2*log(c^2*x^2)*log(x)/(c^2*x^5 - x^3), x) + 16*a*b^
2*c^2*integrate(1/8*x^2*log(x)^2/(c^2*x^5 - x^3), x) - (c^2*log(c*x + 1) +
c^2*log(c*x - 1) - 2*c^2*log(x) + 1/x^2)*a*b^2*log(c)^2 - 16*b^3*integrate(
1/8*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^5 - x^3), x)*log(c)^2 +
```

```
16*b^3*integrate(1/8*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)/(
c^2*x^5 - x^3), x)*log(c) - 32*b^3*integrate(1/8*arctan(1/(sqrt(c*x + 1)*sq
rt(c*x - 1)))*log(x)/(c^2*x^5 - x^3), x)*log(c) + 16*a*b^2*integrate(1/8*lo
g(c^2*x^2)/(c^2*x^5 - x^3), x)*log(c) - 32*a*b^2*integrate(1/8*log(x)/(c^2*
x^5 - x^3), x)*log(c) + 8*b^3*integrate(1/8*sqrt(c*x + 1)*sqrt(c*x - 1)*arc
tan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))^2/(c^2*x^5 - x^3), x) - 2*b^3*integrat
e(1/8*sqrt(c*x + 1)*sqrt(c*x - 1)*log(c^2*x^2)^2/(c^2*x^5 - x^3), x) + 16*b
^3*integrate(1/8*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)*log(x
)/(c^2*x^5 - x^3), x) - 16*b^3*integrate(1/8*arctan(1/(sqrt(c*x + 1)*sqrt(c
*x - 1)))*log(x)^2/(c^2*x^5 - x^3), x) + 16*a*b^2*integrate(1/8*arctan(1/(s
qrt(c*x + 1)*sqrt(c*x - 1)))^2/(c^2*x^5 - x^3), x) - 8*b^3*integrate(1/8*ar
ctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)/(c^2*x^5 - x^3), x) - 4*
a*b^2*integrate(1/8*log(c^2*x^2)^2/(c^2*x^5 - x^3), x) + 16*a*b^2*integrate
(1/8*log(c^2*x^2)*log(x)/(c^2*x^5 - x^3), x) - 16*a*b^2*integrate(1/8*log(x
)^2/(c^2*x^5 - x^3), x))*x^2)/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))^3/x^3,x)
```

```
[Out] int((a + b*asin(1/(c*x)))^3/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{acsc}(cx))^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))**3/x**3,x)
```

```
[Out] Integral((a + b*acsc(c*x))**3/x**3, x)
```

3.31 $\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x^4}\,dx$

Optimal. Leaf size=170

$$\frac{4b^2c^2\left(a+b\csc^{-1}(cx)\right)}{3x}+\frac{2b^2\left(a+b\csc^{-1}(cx)\right)}{9x^3}-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2}{3x^2}-\frac{2}{3}bc^3\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)$$

```
[Out] -2/27*b^3*c^3*(1-1/c^2/x^2)^(3/2)+2/9*b^2*(a+b*arccsc(c*x))/x^3+4/3*b^2*c^2
*(a+b*arccsc(c*x))/x-1/3*(a+b*arccsc(c*x))^3/x^3+14/9*b^3*c^3*(1-1/c^2/x^2)
^(1/2)-2/3*b*c^3*(a+b*arccsc(c*x))^2*(1-1/c^2/x^2)^(1/2)-1/3*b*c*(a+b*arccs
c(c*x))^2*(1-1/c^2/x^2)^(1/2)/x^2
```

Rubi [A] time = 0.15, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5223, 4404, 3311, 3296, 2638, 2633}

$$\frac{4b^2c^2\left(a+b\csc^{-1}(cx)\right)}{3x}+\frac{2b^2\left(a+b\csc^{-1}(cx)\right)}{9x^3}-\frac{2}{3}bc^3\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])^3/x^4,x]
```

```
[Out] (14*b^3*c^3*Sqrt[1 - 1/(c^2*x^2)])/9 - (2*b^3*c^3*(1 - 1/(c^2*x^2))^(3/2))/
27 + (2*b^2*(a + b*ArcCsc[c*x]))/(9*x^3) + (4*b^2*c^2*(a + b*ArcCsc[c*x]))/
(3*x) - (2*b*c^3*Sqrt[1 - 1/(c^2*x^2)]*(a + b*ArcCsc[c*x])^2)/3 - (b*c*Sqrt
[1 - 1/(c^2*x^2)]*(a + b*ArcCsc[c*x])^2)/(3*x^2) - (a + b*ArcCsc[c*x])^3/(3
*x^3)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
```

```
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5223

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x^4}\,dx = -\left(c^3\,\text{Subst}\left(\int (a+bx)^3\cos(x)\sin^2(x)\,dx, x, \csc^{-1}(cx)\right)\right)$$
$$= -\frac{\left(a+b\csc^{-1}(cx)\right)^3}{3x^3} + \left(bc^3\right)\text{Subst}\left(\int (a+bx)^2\sin^3(x)\,dx, x, \csc^{-1}(cx)\right)$$
$$= \frac{2b^2\left(a+b\csc^{-1}(cx)\right)}{9x^3} - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2}{3x^2} - \frac{\left(a+b\csc^{-1}(cx)\right)^3}{3x^3} + \frac{1}{3}\left(2\right.$$
$$= \frac{2b^2\left(a+b\csc^{-1}(cx)\right)}{9x^3} - \frac{2}{3}bc^3\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2 - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc\right.}{3x^2}$$
$$= \frac{2}{9}b^3c^3\sqrt{1-\frac{1}{c^2x^2}} - \frac{2}{27}b^3c^3\left(1-\frac{1}{c^2x^2}\right)^{3/2} + \frac{2b^2\left(a+b\csc^{-1}(cx)\right)}{9x^3} + \frac{4b^2c^2\left(a+b\csc\right.}{3x}$$
$$= \frac{14}{9}b^3c^3\sqrt{1-\frac{1}{c^2x^2}} - \frac{2}{27}b^3c^3\left(1-\frac{1}{c^2x^2}\right)^{3/2} + \frac{2b^2\left(a+b\csc^{-1}(cx)\right)}{9x^3} + \frac{4b^2c^2\left(a+b\csc\right.}{3x}$$

Mathematica [A] time = 0.33, size = 204, normalized size = 1.20

$$\frac{-9a^3+3b\csc^{-1}(cx)\left(-9a^2-6abcx\sqrt{1-\frac{1}{c^2x^2}}\left(2c^2x^2+1\right)+2b^2\left(6c^2x^2+1\right)\right)-9a^2bcx\sqrt{1-\frac{1}{c^2x^2}}\left(2c^2x^2+1\right)+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])^3/x^4,x]
```

```
[Out] (-9*a^3 - 9*a^2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2) + 6*a*b^2*(1 +
6*c^2*x^2) + 2*b^3*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 20*c^2*x^2) + 3*b*(-9*a^2
 - 6*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2) + 2*b^2*(1 + 6*c^2*x^2))
*ArcCsc[c*x] - 9*b^2*(3*a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2))*Ar
cCsc[c*x]^2 - 9*b^3*ArcCsc[c*x]^3)/(27*x^3)
```

fricas [A] time = 0.72, size = 173, normalized size = 1.02

$$\frac{36\,ab^2c^2x^2 - 9\,b^3\,\text{arccsc}\,(cx)^3 - 27\,ab^2\,\text{arccsc}\,(cx)^2 - 9\,a^3 + 6\,ab^2 + 3\left(12\,b^3c^2x^2 - 9\,a^2b + 2\,b^3\right)\text{arccsc}\,(cx) - \left(\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/27*(36*a*b^2*c^2*x^2 - 9*b^3*arccsc(c*x)^3 - 27*a*b^2*arccsc(c*x)^2 - 9*a
^3 + 6*a*b^2 + 3*(12*b^3*c^2*x^2 - 9*a^2*b + 2*b^3)*arccsc(c*x) - (2*(9*a^2
```

```
*b - 20*b^3)*c^2*x^2 + 9*a^2*b - 2*b^3 + 9*(2*b^3*c^2*x^2 + b^3)*arccsc(c*x
)^2 + 18*(2*a*b^2*c^2*x^2 + a*b^2)*arccsc(c*x))*sqrt(c^2*x^2 - 1))/x^3
```

giac [B] time = 0.18, size = 428, normalized size = 2.52

$$\frac{1}{27}\left(9\,b^3c^2\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}\arcsin\left(\frac{1}{cx}\right)^2+18\,ab^2c^2\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}\arcsin\left(\frac{1}{cx}\right)-27\,b^3c^2\sqrt{-\frac{1}{c^2x^2}+1}\,\arcsin\left(\frac{1}{cx}\right)^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3/x^4,x, algorithm="giac")
```

```
[Out] 1/27*(9*b^3*c^2*(-1/(c^2*x^2) + 1)^(3/2)*arcsin(1/(c*x))^2 + 18*a*b^2*c^2*(
-1/(c^2*x^2) + 1)^(3/2)*arcsin(1/(c*x)) - 27*b^3*c^2*sqrt(-1/(c^2*x^2) + 1)
*arcsin(1/(c*x))^2 - 9*b^3*c*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))^3/x + 9*a^2*
b*c^2*(-1/(c^2*x^2) + 1)^(3/2) - 2*b^3*c^2*(-1/(c^2*x^2) + 1)^(3/2) - 54*a*
b^2*c^2*sqrt(-1/(c^2*x^2) + 1)*arcsin(1/(c*x)) - 27*a*b^2*c*(1/(c^2*x^2) -
1)*arcsin(1/(c*x))^2/x - 9*b^3*c*arcsin(1/(c*x))^3/x - 27*a^2*b*c^2*sqrt(-1
/(c^2*x^2) + 1) + 42*b^3*c^2*sqrt(-1/(c^2*x^2) + 1) - 27*a^2*b*c*(1/(c^2*x^
2) - 1)*arcsin(1/(c*x))/x + 6*b^3*c*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))/x - 2
7*a*b^2*c*arcsin(1/(c*x))^2/x + 6*a*b^2*c*(1/(c^2*x^2) - 1)/x - 27*a^2*b*c*
arcsin(1/(c*x))/x + 42*b^3*c*arcsin(1/(c*x))/x + 42*a*b^2*c/x - 9*a^3/(c*x^
3))*c
```

maple [B] time = 0.59, size = 299, normalized size = 1.76

$$c^3\left(-\frac{a^3}{3c^3x^3}+b^3\left(-\frac{\operatorname{arccsc}(cx)^3}{3c^3x^3}-\frac{\operatorname{arccsc}(cx)^2\left(2c^2x^2+1\right)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3c^2x^2}+\frac{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3}+\frac{4\operatorname{arccsc}(cx)}{3cx}+\frac{2\operatorname{arccsc}(cx)}{9c^3x^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))^3/x^4,x)
```

```
[Out] c^3*(-1/3*a^3/c^3/x^3+b^3*(-1/3/c^3/x^3*arccsc(c*x)^3-1/3*arccsc(c*x)^2*(2*
c^2*x^2+1)/c^2/x^2*((c^2*x^2-1)/c^2/x^2)^(1/2)+4/3*((c^2*x^2-1)/c^2/x^2)^(1
/2)+4/3/c/x*arccsc(c*x)+2/9/c^3/x^3*arccsc(c*x)+2/27*(2*c^2*x^2+1)/c^2/x^2*
((c^2*x^2-1)/c^2/x^2)^(1/2))+3*a*b^2*(-1/3/c^3/x^3*arccsc(c*x)^2-2/9*arccsc
(c*x)*(2*c^2*x^2+1)/c^2/x^2*((c^2*x^2-1)/c^2/x^2)^(1/2)+2/27/c^3/x^3+4/9/c/
x)+3*a^2*b*(-1/3/c^3/x^3*arccsc(c*x)-1/9*(c^2*x^2-1)*(2*c^2*x^2+1)/((c^2*x^
2-1)/c^2/x^2)^(1/2)/c^4/x^4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\,a^2b\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c}-\frac{3\,\operatorname{arccsc}(cx)}{x^3}\right)-\frac{ab^2\operatorname{arccsc}(cx)^2}{x^3}+\frac{-\frac{1}{4}\left(12\,x^3\int\frac{16\,c^4x^4\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*b*((c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c
- 3*arccsc(c*x)/x^3) - a*b^2*arccsc(c*x)^2/x^3 + 1/12*(12*x^3*integrate(-1/
4*(12*c^2*x^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c)^2 - 12*arctan2
(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c)^2 + 12*(c^2*x^2*arctan2(1, sqrt(c*x
 + 1)*sqrt(c*x - 1)) - arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)^2 +
```

```
sqrt(c*x + 1)*sqrt(c*x - 1)*(4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2 -
log(c^2*x^2)^2) - 4*((3*c^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c)
- c^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*x^2 - 3*arctan2(1, sqrt(c*x
+ 1)*sqrt(c*x - 1))*log(c) + 3*(c^2*x^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x -
 1)) - arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x) + arctan2(1, sqrt(c*
x + 1)*sqrt(c*x - 1)))*log(c^2*x^2) + 24*(c^2*x^2*arctan2(1, sqrt(c*x + 1)*
sqrt(c*x - 1))*log(c) - arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c))*log
(x))/(c^2*x^6 - x^4), x) - 4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^3 + 3*
arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2)*b^3/x^3 - 1/3*a^3/x
^3 - 2/9*(6*c^5*x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) - 3*c^3*x^2*arc
tan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) - (6*c^3*x^2 + c)*sqrt(c*x + 1)*sqrt(c
*x - 1) - 3*c*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*a*b^2/(sqrt(c*x + 1)
*sqrt(c*x - 1)*c*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))^3/x^4,x)
```

```
[Out] int((a + b*asin(1/(c*x)))^3/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{acsc}\,(cx))^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))**3/x**4,x)
```

```
[Out] Integral((a + b*acsc(c*x))**3/x**4, x)
```

3.32 $\int \frac{(a+b\csc^{-1}(cx))^3}{x^5}\,dx$

Optimal. Leaf size=208

$$\frac{9b^2c^2\left(a+b\csc^{-1}(cx)\right)}{32x^2}+\frac{3b^2\left(a+b\csc^{-1}(cx)\right)}{32x^4}+\frac{3}{32}c^4\left(a+b\csc^{-1}(cx)\right)^3-\frac{3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2}{16x^3}-\frac{9bc^3\sqrt{}}{}$$

```
[Out] -45/256*b^3*c^4*arccsc(c*x)+3/32*b^2*(a+b*arccsc(c*x))/x^4+9/32*b^2*c^2*(a+
b*arccsc(c*x))/x^2+3/32*c^4*(a+b*arccsc(c*x))^3-1/4*(a+b*arccsc(c*x))^3/x^4
+3/128*b^3*c*(1-1/c^2/x^2)^(1/2)/x^3+45/256*b^3*c^3*(1-1/c^2/x^2)^(1/2)/x-3
/16*b*c*(a+b*arccsc(c*x))^2*(1-1/c^2/x^2)^(1/2)/x^3-9/32*b*c^3*(a+b*arccsc(
c*x))^2*(1-1/c^2/x^2)^(1/2)/x
```

Rubi [A] time = 0.18, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5223, 4404, 3311, 32, 2635, 8}

$$\frac{9b^2c^2\left(a+b\csc^{-1}(cx)\right)}{32x^2}+\frac{3b^2\left(a+b\csc^{-1}(cx)\right)}{32x^4}-\frac{9bc^3\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2}{32x}-\frac{3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{16x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])^3/x^5,x]
```

```
[Out] (3*b^3*c*Sqrt[1 - 1/(c^2*x^2)])/(128*x^3) + (45*b^3*c^3*Sqrt[1 - 1/(c^2*x^2
)])/(256*x) - (45*b^3*c^4*ArcCsc[c*x])/256 + (3*b^2*(a + b*ArcCsc[c*x]))/(3
2*x^4) + (9*b^2*c^2*(a + b*ArcCsc[c*x]))/(32*x^2) - (3*b*c*Sqrt[1 - 1/(c^2*
x^2)]*(a + b*ArcCsc[c*x])^2)/(16*x^3) - (9*b*c^3*Sqrt[1 - 1/(c^2*x^2)]*(a +
 b*ArcCsc[c*x])^2)/(32*x) + (3*c^4*(a + b*ArcCsc[c*x])^3)/32 - (a + b*ArcCs
c[c*x])^3/(4*x^4)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5223

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x^5}\,dx = -\left(c^4\,\text{Subst}\left(\int (a+bx)^3\cos(x)\sin^3(x)\,dx, x, \csc^{-1}(cx)\right)\right)$$
$$= -\frac{\left(a+b\csc^{-1}(cx)\right)^3}{4x^4} + \frac{1}{4}\left(3bc^4\right)\text{Subst}\left(\int (a+bx)^2\sin^4(x)\,dx, x, \csc^{-1}(cx)\right)$$
$$= \frac{3b^2\left(a+b\csc^{-1}(cx)\right)}{32x^4} - \frac{3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2}{16x^3} - \frac{\left(a+b\csc^{-1}(cx)\right)^3}{4x^4} + \frac{1}{16}$$
$$= \frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{128x^3} + \frac{3b^2\left(a+b\csc^{-1}(cx)\right)}{32x^4} + \frac{9b^2c^2\left(a+b\csc^{-1}(cx)\right)}{32x^2} - \frac{3bc\sqrt{1-\frac{1}{c^2x^2}}\,(}{1}$$
$$= \frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{128x^3} + \frac{45b^3c^3\sqrt{1-\frac{1}{c^2x^2}}}{256x} + \frac{3b^2\left(a+b\csc^{-1}(cx)\right)}{32x^4} + \frac{9b^2c^2\left(a+b\csc^{-1}(cx\right.}{32x^2}$$
$$= \frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{128x^3} + \frac{45b^3c^3\sqrt{1-\frac{1}{c^2x^2}}}{256x} - \frac{45}{256}b^3c^4\csc^{-1}(cx) + \frac{3b^2\left(a+b\csc^{-1}(cx)\right)}{32x^4} +$$

Mathematica [A] time = 0.36, size = 283, normalized size = 1.36

$$\frac{-64a^3 + 9bc^4x^4\left(8a^2-5b^2\right)\sin^{-1}\left(\frac{1}{cx}\right) + 24b\csc^{-1}(cx)\left(-8a^2 - 2abcx\sqrt{1-\frac{1}{c^2x^2}}\left(3c^2x^2+2\right) + b^2\left(3c^2x^2+1\right)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])^3/x^5,x]
```

```
[Out] (-64*a^3 + 24*a*b^2 - 48*a^2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x + 6*b^3*c*Sqrt[1 -
 1/(c^2*x^2)]*x + 72*a*b^2*c^2*x^2 - 72*a^2*b*c^3*Sqrt[1 - 1/(c^2*x^2)]*x^3
 + 45*b^3*c^3*Sqrt[1 - 1/(c^2*x^2)]*x^3 + 24*b*(-8*a^2 + b^2*(1 + 3*c^2*x^2
) - 2*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(2 + 3*c^2*x^2))*ArcCsc[c*x] - 24*b^2*(
b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(2 + 3*c^2*x^2) + a*(8 - 3*c^4*x^4))*ArcCsc[c*x
]^2 + 8*b^3*(-8 + 3*c^4*x^4)*ArcCsc[c*x]^3 + 9*b*(8*a^2 - 5*b^2)*c^4*x^4*Ar
cSin[1/(c*x)])/(256*x^4)
```

fricas [A] time = 0.93, size = 225, normalized size = 1.08

$$\frac{72\,ab^2c^2x^2 + 8\left(3\,b^3c^4x^4 - 8\,b^3\right)\text{arccsc}\,(cx)^3 - 64\,a^3 + 24\,ab^2 + 24\left(3\,ab^2c^4x^4 - 8\,ab^2\right)\text{arccsc}\,(cx)^2 + 3\left(3\left(8\,a^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3/x^5,x, algorithm="fricas")
```

```
[Out] 1/256*(72*a*b^2*c^2*x^2 + 8*(3*b^3*c^4*x^4 - 8*b^3)*arccsc(c*x)^3 - 64*a^3
+ 24*a*b^2 + 24*(3*a*b^2*c^4*x^4 - 8*a*b^2)*arccsc(c*x)^2 + 3*(3*(8*a^2*b -
 5*b^3)*c^4*x^4 + 24*b^3*c^2*x^2 - 64*a^2*b + 8*b^3)*arccsc(c*x) - 3*(3*(8*
a^2*b - 5*b^3)*c^2*x^2 + 16*a^2*b - 2*b^3 + 8*(3*b^3*c^2*x^2 + 2*b^3)*arccs
c(c*x)^2 + 16*(3*a*b^2*c^2*x^2 + 2*a*b^2)*arccsc(c*x))*sqrt(c^2*x^2 - 1))/x
^4
```

giac [B] time = 0.18, size = 576, normalized size = 2.77

$$-\frac{1}{256}\left(64\,b^3c^3\left(\frac{1}{c^2x^2}-1\right)^2\arcsin\left(\frac{1}{cx}\right)^3+192\,ab^2c^3\left(\frac{1}{c^2x^2}-1\right)^2\arcsin\left(\frac{1}{cx}\right)^2+128\,b^3c^3\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3/x^5,x, algorithm="giac")
```

```
[Out] -1/256*(64*b^3*c^3*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*x))^3 + 192*a*b^2*c^3*(1
/(c^2*x^2) - 1)^2*arcsin(1/(c*x))^2 + 128*b^3*c^3*(1/(c^2*x^2) - 1)*arcsin(
1/(c*x))^3 + 192*a^2*b*c^3*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*x)) - 24*b^3*c^3
*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*x)) + 384*a*b^2*c^3*(1/(c^2*x^2) - 1)*arcs
in(1/(c*x))^2 + 40*b^3*c^3*arcsin(1/(c*x))^3 - 24*a*b^2*c^3*(1/(c^2*x^2) -
1)^2 + 384*a^2*b*c^3*(1/(c^2*x^2) - 1)*arcsin(1/(c*x)) - 120*b^3*c^3*(1/(c^
2*x^2) - 1)*arcsin(1/(c*x)) + 120*a*b^2*c^3*arcsin(1/(c*x))^2 - 48*b^3*c^2*
(-1/(c^2*x^2) + 1)^(3/2)*arcsin(1/(c*x))^2/x - 120*a*b^2*c^3*(1/(c^2*x^2) -
 1) + 120*a^2*b*c^3*arcsin(1/(c*x)) - 51*b^3*c^3*arcsin(1/(c*x)) - 96*a*b^2
*c^2*(-1/(c^2*x^2) + 1)^(3/2)*arcsin(1/(c*x))/x + 120*b^3*c^2*sqrt(-1/(c^2*
x^2) + 1)*arcsin(1/(c*x))^2/x - 51*a*b^2*c^3 - 48*a^2*b*c^2*(-1/(c^2*x^2) +
 1)^(3/2)/x + 6*b^3*c^2*(-1/(c^2*x^2) + 1)^(3/2)/x + 240*a*b^2*c^2*sqrt(-1/
(c^2*x^2) + 1)*arcsin(1/(c*x))/x + 120*a^2*b*c^2*sqrt(-1/(c^2*x^2) + 1)/x -
 51*b^3*c^2*sqrt(-1/(c^2*x^2) + 1)/x + 64*a^3/(c*x^4))*c
```

maple [B] time = 0.76, size = 472, normalized size = 2.27

$$-\frac{a^3}{4x^4}-\frac{b^3\mathrm{arccsc}\left(cx\right)^3}{4x^4}+\frac{3c^4b^3\mathrm{arccsc}\left(cx\right)^3}{32}-\frac{9c^3b^3\mathrm{arccsc}\left(cx\right)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{32x}-\frac{3c\,b^3\mathrm{arccsc}\left(cx\right)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{16x^3}+\frac{3b^3\mathrm{arccsc}\left(cx\right)}{32x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))^3/x^5,x)
```

```
[Out] -1/4*a^3/x^4-1/4*b^3/x^4*arccsc(c*x)^3+3/32*c^4*b^3*arccsc(c*x)^3-9/32*c^3*
b^3*arccsc(c*x)^2/x*((c^2*x^2-1)/c^2/x^2)^(1/2)-3/16*c*b^3*arccsc(c*x)^2/x^
3*((c^2*x^2-1)/c^2/x^2)^(1/2)+3/32*b^3/x^4*arccsc(c*x)+45/256*c^3*b^3*((c^2
*x^2-1)/c^2/x^2)^(1/2)/x+3/128*c*b^3/x^3*((c^2*x^2-1)/c^2/x^2)^(1/2)-45/256
*b^3*c^4*arccsc(c*x)+9/32*c^2*b^3/x^2*arccsc(c*x)-3/4*a*b^2/x^4*arccsc(c*x)
^2+9/32*c^4*a*b^2*arccsc(c*x)^2-9/16*c^3*a*b^2*arccsc(c*x)/x*((c^2*x^2-1)/c
^2/x^2)^(1/2)-3/8*c*a*b^2*arccsc(c*x)/x^3*((c^2*x^2-1)/c^2/x^2)^(1/2)+3/32*
a*b^2/x^4+9/32*c^2*a*b^2/x^2-3/4*a^2*b/x^4*arccsc(c*x)+9/32*c^3*a^2*b*(c^2*
x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*arctan(1/(c^2*x^2-1)^(1/2))-9/32
*c^3*a^2*b/((c^2*x^2-1)/c^2/x^2)^(1/2)/x+3/32*c*a^2*b/((c^2*x^2-1)/c^2/x^2)
^(1/2)/x^3+3/16/c*a^2*b/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3/x^5,x, algorithm="maxima")
```

```
[Out] -3/32*a^2*b*((3*c^5*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)) + (3*c^8*x^3*(-1/(c^
2*x^2) + 1)^(3/2) + 5*c^6*x*sqrt(-1/(c^2*x^2) + 1))/(c^4*x^4*(1/(c^2*x^2) -
 1)^2 - 2*c^2*x^2*(1/(c^2*x^2) - 1) + 1))/c + 8*arccsc(c*x)/x^4) - 1/4*a^3/
x^4 - 1/16*(4*b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 3*b^3*arctan2
(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 + 12*(2*(c^2*log(c*x + 1) +
 c^2*log(c*x - 1) - 2*c^2*log(x) + 1/x^2)*a*b^2*c^2*log(c)^2 + 64*b^3*c^2*i
ntegrate(1/16*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^7 - x^5),
x)*log(c)^2 - 64*b^3*c^2*integrate(1/16*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*
x - 1)))*log(c^2*x^2)/(c^2*x^7 - x^5), x)*log(c) + 128*b^3*c^2*integrate(1/
16*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)/(c^2*x^7 - x^5), x)*l
og(c) - 64*a*b^2*c^2*integrate(1/16*x^2*log(c^2*x^2)/(c^2*x^7 - x^5), x)*lo
g(c) + 128*a*b^2*c^2*integrate(1/16*x^2*log(x)/(c^2*x^7 - x^5), x)*log(c) -
 64*b^3*c^2*integrate(1/16*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(
c^2*x^2)*log(x)/(c^2*x^7 - x^5), x) + 64*b^3*c^2*integrate(1/16*x^2*arctan(
1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)^2/(c^2*x^7 - x^5), x) - 64*a*b^2*c^
2*integrate(1/16*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))^2/(c^2*x^7 - x
^5), x) + 16*b^3*c^2*integrate(1/16*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x -
1)))*log(c^2*x^2)/(c^2*x^7 - x^5), x) + 16*a*b^2*c^2*integrate(1/16*x^2*log
(c^2*x^2)^2/(c^2*x^7 - x^5), x) - 64*a*b^2*c^2*integrate(1/16*x^2*log(c^2*x
^2)*log(x)/(c^2*x^7 - x^5), x) + 64*a*b^2*c^2*integrate(1/16*x^2*log(x)^2/(
c^2*x^7 - x^5), x) - (2*c^4*log(c*x + 1) + 2*c^4*log(c*x - 1) - 4*c^4*log(x
) + (2*c^2*x^2 + 1)/x^4)*a*b^2*log(c)^2 - 64*b^3*integrate(1/16*arctan(1/(s
qrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^7 - x^5), x)*log(c)^2 + 64*b^3*integrat
e(1/16*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)/(c^2*x^7 - x^5)
, x)*log(c) - 128*b^3*integrate(1/16*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1))
)*log(x)/(c^2*x^7 - x^5), x)*log(c) + 64*a*b^2*integrate(1/16*log(c^2*x^2)/
(c^2*x^7 - x^5), x)*log(c) - 128*a*b^2*integrate(1/16*log(x)/(c^2*x^7 - x^5
), x)*log(c) + 16*b^3*integrate(1/16*sqrt(c*x + 1)*sqrt(c*x - 1)*arctan(1/(
sqrt(c*x + 1)*sqrt(c*x - 1)))^2/(c^2*x^7 - x^5), x) - 4*b^3*integrate(1/16*
sqrt(c*x + 1)*sqrt(c*x - 1)*log(c^2*x^2)^2/(c^2*x^7 - x^5), x) + 64*b^3*int
egrate(1/16*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)*log(x)/(c^
2*x^7 - x^5), x) - 64*b^3*integrate(1/16*arctan(1/(sqrt(c*x + 1)*sqrt(c*x -
 1)))*log(x)^2/(c^2*x^7 - x^5), x) + 64*a*b^2*integrate(1/16*arctan(1/(sqrt
(c*x + 1)*sqrt(c*x - 1)))^2/(c^2*x^7 - x^5), x) - 16*b^3*integrate(1/16*arc
tan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)/(c^2*x^7 - x^5), x) - 16*
a*b^2*integrate(1/16*log(c^2*x^2)^2/(c^2*x^7 - x^5), x) + 64*a*b^2*integrat
e(1/16*log(c^2*x^2)*log(x)/(c^2*x^7 - x^5), x) - 64*a*b^2*integrate(1/16*lo
g(x)^2/(c^2*x^7 - x^5), x))*x^4)/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))^3/x^5,x)
```

```
[Out] int((a + b*asin(1/(c*x)))^3/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\,\mathrm{acsc}\,(cx)\right)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))**3/x**5,x)
```

```
[Out] Integral((a + b*acsc(c*x))**3/x**5, x)
```

3.33 $\int \frac{x}{a+b\csc^{-1}(cx)}\,dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{x}{a+b\csc^{-1}(cx)},x\right)$$

```
[Out] Unintegrable(x/(a+b*arccsc(c*x)),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{a+b\csc^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/(a + b*ArcCsc[c*x]),x]
[Out] Defer[Int][x/(a + b*ArcCsc[c*x]), x]
```

Rubi steps

$$\int \frac{x}{a+b\csc^{-1}(cx)}\,dx=\int \frac{x}{a+b\csc^{-1}(cx)}\,dx$$

Mathematica [A] time = 2.91, size = 0, normalized size = 0.00

$$\int \frac{x}{a+b\csc^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/(a + b*ArcCsc[c*x]),x]
[Out] Integrate[x/(a + b*ArcCsc[c*x]), x]
```

fricas [A] time = 1.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{b\,\text{arccsc}(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccsc(c*x)),x, algorithm="fricas")
[Out] integral(x/(b*arccsc(c*x) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{b\,\text{arccsc}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccsc(c*x)),x, algorithm="giac")
[Out] integrate(x/(b*arccsc(c*x) + a), x)
```

maple [A] time = 2.29, size = 0, normalized size = 0.00

$$\int \frac{x}{a+b\,\text{arccsc}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arccsc(c*x)),x)
```

```
[Out] int(x/(a+b*arccsc(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{b \operatorname{arccsc}(cx) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*arccsc(c*x) + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{x}{a + b \operatorname{asin}\left(\frac{1}{c\,x}\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*asin(1/(c*x))),x)
```

```
[Out] int(x/(a + b*asin(1/(c*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a + b \operatorname{acsc}(cx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*acsc(c*x)),x)
```

```
[Out] Integral(x/(a + b*acsc(c*x)), x)
```

3.34 $\int \frac{1}{a+b\csc^{-1}(cx)}\,dx$

Optimal. Leaf size=13

$$\mathrm{Int}\left(\frac{1}{a+b\csc^{-1}(cx)},x\right)$$

```
[Out] Unintegrable(1/(a+b*arccsc(c*x)),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{a+b\csc^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCsc[c*x])^(-1),x]
[Out] Defer[Int][(a + b*ArcCsc[c*x])^(-1), x]
```

Rubi steps

$$\int \frac{1}{a+b\csc^{-1}(cx)}\,dx=\int \frac{1}{a+b\csc^{-1}(cx)}\,dx$$

Mathematica [A] time = 3.49, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\csc^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCsc[c*x])^(-1),x]
[Out] Integrate[(a + b*ArcCsc[c*x])^(-1), x]
```

fricas [A] time = 1.08, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{b\,\mathrm{arccsc}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccsc(c*x)),x, algorithm="fricas")
[Out] integral(1/(b*arccsc(c*x) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\,\mathrm{arccsc}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccsc(c*x)),x, algorithm="giac")
[Out] integrate(1/(b*arccsc(c*x) + a), x)
```

maple [A] time = 0.90, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\,\mathrm{arccsc}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccsc(c*x)),x)
```

```
[Out] int(1/(a+b*arccsc(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b \operatorname{arccsc}(cx) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*arccsc(c*x) + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{a + b \operatorname{asin}\left(\frac{1}{c x}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asin(1/(c*x))),x)
```

```
[Out] int(1/(a + b*asin(1/(c*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b \operatorname{acsc}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acsc(c*x)),x)
```

```
[Out] Integral(1/(a + b*acsc(c*x)), x)
```

3.35 $\int \frac{1}{x\left(a+b\csc^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=17

$$\operatorname{Int}\left(\frac{1}{x\left(a+b\csc^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*arccsc(c*x)),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(a+b\csc^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(a + b*ArcCsc[c*x])),x]
[Out] Defer[Int][1/(x*(a + b*ArcCsc[c*x])), x]
```

Rubi steps

$$\int \frac{1}{x\left(a+b\csc^{-1}(cx)\right)}\,dx=\int \frac{1}{x\left(a+b\csc^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\csc^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(a + b*ArcCsc[c*x])),x]
[Out] Integrate[1/(x*(a + b*ArcCsc[c*x])), x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{1}{bx\operatorname{arccsc}(cx)+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccsc(c*x)),x, algorithm="fricas")
[Out] integral(1/(b*x*arccsc(c*x) + a*x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\operatorname{arccsc}(cx)+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccsc(c*x)),x, algorithm="giac")
[Out] integrate(1/((b*arccsc(c*x) + a)*x), x)
```

maple [A] time = 1.05, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\,\mathrm{arccsc}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*arccsc(c*x)),x)
[Out] int(1/x/(a+b*arccsc(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,\mathrm{arccsc}\left(cx\right)+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccsc(c*x)),x, algorithm="maxima")
[Out] integrate(1/((b*arccsc(c*x) + a)*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{x\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*asin(1/(c*x)))),x)
[Out] int(1/(x*(a + b*asin(1/(c*x)))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\,\mathrm{acsc}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*acsc(c*x)),x)
[Out] Integral(1/(x*(a + b*acsc(c*x))), x)
```

3.36 $\int \frac{1}{x^2\left(a+b\csc^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=47

$$-\frac{c\cos\left(\frac{a}{b}\right)\text{Ci}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{b}-\frac{c\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{b}$$

```
[Out] -c*Ci(a/b+arccsc(c*x))*cos(a/b)/b-c*Si(a/b+arccsc(c*x))*sin(a/b)/b
```

Rubi [A] time = 0.11, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5223, 3303, 3299, 3302}

$$-\frac{c\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{b}-\frac{c\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*ArcCsc[c*x])),x]
```

```
[Out] -((c*Cos[a/b]*CosIntegral[a/b + ArcCsc[c*x]])/b) - (c*Sin[a/b]*SinIntegral[
a/b + ArcCsc[c*x]])/b
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5223

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a+b\csc^{-1}(cx)\right)}\,dx &= -\left(c\,\text{Subst}\left(\int \frac{\cos(x)}{a+bx}\,dx, x, \csc^{-1}(cx)\right)\right)\\ &= -\left(\left(c\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \csc^{-1}(cx)\right)\right)-\left(c\sin\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sin(}{a\cdot}\right.\\ &= -\frac{c\cos\left(\frac{a}{b}\right)\text{Ci}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{b}-\frac{c\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.08, size = 43, normalized size = 0.91

$$-\frac{c\left(\cos\left(\frac{a}{b}\right)\text{Ci}\left(\frac{a}{b}+\csc^{-1}(cx)\right)+\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\csc^{-1}(cx)\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*ArcCsc[c*x])),x]
```

```
[Out] -((c*(Cos[a/b]*CosIntegral[a/b + ArcCsc[c*x]] + Sin[a/b]*SinIntegral[a/b +
ArcCsc[c*x]]))/b)
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{bx^2 \operatorname{arccsc}(cx) + ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*x^2*arccsc(c*x) + a*x^2), x)
```

giac [A] time = 0.12, size = 54, normalized size = 1.15

$$-c\left(\frac{\cos\left(\frac{a}{b}\right)\text{Ci}\left(\frac{a}{b}+\arcsin\left(\frac{1}{cx}\right)\right)}{b}+\frac{\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\arcsin\left(\frac{1}{cx}\right)\right)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] -c*(cos(a/b)*cos_integral(a/b + arcsin(1/(c*x)))/b + sin(a/b)*sin_integral(
a/b + arcsin(1/(c*x)))/b)
```

maple [A] time = 0.18, size = 48, normalized size = 1.02

$$c\left(-\frac{\text{Si}\left(\frac{a}{b}+\operatorname{arccsc}(cx)\right)\sin\left(\frac{a}{b}\right)}{b}-\frac{\text{Ci}\left(\frac{a}{b}+\operatorname{arccsc}(cx)\right)\cos\left(\frac{a}{b}\right)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*arccsc(c*x)),x)
```

```
[Out] c*(-Si(a/b+arccsc(c*x))*sin(a/b)/b-Ci(a/b+arccsc(c*x))*cos(a/b)/b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{arccsc}(cx) + a)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*arccsc(c*x) + a)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*asin(1/(c*x)))),x)
```

```
[Out] int(1/(x^2*(a + b*asin(1/(c*x)))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+b\,\mathrm{acsc}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*acsc(c*x)),x)
```

```
[Out] Integral(1/(x**2*(a + b*acsc(c*x))), x)
```

3.37 $\int \frac{1}{x^3\left(a+b\csc^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=63

$$\frac{c^2\sin\left(\frac{2a}{b}\right)\text{Ci}\left(\frac{2a}{b}+2\csc^{-1}(cx)\right)}{2b}-\frac{c^2\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\csc^{-1}(cx)\right)}{2b}$$

```
[Out] -1/2*c^2*cos(2*a/b)*Si(2*a/b+2*arccsc(c*x))/b+1/2*c^2*Ci(2*a/b+2*arccsc(c*x
))*sin(2*a/b)/b
```

Rubi [A] time = 0.13, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5223, 4406, 12, 3303, 3299, 3302}

$$\frac{c^2\sin\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2a}{b}+2\csc^{-1}(cx)\right)}{2b}-\frac{c^2\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\csc^{-1}(cx)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*ArcCsc[c*x])),x]
```

```
[Out] (c^2*CosIntegral[(2*a)/b + 2*ArcCsc[c*x]]*Sin[(2*a)/b])/(2*b) - (c^2*Cos[(2
*a)/b]*SinIntegral[(2*a)/b + 2*ArcCsc[c*x]])/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5223

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
```

```
0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a+b\csc^{-1}(cx)\right)}\,dx &= -\left(c^2\,\text{Subst}\left(\int\frac{\cos(x)\sin(x)}{a+bx}\,dx,x,\csc^{-1}(cx)\right)\right)\\ &= -\left(c^2\,\text{Subst}\left(\int\frac{\sin(2x)}{2(a+bx)}\,dx,x,\csc^{-1}(cx)\right)\right)\\ &= -\left(\frac{1}{2}c^2\,\text{Subst}\left(\int\frac{\sin(2x)}{a+bx}\,dx,x,\csc^{-1}(cx)\right)\right)\\ &= -\left(\frac{1}{2}\left(c^2\cos\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int\frac{\sin\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx,x,\csc^{-1}(cx)\right)\right)+\frac{1}{2}\left(c^2\sin\left(\frac{2a}{b}\right)\right)\text{Sub}\\ &= \frac{c^2\text{Ci}\left(\frac{2a}{b}+2\csc^{-1}(cx)\right)\sin\left(\frac{2a}{b}\right)}{2b}-\frac{c^2\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\csc^{-1}(cx)\right)}{2b}\end{aligned}$$

Mathematica [A] time = 0.08, size = 56, normalized size = 0.89

$$-\frac{c^2\left(\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\csc^{-1}(cx)\right)-\sin\left(\frac{2a}{b}\right)\text{Ci}\left(\frac{2a}{b}+2\csc^{-1}(cx)\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*ArcCsc[c*x])),x]
[Out] -1/2*(c^2*(-(CosIntegral[(2*a)/b + 2*ArcCsc[c*x]]*Sin[(2*a)/b]) + Cos[(2*a)
/b]*SinIntegral[(2*a)/b + 2*ArcCsc[c*x]]))/b
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{bx^3\,\text{arccsc}\,(cx)+ax^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*arccsc(c*x)),x, algorithm="fricas")
[Out] integral(1/(b*x^3*arccsc(c*x) + a*x^3), x)
```

giac [A] time = 0.16, size = 95, normalized size = 1.51

$$\frac{1}{2}\left(\frac{2\,c\cos\left(\frac{a}{b}\right)\text{Ci}\left(\frac{2\,a}{b}+2\,\arcsin\left(\frac{1}{cx}\right)\right)\sin\left(\frac{a}{b}\right)}{b}-\frac{2\,c\cos\left(\frac{a}{b}\right)^2\text{Si}\left(\frac{2\,a}{b}+2\,\arcsin\left(\frac{1}{cx}\right)\right)}{b}+\frac{c\,\text{Si}\left(\frac{2\,a}{b}+2\,\arcsin\left(\frac{1}{cx}\right)\right)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*arccsc(c*x)),x, algorithm="giac")
[Out] 1/2*(2*c*cos(a/b)*cos_integral(2*a/b + 2*arcsin(1/(c*x)))*sin(a/b)/b - 2*c*
cos(a/b)^2*sin_integral(2*a/b + 2*arcsin(1/(c*x)))/b + c*sin_integral(2*a/b
 + 2*arcsin(1/(c*x)))/b)*c
```

maple [A] time = 0.10, size = 58, normalized size = 0.92

$$c^2\left(-\frac{\mathrm{Si}\left(\frac{2a}{b}+2\,\mathrm{arccsc}\,(cx)\right)\cos\left(\frac{2a}{b}\right)}{2b}+\frac{\mathrm{Ci}\left(\frac{2a}{b}+2\,\mathrm{arccsc}\,(cx)\right)\sin\left(\frac{2a}{b}\right)}{2b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b*arccsc(c*x)),x)
```

```
[Out] c^2*(-1/2*Si(2*a/b+2*arccsc(c*x))*cos(2*a/b)/b+1/2*Ci(2*a/b+2*arccsc(c*x))*
sin(2*a/b)/b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,\mathrm{arccsc}\,(cx)+a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*arccsc(c*x) + a)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*asin(1/(c*x)))),x)
```

```
[Out] int(1/(x^3*(a + b*asin(1/(c*x)))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\,(a+b\,\mathrm{acsc}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*acsc(c*x)),x)
```

```
[Out] Integral(1/(x**3*(a + b*acsc(c*x))), x)
```

3.38 $\int \frac{1}{x^4\left(a+b\csc^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=117

$$-\frac{c^3\cos\left(\frac{a}{b}\right)\text{Ci}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{4b}+\frac{c^3\cos\left(\frac{3a}{b}\right)\text{Ci}\left(\frac{3a}{b}+3\csc^{-1}(cx)\right)}{4b}-\frac{c^3\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{4b}+\frac{c^3\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3a}{b}\right.}{4b}$$

```
[Out] -1/4*c^3*Ci(a/b+arccsc(c*x))*cos(a/b)/b+1/4*c^3*Ci(3*a/b+3*arccsc(c*x))*cos
(3*a/b)/b-1/4*c^3*Si(a/b+arccsc(c*x))*sin(a/b)/b+1/4*c^3*Si(3*a/b+3*arccsc(
c*x))*sin(3*a/b)/b
```

Rubi [A] time = 0.23, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5223, 4406, 3303, 3299, 3302}

$$-\frac{c^3\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{4b}+\frac{c^3\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3a}{b}+3\csc^{-1}(cx)\right)}{4b}-\frac{c^3\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\csc^{-1}(cx\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*ArcCsc[c*x])),x]
```

```
[Out] -(c^3*Cos[a/b]*CosIntegral[a/b + ArcCsc[c*x]])/(4*b) + (c^3*Cos[(3*a)/b]*Co
sIntegral[(3*a)/b + 3*ArcCsc[c*x]])/(4*b) - (c^3*Sin[a/b]*SinIntegral[a/b +
 ArcCsc[c*x]])/(4*b) + (c^3*Sin[(3*a)/b]*SinIntegral[(3*a)/b + 3*ArcCsc[c*x
]])/(4*b)
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5223

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{1}{x^4\left(a+b\csc^{-1}(cx)\right)}\,dx = -\left(c^3\,\text{Subst}\left(\int \frac{\cos(x)\sin^2(x)}{a+bx}\,dx, x, \csc^{-1}(cx)\right)\right)$$

$$= -\left(c^3\,\text{Subst}\left(\int \left(\frac{\cos(x)}{4(a+bx)} - \frac{\cos(3x)}{4(a+bx)}\right)dx, x, \csc^{-1}(cx)\right)\right)$$

$$= -\left(\frac{1}{4}c^3\,\text{Subst}\left(\int \frac{\cos(x)}{a+bx}\,dx, x, \csc^{-1}(cx)\right)\right) + \frac{1}{4}c^3\,\text{Subst}\left(\int \frac{\cos(3x)}{a+bx}\,dx, x, \csc^{-1}(c\right.$$

$$= -\left(\frac{1}{4}\left(c^3\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \csc^{-1}(cx)\right)\right) + \frac{1}{4}\left(c^3\cos\left(\frac{3a}{b}\right)\right)\text{Sub}$$

$$= -\frac{c^3\cos\left(\frac{a}{b}\right)\text{Ci}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{4b} + \frac{c^3\cos\left(\frac{3a}{b}\right)\text{Ci}\left(\frac{3a}{b}+3\csc^{-1}(cx)\right)}{4b} - \frac{c^3\sin\left(\frac{a}{b}\right)\text{Si}}{}$$

Mathematica [A] time = 0.18, size = 91, normalized size = 0.78

$$-\frac{c^3\left(\cos\left(\frac{a}{b}\right)\text{Ci}\left(\frac{a}{b}+\csc^{-1}(cx)\right) - \cos\left(\frac{3a}{b}\right)\text{Ci}\left(3\left(\frac{a}{b}+\csc^{-1}(cx)\right)\right) + \sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\csc^{-1}(cx)\right) - \sin\left(\frac{3a}{b}\right)\text{Si}\left(3\left(\right.\right.\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*ArcCsc[c*x])),x]
```

```
[Out] -1/4*(c^3*(Cos[a/b]*CosIntegral[a/b + ArcCsc[c*x]] - Cos[(3*a)/b]*CosIntegr
al[3*(a/b + ArcCsc[c*x])] + Sin[a/b]*SinIntegral[a/b + ArcCsc[c*x]] - Sin[(
3*a)/b]*SinIntegral[3*(a/b + ArcCsc[c*x])]))/b
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{bx^4\,\text{arccsc}\,(cx) + ax^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*x^4*arccsc(c*x) + a*x^4), x)
```

giac [A] time = 0.15, size = 200, normalized size = 1.71

$$\frac{1}{4}\left(\frac{4\,c^2\cos\left(\frac{a}{b}\right)^3\text{Ci}\left(\frac{3\,a}{b}+3\,\arcsin\left(\frac{1}{cx}\right)\right)}{b} + \frac{4\,c^2\cos\left(\frac{a}{b}\right)^2\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{3\,a}{b}+3\,\arcsin\left(\frac{1}{cx}\right)\right)}{b} - \frac{3\,c^2\cos\left(\frac{a}{b}\right)\text{Ci}\left(\frac{3\,a}{b}+\right.}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/4*(4*c^2*cos(a/b)^3*cos_integral(3*a/b + 3*arcsin(1/(c*x)))/b + 4*c^2*cos
(a/b)^2*sin(a/b)*sin_integral(3*a/b + 3*arcsin(1/(c*x)))/b - 3*c^2*cos(a/b)
*cos_integral(3*a/b + 3*arcsin(1/(c*x)))/b - c^2*cos(a/b)*cos_integral(a/b
+ arcsin(1/(c*x)))/b - c^2*sin(a/b)*sin_integral(3*a/b + 3*arcsin(1/(c*x)))
/b - c^2*sin(a/b)*sin_integral(a/b + arcsin(1/(c*x)))/b)*c
```

maple [A] time = 0.10, size = 102, normalized size = 0.87

$$c^3\left(-\frac{\mathrm{Si}\left(\frac{a}{b}+\operatorname{arccsc}(cx)\right)\sin\left(\frac{a}{b}\right)}{4b}-\frac{\mathrm{Ci}\left(\frac{a}{b}+\operatorname{arccsc}(cx)\right)\cos\left(\frac{a}{b}\right)}{4b}+\frac{\mathrm{Si}\left(\frac{3a}{b}+3\operatorname{arccsc}(cx)\right)\sin\left(\frac{3a}{b}\right)}{4b}+\frac{\mathrm{Ci}\left(\frac{3a}{b}+3\operatorname{ar}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a+b*arccsc(c*x)),x)
```

```
[Out] c^3*(-1/4*Si(a/b+arccsc(c*x))*sin(a/b)/b-1/4*Ci(a/b+arccsc(c*x))*cos(a/b)/b
+1/4*Si(3*a/b+3*arccsc(c*x))*sin(3*a/b)/b+1/4*Ci(3*a/b+3*arccsc(c*x))*cos(3
*a/b)/b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\operatorname{arccsc}(cx)+a)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*arccsc(c*x) + a)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\left(a+b\operatorname{asin}\left(\frac{1}{c\,x}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*asin(1/(c*x)))),x)
```

```
[Out] int(1/(x^4*(a + b*asin(1/(c*x)))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a+b\operatorname{acsc}(cx)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a+b*acsc(c*x)),x)
```

```
[Out] Integral(1/(x**4*(a + b*acsc(c*x))), x)
```

3.39 $\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^3\,dx$

Optimal. Leaf size=19

$$\text{Int}\left((dx)^m \left(a+b\csc^{-1}(cx)\right)^3,x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arccsc(c*x))^3,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^3\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcCsc[c*x])^3,x]
[Out] Defer[Int][(d*x)^m*(a + b*ArcCsc[c*x])^3, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^3\,dx = \int (dx)^m \left(a+b\csc^{-1}(cx)\right)^3\,dx$$

Mathematica [A] time = 7.35, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^3\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcCsc[c*x])^3,x]
[Out] Integrate[(d*x)^m*(a + b*ArcCsc[c*x])^3, x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3\,\text{arccsc}\,(cx)^3+3\,ab^2\,\text{arccsc}\,(cx)^2+3\,a^2b\,\text{arccsc}\,(cx)+a^3\right)(dx)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccsc(c*x))^3,x, algorithm="fricas")
[Out] integral((b^3*arccsc(c*x)^3 + 3*a*b^2*arccsc(c*x)^2 + 3*a^2*b*arccsc(c*x) +
 a^3)*(d*x)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{arccsc}\,(cx)+a)^3\,(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccsc(c*x))^3,x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)^3*(d*x)^m, x)
```

maple [A] time = 4.30, size = 0, normalized size = 0.00

$$\int (dx)^m\,(a+b\,\text{arccsc}\,(cx))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arccsc(c*x))^3,x)
```

```
[Out] int((d*x)^m*(a+b*arccsc(c*x))^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccsc(c*x))^3,x, algorithm="maxima")
```

```
[Out] (d*x)^(m + 1)*a^3/(d*(m + 1)) + 1/4*(4*b^3*d^m*x*x^m*arctan2(1, sqrt(c*x +
1)*sqrt(c*x - 1))^3 - 3*b^3*d^m*x*x^m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1
))*log(c^2*x^2)^2 - 4*(m + 1)*integrate(-3/4*((a*b^2*d^m*m + a*b^2*d^m - (a
*b^2*c^2*d^m*m + a*b^2*c^2*d^m)*x^2)*x^m*log(c^2*x^2)^2 + 4*((b^3*arctan2(1
, sqrt(c*x + 1)*sqrt(c*x - 1)) + a*b^2)*d^m*m - ((b^3*arctan2(1, sqrt(c*x +
 1)*sqrt(c*x - 1)) + a*b^2)*c^2*d^m*m + (b^3*arctan2(1, sqrt(c*x + 1)*sqrt(
c*x - 1)) + a*b^2)*c^2*d^m)*x^2 + (b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x -
1)) + a*b^2)*d^m)*x^m*log(x)^2 + 8*((b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x
- 1)) + a*b^2)*d^m*m*log(c) - ((b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))
 + a*b^2)*c^2*d^m*m*log(c) + (b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) +
 a*b^2)*c^2*d^m*log(c))*x^2 + (b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))
+ a*b^2)*d^m*log(c))*x^m*log(x) + (4*b^3*d^m*x^m*arctan2(1, sqrt(c*x + 1)*s
qrt(c*x - 1))^2 - b^3*d^m*x^m*log(c^2*x^2)^2)*sqrt(c*x + 1)*sqrt(c*x - 1) -
 4*((a*b^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2 + a^2*b*arctan2(1, sqr
t(c*x + 1)*sqrt(c*x - 1)) - (b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) +
a*b^2)*log(c)^2)*d^m*m + (((b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + a
*b^2)*c^2*log(c)^2 - (a*b^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2 + a^2
*b*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*c^2)*d^m*m + ((b^3*arctan2(1, s
qrt(c*x + 1)*sqrt(c*x - 1)) + a*b^2)*c^2*log(c)^2 - (a*b^2*arctan2(1, sqrt(
c*x + 1)*sqrt(c*x - 1))^2 + a^2*b*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*
c^2)*d^m)*x^2 + (a*b^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2 + a^2*b*ar
ctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) - (b^3*arctan2(1, sqrt(c*x + 1)*sqrt(
c*x - 1)) + a*b^2)*log(c)^2)*d^m)*x^m - 4*(((b^3*arctan2(1, sqrt(c*x + 1)*s
qrt(c*x - 1)) + a*b^2)*d^m*m - ((b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)
) + a*b^2)*c^2*d^m*m + (b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + a*b^2
)*c^2*d^m)*x^2 + (b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + a*b^2)*d^m)
*x^m*log(x) + ((b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + a*b^2)*d^m*m*
log(c) - ((b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + a*b^2)*c^2*d^m*m*l
og(c) + (b^3*c^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + (b^3*arctan2(1,
sqrt(c*x + 1)*sqrt(c*x - 1)) + a*b^2)*c^2*log(c))*d^m)*x^2 + (b^3*arctan2(1
, sqrt(c*x + 1)*sqrt(c*x - 1)) + (b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1
)) + a*b^2)*log(c))*d^m)*x^m)*log(c^2*x^2))/((c^2*m + c^2)*x^2 - m - 1), x)
)/(m + 1)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int {\left(d\,x\right)}^m\,{\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*asin(1/(c*x)))^3,x)
```

```
[Out] int((d*x)^m*(a + b*asin(1/(c*x)))^3, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(dx\right)^{m} \left(a + b \operatorname{acsc}{\left(cx \right)}\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*acsc(c*x))**3,x)
```

```
[Out] Integral((d*x)**m*(a + b*acsc(c*x))**3, x)
```

3.40 $\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^2 \,dx$

Optimal. Leaf size=19

$$\text{Int}\left((dx)^m \left(a+b\csc^{-1}(cx)\right)^2,x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arccsc(c*x))^2,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^2 \,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcCsc[c*x])^2,x]
```

```
[Out] Defer[Int][(d*x)^m*(a + b*ArcCsc[c*x])^2, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^2 \,dx=\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^2 \,dx$$

Mathematica [A] time = 4.86, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^2 \,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcCsc[c*x])^2,x]
```

```
[Out] Integrate[(d*x)^m*(a + b*ArcCsc[c*x])^2, x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2 \operatorname{arccsc}(cx)^2+2\,ab \operatorname{arccsc}(cx)+a^2\right)(dx)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccsc(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arccsc(c*x)^2 + 2*a*b*arccsc(c*x) + a^2)*(d*x)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arccsc}(cx)+a)^2 (dx)^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccsc(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)^2*(d*x)^m, x)
```

maple [A] time = 3.97, size = 0, normalized size = 0.00

$$\int (dx)^m (a+b \operatorname{arccsc}(cx))^2 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arccsc(c*x))^2,x)
[Out] int((d*x)^m*(a+b*arccsc(c*x))^2,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccsc(c*x))^2,x, algorithm="maxima")
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int (d\,x)^m\,{\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*asin(1/(c*x)))^2,x)
[Out] int((d*x)^m*(a + b*asin(1/(c*x)))^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d x\right)^{m} \left(a + b \operatorname{acsc}{\left(c x \right)}\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*acsc(c*x))**2,x)
[Out] Integral((d*x)**m*(a + b*acsc(c*x))**2, x)
```

3.41 $\int (dx)^m \left(a+b\csc^{-1}(cx)\right)\,dx$

Optimal. Leaf size=66

$$\frac{(dx)^{m+1}\left(a+b\csc^{-1}(cx)\right)}{d(m+1)}+\frac{b(dx)^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{c^2x^2}\right)}{cm(m+1)}$$

```
[Out] (d*x)^(1+m)*(a+b*arccsc(c*x))/d/(1+m)+b*(d*x)^m*hypergeom([1/2, -1/2*m],[1-
1/2*m],1/c^2/x^2)/c/m/(1+m)
```

Rubi [A] time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5221, 339, 364}

$$\frac{(dx)^{m+1}\left(a+b\csc^{-1}(cx)\right)}{d(m+1)}+\frac{b(dx)^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{c^2x^2}\right)}{cm(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*ArcCsc[c*x]),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*ArcCsc[c*x]))/(d*(1 + m)) + (b*(d*x)^m*Hypergeometric
2F1[1/2, -m/2, 1 - m/2, 1/(c^2*x^2)])/(c*m*(1 + m))
```

Rule 339

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Dist[((c
*x)^(m + 1)*(1/x)^(m + 1))/c, Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x
], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5221

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((d*x)^(m + 1)*(a + b*ArcCsc[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m +
1)), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (dx)^m\left(a+b\csc^{-1}(cx)\right)\,dx&=\frac{(dx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{d(1+m)}+\frac{(bd)\int\frac{(dx)^{-1+m}}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{c(1+m)}\\&=\frac{(dx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{d(1+m)}-\frac{\left(b\left(\frac{1}{x}\right)^m(dx)^m\right)\operatorname{Subst}\left(\int\frac{x^{-1-m}}{\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{c(1+m)}\\&=\frac{(dx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{d(1+m)}+\frac{b(dx)^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{c^2x^2}\right)}{cm(1+m)}\end{aligned}$$

Mathematica [A] time = 0.19, size = 83, normalized size = 1.26

$$\frac{(dx)^m\left((m+1)x\left(a+b\csc^{-1}(cx)\right)+\frac{b\sqrt{1-c^2x^2}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};c^2x^2\right)}{c\sqrt{1-\frac{1}{c^2x^2}}}\right)}{(m+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*ArcCsc[c*x]),x]
```

```
[Out] ((d*x)^m*((1 + m)*x*(a + b*ArcCsc[c*x]) + (b*Sqrt[1 - c^2*x^2]*Hypergeometr
ic2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(c*Sqrt[1 - 1/(c^2*x^2)])))/(1 +
 m)^2
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((b\,\mathrm{arccsc}\,(cx)+a)\,(dx)^m\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)*(d*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(b\,\mathrm{arccsc}\,(cx)+a)\,(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*(d*x)^m, x)
```

maple [F] time = 3.87, size = 0, normalized size = 0.00

$$\int(dx)^m\,(a+b\,\mathrm{arccsc}\,(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arccsc(c*x)),x)
```

```
[Out] int((d*x)^m*(a+b*arccsc(c*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(d^m x x^m\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right)-\left(c^2d^m m+c^2d^m\right)\int\frac{\sqrt{cx+1}\,\sqrt{cx-1}\,x^m}{c^2m-\left(c^4m+c^4\right)x^2+c^2}\,dx\right)b}{m+1}+\frac{(dx)^{m+1}a}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] (d^m*x*x^m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + (c^2*d^m*m + c^2*d^m)*
integrate(-sqrt(c*x + 1)*sqrt(c*x - 1)*x^m/(c^2*m - (c^4*m + c^4)*x^2 + c^2
), x))*b/(m + 1) + (d*x)^(m + 1)*a/(d*(m + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int(d\,x)^m\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*asin(1/(c*x))),x)
```

```
[Out] int((d*x)^m*(a + b*asin(1/(c*x))), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b \operatorname{acsc}(cx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*acsc(c*x)),x)
```

```
[Out] Integral((d*x)**m*(a + b*acsc(c*x)), x)
```

3.42 $\int \frac{(dx)^m}{a+b\csc^{-1}(cx)}\,dx$

Optimal. Leaf size=19

$$\mathrm{Int}\left(\frac{(dx)^m}{a+b\csc^{-1}(cx)},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arccsc(c*x)),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(dx)^m}{a+b\csc^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcCsc[c*x]),x]
```

```
[Out] Defer[Int][(d*x)^m/(a + b*ArcCsc[c*x]), x]
```

Rubi steps

$$\int \frac{(dx)^m}{a+b\csc^{-1}(cx)}\,dx = \int \frac{(dx)^m}{a+b\csc^{-1}(cx)}\,dx$$

Mathematica [A] time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a+b\csc^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcCsc[c*x]),x]
```

```
[Out] Integrate[(d*x)^m/(a + b*ArcCsc[c*x]), x]
```

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(dx)^m}{b\,\mathrm{arccsc}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b*arccsc(c*x) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{b\,\mathrm{arccsc}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arccsc(c*x) + a), x)
```

maple [A] time = 2.83, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a + b \operatorname{arccsc}(cx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arccsc(c*x)),x)
```

```
[Out] int((d*x)^m/(a+b*arccsc(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{b \operatorname{arccsc}(cx) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(b*arccsc(c*x) + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d\,x)^m}{a + b \operatorname{asin}\left(\frac{1}{c\,x}\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*asin(1/(c*x))),x)
```

```
[Out] int((d*x)^m/(a + b*asin(1/(c*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a + b \operatorname{acsc}(cx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*acsc(c*x)),x)
```

```
[Out] Integral((d*x)**m/(a + b*acsc(c*x)), x)
```

3.43 $\int \frac{(dx)^m}{\left(a+b\csc^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{(dx)^m}{\left(a+b\csc^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arccsc(c*x))^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(dx)^m}{\left(a+b\csc^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcCsc[c*x])^2,x]
[Out] Defer[Int][(d*x)^m/(a + b*ArcCsc[c*x])^2, x]
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a+b\csc^{-1}(cx)\right)^2}\,dx=\int \frac{(dx)^m}{\left(a+b\csc^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 1.78, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(a+b\csc^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcCsc[c*x])^2,x]
[Out] Integrate[(d*x)^m/(a + b*ArcCsc[c*x])^2, x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx)^m}{b^2\,\text{arccsc}\,(cx)^2+2\,ab\,\text{arccsc}\,(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arccsc(c*x))^2,x, algorithm="fricas")
[Out] integral((d*x)^m/(b^2*arccsc(c*x)^2 + 2*a*b*arccsc(c*x) + a^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(b\,\text{arccsc}\,(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arccsc(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arccsc(c*x) + a)^2, x)
```

maple [A] time = 2.92, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(a + b\,\mathrm{arccsc}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arccsc(c*x))^2,x)
```

```
[Out] int((d*x)^m/(a+b*arccsc(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{4\sqrt{cx+1}\,\sqrt{cx-1}\left(b\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right)+a\right)d^m x x^m - 4\left(4\,b^3\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right)^2 + b^3\log\left(c^2x\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arccsc(c*x))^2,x, algorithm="maxima")
```

```
[Out] (4*sqrt(c*x + 1)*sqrt(c*x - 1)*(b*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) +
 a)*d^m*x*x^m - (4*b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*log(
c^2*x^2)^2 + 4*b^3*log(c)^2 + 8*b^3*log(c)*log(x) + 4*b^3*log(x)^2 + 8*a*b^
2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*a^2*b - 4*(b^3*log(c) + b^3*l
og(x))*log(c^2*x^2))*integrate(4*((b*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)
) + a)*d^m*m - ((b*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + a)*c^2*d^m*m +
 2*(b*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + a)*c^2*d^m)*x^2 + (b*arctan
2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + a)*d^m)*sqrt(c*x + 1)*sqrt(c*x - 1)*x^m
/(4*b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 4*b^3*log(c)^2 + 8*a*b^
2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*a^2*b - 4*(b^3*c^2*log(c)^2 +
 (b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 2*a*b^2*arctan2(1, sqrt(c
*x + 1)*sqrt(c*x - 1)) + a^2*b)*c^2)*x^2 - (b^3*c^2*x^2 - b^3)*log(c^2*x^2)
^2 - 4*(b^3*c^2*x^2 - b^3)*log(x)^2 + 4*(b^3*c^2*x^2*log(c) - b^3*log(c) +
(b^3*c^2*x^2 - b^3)*log(x))*log(c^2*x^2) - 8*(b^3*c^2*x^2*log(c) - b^3*log(
c))*log(x)), x))/(4*b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*log
(c^2*x^2)^2 + 4*b^3*log(c)^2 + 8*b^3*log(c)*log(x) + 4*b^3*log(x)^2 + 8*a*b
^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*a^2*b - 4*(b^3*log(c) + b^3*
log(x))*log(c^2*x^2))
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d\,x)^m}{\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*asin(1/(c*x)))^2,x)
```

```
[Out] int((d*x)^m/(a + b*asin(1/(c*x)))^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(a + b\,\mathrm{acsc}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*acsc(c*x))**2,x)
```

```
[Out] Integral((d*x)**m/(a + b*acsc(c*x))**2, x)
```

3.44 $\int (d+ex)^3 \left(a+b\csc^{-1}(cx)\right) dx$

Optimal. Leaf size=167

$$\frac{(d+ex)^4\left(a+b\csc^{-1}(cx)\right)}{4e}+\frac{bde^2x^2\sqrt{1-\frac{1}{c^2x^2}}}{2c}+\frac{be^3x^3\sqrt{1-\frac{1}{c^2x^2}}}{12c}+\frac{bex\sqrt{1-\frac{1}{c^2x^2}}\left(9c^2d^2+e^2\right)}{6c^3}+\frac{bd\left(2c^2d^2+e^2\right)\mathrm{t}}{2}$$

```
[Out] -1/4*b*d^4*arccsc(c*x)/e+1/4*(e*x+d)^4*(a+b*arccsc(c*x))/e+1/2*b*d*(2*c^2*d
^2+e^2)*arctanh((1-1/c^2/x^2)^(1/2))/c^3+1/6*b*e*(9*c^2*d^2+e^2)*x*(1-1/c^2
/x^2)^(1/2)/c^3+1/2*b*d*e^2*x^2*(1-1/c^2/x^2)^(1/2)/c+1/12*b*e^3*x^3*(1-1/c
^2/x^2)^(1/2)/c
```

Rubi [A] time = 0.40, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {5227, 1568, 1475, 1807, 844, 216, 266, 63, 208}

$$\frac{(d+ex)^4\left(a+b\csc^{-1}(cx)\right)}{4e}+\frac{bex\sqrt{1-\frac{1}{c^2x^2}}\left(9c^2d^2+e^2\right)}{6c^3}+\frac{bd\left(2c^2d^2+e^2\right)\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{2c^3}+\frac{bde^2x^2\sqrt{1-\frac{1}{c^2x^2}}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*e*(9*c^2*d^2 + e^2)*Sqrt[1 - 1/(c^2*x^2)]*x)/(6*c^3) + (b*d*e^2*Sqrt[1 -
 1/(c^2*x^2)]*x^2)/(2*c) + (b*e^3*Sqrt[1 - 1/(c^2*x^2)]*x^3)/(12*c) - (b*d^
4*ArcCsc[c*x])/(4*e) + ((d + e*x)^4*(a + b*ArcCsc[c*x]))/(4*e) + (b*d*(2*c^
2*d^2 + e^2)*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/(2*c^3)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1475

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1568

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 5227

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCsc[c*x]))/(e*(m + 1)), x] + Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)^3\left(a+b\csc^{-1}(cx)\right)dx = \frac{(d+ex)^4\left(a+b\csc^{-1}(cx)\right)}{4e}+\frac{b\int \frac{(d+ex)^4}{\sqrt{1-\frac{1}{c^2x^2}}\,x^2}dx}{4ce}$$

$$= \frac{(d+ex)^4\left(a+b\csc^{-1}(cx)\right)}{4e}+\frac{b\int \frac{\left(e+\frac{d}{x}\right)^4x^2}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{4ce}$$

$$= \frac{(d+ex)^4\left(a+b\csc^{-1}(cx)\right)}{4e}-\frac{b\,\text{Subst}\left(\int \frac{(e+dx)^4}{x^4\sqrt{1-\frac{x^2}{c^2}}}dx,x,\frac{1}{x}\right)}{4ce}$$

$$= \frac{be^3\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c}+\frac{(d+ex)^4\left(a+b\csc^{-1}(cx)\right)}{4e}+\frac{b\,\text{Subst}\left(\int \frac{-12de^3-2e^2\left(9d^2+\frac{e^2}{c^2}\ldots\right.}{x^3\sqrt{\ldots}}\ldots\right.}{12c\ldots}$$

$$= \frac{bde^2\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{2c}+\frac{be^3\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c}+\frac{(d+ex)^4\left(a+b\csc^{-1}(cx)\right)}{4e}-\frac{b\,\text{Subs}\ldots}{\ldots}$$

$$= \frac{be\left(9c^2d^2+e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x}{6c^3}+\frac{bde^2\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{2c}+\frac{be^3\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c}+\frac{(d+e\ldots}{\ldots}$$

$$= \frac{be\left(9c^2d^2+e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x}{6c^3}+\frac{bde^2\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{2c}+\frac{be^3\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c}+\frac{(d+e\ldots}{\ldots}$$

$$= \frac{be\left(9c^2d^2+e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x}{6c^3}+\frac{bde^2\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{2c}+\frac{be^3\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c}-\frac{bd^4\,\text{cs}\ldots}{4\ldots}$$

$$= \frac{be\left(9c^2d^2+e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x}{6c^3}+\frac{bde^2\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{2c}+\frac{be^3\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c}-\frac{bd^4\,\text{cs}\ldots}{4\ldots}$$

$$= \frac{be\left(9c^2d^2+e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x}{6c^3}+\frac{bde^2\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{2c}+\frac{be^3\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c}-\frac{bd^4\,\text{cs}\ldots}{4\ldots}$$

Mathematica [A] time = 0.31, size = 165, normalized size = 0.99

$$\frac{3ac^3x\left(4d^3+6d^2ex+4de^2x^2+e^3x^3\right)+3bc^3x\csc^{-1}(cx)\left(4d^3+6d^2ex+4de^2x^2+e^3x^3\right)+bex\sqrt{1-\frac{1}{c^2x^2}}\left(c^2\left(18\ldots\right.\right.}{12c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (3*a*c^3*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) + b*e*Sqrt[1 - 1/(c^
2*x^2)]*x*(2*e^2 + c^2*(18*d^2 + 6*d*e*x + e^2*x^2)) + 3*b*c^3*x*(4*d^3 + 6
*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3)*ArcCsc[c*x] + 6*b*d*(2*c^2*d^2 + e^2)*Log
[(1 + Sqrt[1 - 1/(c^2*x^2)])*x])/(12*c^3)
```

fricas [A] time = 0.61, size = 290, normalized size = 1.74

$$\frac{3\,ac^4e^3x^4+12\,ac^4de^2x^3+18\,ac^4d^2ex^2+12\,ac^4d^3x+3\left(bc^4e^3x^4+4\,bc^4de^2x^3+6\,bc^4d^2ex^2+4\,bc^4d^3x-4\,bc^4d^3-\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*c^4*e^3*x^4 + 12*a*c^4*d*e^2*x^3 + 18*a*c^4*d^2*e*x^2 + 12*a*c^4*
d^3*x + 3*(b*c^4*e^3*x^4 + 4*b*c^4*d*e^2*x^3 + 6*b*c^4*d^2*e*x^2 + 4*b*c^4*
d^3*x - 4*b*c^4*d^3 - 6*b*c^4*d^2*e - 4*b*c^4*d*e^2 - b*c^4*e^3)*arccsc(c*x
) - 6*(4*b*c^4*d^3 + 6*b*c^4*d^2*e + 4*b*c^4*d*e^2 + b*c^4*e^3)*arctan(-c*x
 + sqrt(c^2*x^2 - 1)) - 6*(2*b*c^3*d^3 + b*c*d*e^2)*log(-c*x + sqrt(c^2*x^2
 - 1)) + (b*c^2*e^3*x^2 + 6*b*c^2*d*e^2*x + 18*b*c^2*d^2*e + 2*b*e^3)*sqrt(
c^2*x^2 - 1))/c^4
```

giac [B] time = 2.01, size = 1112, normalized size = 6.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/192*(3*b*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))*e^3/c + 3*a*x
^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*e^3/c + 24*b*d*x^3*(sqrt(-1/(c^2*x^2) + 1
) + 1)^3*arcsin(1/(c*x))*e^2/c + 24*a*d*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*
e^2/c + 72*b*d^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))*e/c + 7
2*a*d^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*e/c + 96*b*d^3*x*(sqrt(-1/(c^2*x
^2) + 1) + 1)*arcsin(1/(c*x))/c + 2*b*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*e^
3/c^2 + 96*a*d^3*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c + 24*b*d*x^2*(sqrt(-1/(c^
2*x^2) + 1) + 1)^2*e^2/c^2 + 12*b*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin
(1/(c*x))*e^3/c^3 + 144*b*d^2*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*e/c^2 + 12*a*x
^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*e^3/c^3 + 72*b*d*x*(sqrt(-1/(c^2*x^2) + 1
) + 1)*arcsin(1/(c*x))*e^2/c^3 + 192*b*d^3*log(sqrt(-1/(c^2*x^2) + 1) + 1)/
c^2 - 192*b*d^3*log(1/(abs(c)*abs(x)))/c^2 + 72*a*d*x*(sqrt(-1/(c^2*x^2) +
1) + 1)*e^2/c^3 + 144*b*d^2*arcsin(1/(c*x))*e/c^3 + 144*a*d^2*e/c^3 + 96*b*
d^3*arcsin(1/(c*x))/(c^3*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 18*b*x*(sqrt(-1/
(c^2*x^2) + 1) + 1)*e^3/c^4 + 96*b*d*e^2*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^
4 - 96*b*d*e^2*log(1/(abs(c)*abs(x)))/c^4 + 96*a*d^3/(c^3*x*(sqrt(-1/(c^2*x
^2) + 1) + 1)) + 18*b*arcsin(1/(c*x))*e^3/c^5 - 144*b*d^2*e/(c^4*x*(sqrt(-1
/(c^2*x^2) + 1) + 1)) + 18*a*e^3/c^5 + 72*b*d*arcsin(1/(c*x))*e^2/(c^5*x*(s
qrt(-1/(c^2*x^2) + 1) + 1)) + 72*a*d*e^2/(c^5*x*(sqrt(-1/(c^2*x^2) + 1) + 1
)) + 72*b*d^2*arcsin(1/(c*x))*e/(c^5*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) +
72*a*d^2*e/(c^5*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) - 18*b*e^3/(c^6*x*(sqrt
(-1/(c^2*x^2) + 1) + 1)) - 24*b*d*e^2/(c^6*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)
^2) + 12*b*arcsin(1/(c*x))*e^3/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 1
2*a*e^3/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 24*b*d*arcsin(1/(c*x))*e
^2/(c^7*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 24*a*d*e^2/(c^7*x^3*(sqrt(-1/
(c^2*x^2) + 1) + 1)^3) - 2*b*e^3/(c^8*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) +
 3*b*arcsin(1/(c*x))*e^3/(c^9*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) + 3*a*e^3
/(c^9*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4))*c
```

maple [B] time = 0.06, size = 485, normalized size = 2.90

$$\frac{a\,e^3x^4}{4}+a\,e^2x^3d+\frac{3ae\,x^2d^2}{2}+ax\,d^3+\frac{a\,d^4}{4e}+\frac{b\,e^3\mathrm{arccsc}\,(cx)\,x^4}{4}+b\,e^2\mathrm{arccsc}\,(cx)\,x^3d+\frac{3be\,\mathrm{arccsc}\,(cx)\,x^2d^2}{2}+b\,\mathrm{arccsc}\,(cx\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a+b*arccsc(c*x)),x)
```

```
[Out] 1/4*a*e^3*x^4+a*e^2*x^3*d+3/2*a*e*x^2*d^2+a*x*d^3+1/4*a/e*d^4+1/4*b*e^3*arc
csc(c*x)*x^4+b*e^2*arccsc(c*x)*x^3*d+3/2*b*e*arccsc(c*x)*x^2*d^2+b*arccsc(c
*x)*x*d^3+1/4*b*d^4*arccsc(c*x)/e-1/4/c*b/e*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/
c^2/x^2)^(1/2)/x*d^4*arctan(1/(c^2*x^2-1)^(1/2))+1/c^2*b*(c^2*x^2-1)^(1/2)/
((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d^3*ln(c*x+(c^2*x^2-1)^(1/2))+1/12/c*b*e^3/((
c^2*x^2-1)/c^2/x^2)^(1/2)*x^3+1/12/c^3*b*e^3/((c^2*x^2-1)/c^2/x^2)^(1/2)*x+
1/2/c*b*e^2/((c^2*x^2-1)/c^2/x^2)^(1/2)*d*x^2-1/2/c^3*b*e^2/((c^2*x^2-1)/c^
2/x^2)^(1/2)*d+3/2/c*b*e/((c^2*x^2-1)/c^2/x^2)^(1/2)*x*d^2-3/2/c^3*b*e/((c^
2*x^2-1)/c^2/x^2)^(1/2)/x*d^2+1/2/c^4*b*e^2*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/
c^2/x^2)^(1/2)/x*d*ln(c*x+(c^2*x^2-1)^(1/2))-1/6/c^5*b*e^3/((c^2*x^2-1)/c^2
/x^2)^(1/2)/x
```

maxima [A] time = 0.38, size = 269, normalized size = 1.61

$$\frac{1}{4}ae^3x^4+ade^2x^3+\frac{3}{2}ad^2ex^2+\frac{3}{2}\left(x^2\operatorname{arccsc}(cx)+\frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)bd^2e+\frac{1}{4}\left(4x^3\operatorname{arccsc}(cx)+\frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\log\left(\sqrt{\cdots}\right.}{\cdots}}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*e^3*x^4 + a*d*e^2*x^3 + 3/2*a*d^2*e*x^2 + 3/2*(x^2*arccsc(c*x) + x*sq
rt(-1/(c^2*x^2) + 1)/c)*b*d^2*e + 1/4*(4*x^3*arccsc(c*x) + (2*sqrt(-1/(c^2*
x^2) + 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c
^2 - log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d*e^2 + 1/12*(3*x^4*arccsc(c
*x) + (c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*
b*e^3 + a*d^3*x + 1/2*(2*c*x*arccsc(c*x) + log(sqrt(-1/(c^2*x^2) + 1) + 1)
- log(-sqrt(-1/(c^2*x^2) + 1) + 1))*b*d^3/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,{\left(d+e\,x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))*(d + e*x)^3,x)
```

```
[Out] int((a + b*asin(1/(c*x)))*(d + e*x)^3, x)
```

sympy [A] time = 8.42, size = 362, normalized size = 2.17

$$ad^3x+\frac{3ad^2ex^2}{2}+ade^2x^3+\frac{ae^3x^4}{4}+bd^3x\operatorname{acsc}(cx)+\frac{3bd^2ex^2\operatorname{acsc}(cx)}{2}+bde^2x^3\operatorname{acsc}(cx)+\frac{be^3x^4\operatorname{acsc}(cx)}{4}+\frac{bd^3\left(\left\{\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a+b*acsc(c*x)),x)
```

```
[Out] a*d**3*x + 3*a*d**2*e*x**2/2 + a*d*e**2*x**3 + a*e**3*x**4/4 + b*d**3*x*acs
c(c*x) + 3*b*d**2*e*x**2*acsc(c*x)/2 + b*d*e**2*x**3*acsc(c*x) + b*e**3*x**
4*acsc(c*x)/4 + b*d**3*Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin
(c*x), True))/c + 3*b*d**2*e*Piecewise((sqrt(c**2*x**2 - 1)/c, Abs(c**2*x**
```

```
2) > 1), (I*sqrt(-c**2*x**2 + 1)/c, True))/(2*c) + b*d*e**2*Piecewise((x*sq
rt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1), (-I*c*x
**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*asin(c*x)
/(2*c**2), True))/c + b*e**3*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3*c) + 2*
sqrt(c**2*x**2 - 1)/(3*c**3), Abs(c**2*x**2) > 1), (I*x**2*sqrt(-c**2*x**2
+ 1)/(3*c) + 2*I*sqrt(-c**2*x**2 + 1)/(3*c**3), True))/(4*c)
```

3.45 $\int (d+ex)^2 \left(a+b\csc^{-1}(cx)\right) dx$

Optimal. Leaf size=123

$$\frac{(d+ex)^3\left(a+b\csc^{-1}(cx)\right)}{3e}+\frac{bdex\sqrt{1-\frac{1}{c^2x^2}}}{c}+\frac{be^2x^2\sqrt{1-\frac{1}{c^2x^2}}}{6c}+\frac{b\left(6c^2d^2+e^2\right)\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c^3}-\frac{bd^3\csc^{-1}(c$$

```
[Out] -1/3*b*d^3*arccsc(c*x)/e+1/3*(e*x+d)^3*(a+b*arccsc(c*x))/e+1/6*b*(6*c^2*d^2
+e^2)*arctanh((1-1/c^2/x^2)^(1/2))/c^3+b*d*e*x*(1-1/c^2/x^2)^(1/2)/c+1/6*b*
e^2*x^2*(1-1/c^2/x^2)^(1/2)/c
```

Rubi [A] time = 0.28, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {5227, 1568, 1475, 1807, 844, 216, 266, 63, 208}

$$\frac{(d+ex)^3\left(a+b\csc^{-1}(cx)\right)}{3e}+\frac{b\left(6c^2d^2+e^2\right)\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c^3}+\frac{bdex\sqrt{1-\frac{1}{c^2x^2}}}{c}+\frac{be^2x^2\sqrt{1-\frac{1}{c^2x^2}}}{6c}-\frac{bd^3\csc^{-1}(c$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*d*e*Sqrt[1 - 1/(c^2*x^2)]*x)/c + (b*e^2*Sqrt[1 - 1/(c^2*x^2)]*x^2)/(6*c)
 - (b*d^3*ArcCsc[c*x])/(3*e) + ((d + e*x)^3*(a + b*ArcCsc[c*x]))/(3*e) + (b
*(6*c^2*d^2 + e^2)*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/(6*c^3)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1475

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1568

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 5227

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCsc[c*x]))/(e*(m + 1)), x] + Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)^2\left(a+b\csc^{-1}(cx)\right)dx = \frac{(d+ex)^3\left(a+b\csc^{-1}(cx)\right)}{3e}+\frac{b\int \frac{(d+ex)^3}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{3ce}$$

$$= \frac{(d+ex)^3\left(a+b\csc^{-1}(cx)\right)}{3e}+\frac{b\int \frac{\left(e+\frac{d}{x}\right)^3x}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{3ce}$$

$$= \frac{(d+ex)^3\left(a+b\csc^{-1}(cx)\right)}{3e}-\frac{b\,\text{Subst}\left(\int \frac{(e+dx)^3}{x^3\sqrt{1-\frac{x^2}{c^2}}}dx,x,\frac{1}{x}\right)}{3ce}$$

$$= \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}x^2}{6c}+\frac{(d+ex)^3\left(a+b\csc^{-1}(cx)\right)}{3e}+\frac{b\,\text{Subst}\left(\int \frac{-6de^2-e\left(6d^2+\frac{e^2}{c^2}\right)x\cdots}{x^2\sqrt{1-\frac{x^2}{c^2}}}\cdots\right.}{6ce}$$

$$= \frac{bde\sqrt{1-\frac{1}{c^2x^2}}x}{c}+\frac{be^2\sqrt{1-\frac{1}{c^2x^2}}x^2}{6c}+\frac{(d+ex)^3\left(a+b\csc^{-1}(cx)\right)}{3e}-\frac{b\,\text{Subst}\cdots}{\cdots}$$

$$= \frac{bde\sqrt{1-\frac{1}{c^2x^2}}x}{c}+\frac{be^2\sqrt{1-\frac{1}{c^2x^2}}x^2}{6c}+\frac{(d+ex)^3\left(a+b\csc^{-1}(cx)\right)}{3e}-\frac{\left(bd^3\right)\text{Su}\cdots}{\cdots}$$

$$= \frac{bde\sqrt{1-\frac{1}{c^2x^2}}x}{c}+\frac{be^2\sqrt{1-\frac{1}{c^2x^2}}x^2}{6c}-\frac{bd^3\csc^{-1}(cx)}{3e}+\frac{(d+ex)^3\left(a+b\csc^{-1}\cdots\right.}{3e}$$

$$= \frac{bde\sqrt{1-\frac{1}{c^2x^2}}x}{c}+\frac{be^2\sqrt{1-\frac{1}{c^2x^2}}x^2}{6c}-\frac{bd^3\csc^{-1}(cx)}{3e}+\frac{(d+ex)^3\left(a+b\csc^{-1}\cdots\right.}{3e}$$

$$= \frac{bde\sqrt{1-\frac{1}{c^2x^2}}x}{c}+\frac{be^2\sqrt{1-\frac{1}{c^2x^2}}x^2}{6c}-\frac{bd^3\csc^{-1}(cx)}{3e}+\frac{(d+ex)^3\left(a+b\csc^{-1}\cdots\right.}{3e}$$

Mathematica [A] time = 0.19, size = 122, normalized size = 0.99

$$\frac{c^2x\left(2ac\left(3d^2+3dex+e^2x^2\right)+be\sqrt{1-\frac{1}{c^2x^2}}(6d+ex)\right)+2bc^3x\csc^{-1}(cx)\left(3d^2+3dex+e^2x^2\right)+b\left(6c^2d^2+e^2\right)\cdots}{6c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (c^2*x*(b*e*Sqrt[1 - 1/(c^2*x^2)]*(6*d + e*x) + 2*a*c*(3*d^2 + 3*d*e*x + e^
2*x^2)) + 2*b*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2)*ArcCsc[c*x] + b*(6*c^2*d^2
+ e^2)*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x])/(6*c^3)
```

fricas [A] time = 0.55, size = 209, normalized size = 1.70

$$\frac{2\,ac^3e^2x^3+6\,ac^3dex^2+6\,ac^3d^2x+2\left(bc^3e^2x^3+3\,bc^3dex^2+3\,bc^3d^2x-3\,bc^3d^2-3\,bc^3de-bc^3e^2\right)\text{arccsc}(cx)-\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*e^2*x^3 + 6*a*c^3*d*e*x^2 + 6*a*c^3*d^2*x + 2*(b*c^3*e^2*x^3 +
 3*b*c^3*d*e*x^2 + 3*b*c^3*d^2*x - 3*b*c^3*d^2 - 3*b*c^3*d*e - b*c^3*e^2)*a
rccsc(c*x) - 4*(3*b*c^3*d^2 + 3*b*c^3*d*e + b*c^3*e^2)*arctan(-c*x + sqrt(c
^2*x^2 - 1)) - (6*b*c^2*d^2 + b*e^2)*log(-c*x + sqrt(c^2*x^2 - 1)) + (b*c*e
^2*x + 6*b*c*d*e)*sqrt(c^2*x^2 - 1))/c^3
```

giac [B] time = 2.93, size = 595, normalized size = 4.84

$$\frac{1}{24}\left(\frac{bx^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3\arcsin\left(\frac{1}{cx}\right)e^2}{c}+\frac{ax^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3e^2}{c}-\frac{24\,bdx^2\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)e}{c}+\frac{12\,bd^2x}{\quad}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/24*(b*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x))*e^2/c + a*x^3*(s
qrt(-1/(c^2*x^2) + 1) + 1)^3*e^2/c - 24*b*d*x^2*(1/(c^2*x^2) - 1)*arcsin(1/
(c*x))*e/c + 12*b*d^2*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c - 24
*a*d*x^2*(1/(c^2*x^2) - 1)*e/c + 12*a*d^2*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c
+ b*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*e^2/c^2 + 24*b*d*x*sqrt(-1/(c^2*x^2)
 + 1)*e/c^2 + 3*b*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))*e^2/c^3 +
24*b*d^2*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - 24*b*d^2*log(1/(abs(c)*abs(x
)))/c^2 + 3*a*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*e^2/c^3 + 24*b*d*arcsin(1/(c*x
))*e/c^3 + 24*a*d*e/c^3 + 12*b*d^2*arcsin(1/(c*x))/(c^3*x*(sqrt(-1/(c^2*x^2
) + 1) + 1)) + 4*b*e^2*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 - 4*b*e^2*log(1/
(abs(c)*abs(x)))/c^4 + 12*a*d^2/(c^3*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3*b*
arcsin(1/(c*x))*e^2/(c^5*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3*a*e^2/(c^5*x*(
sqrt(-1/(c^2*x^2) + 1) + 1)) - b*e^2/(c^6*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^
2) + b*arcsin(1/(c*x))*e^2/(c^7*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + a*e^2
/(c^7*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3))*c
```

maple [B] time = 0.05, size = 361, normalized size = 2.93

$$\frac{a\,e^2x^3}{3}+aed\,x^2+ax\,d^2+\frac{a\,d^3}{3e}+\frac{b\,e^2\mathrm{arccsc}\,(cx)\,x^3}{3}+be\,\mathrm{arccsc}\,(cx)\,x^2d+b\,\mathrm{arccsc}\,(cx)\,x\,d^2+\frac{b\,d^3\mathrm{arccsc}\,(cx)}{3e}-\frac{b\sqrt{c^2x^2-\cdots}}{3\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(a+b*arccsc(c*x)),x)
```

```
[Out] 1/3*a*e^2*x^3+a*e*d*x^2+a*x*d^2+1/3*a/e*d^3+1/3*b*e^2*arccsc(c*x)*x^3+b*e*a
rccsc(c*x)*x^2*d+b*arccsc(c*x)*x*d^2+1/3*b*d^3*arccsc(c*x)/e-1/3/c*b/e*(c^2
*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d^3*arctan(1/(c^2*x^2-1)^(1/2))
+1/c^2*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d^2*ln(c*x+(c^2*x^
2-1)^(1/2))+1/6/c*b*e^2/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^2-1/6/c^3*b*e^2/((c^2
*x^2-1)/c^2/x^2)^(1/2)+1/c*b*e/((c^2*x^2-1)/c^2/x^2)^(1/2)*x*d-1/c^3*b*e/((
c^2*x^2-1)/c^2/x^2)^(1/2)/x*d+1/6/c^4*b*e^2*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/
c^2/x^2)^(1/2)/x*ln(c*x+(c^2*x^2-1)^(1/2))
```

maxima [A] time = 0.37, size = 198, normalized size = 1.61

$$\frac{1}{3}\,ae^2x^3+adex^2+\left(x^2\,\mathrm{arccsc}\,(cx)+\frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)bde+\frac{1}{12}\left(4\,x^3\,\mathrm{arccsc}\,(cx)+\frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}-\frac{\log(\cdots)}{\cdots}}{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*e^2*x^3 + a*d*e*x^2 + (x^2*arccsc(c*x) + x*sqrt(-1/(c^2*x^2) + 1)/c)*
b*d*e + 1/12*(4*x^3*arccsc(c*x) + (2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^
2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c^2*x^2
) + 1) - 1)/c^2)/c)*b*e^2 + a*d^2*x + 1/2*(2*c*x*arccsc(c*x) + log(sqrt(-1/
(c^2*x^2) + 1) + 1) - log(-sqrt(-1/(c^2*x^2) + 1) + 1))*b*d^2/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,(d + e\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))*(d + e*x)^2,x)
```

```
[Out] int((a + b*asin(1/(c*x)))*(d + e*x)^2, x)
```

sympy [A] time = 7.03, size = 228, normalized size = 1.85

$$ad^2x+adex^2+\frac{ae^2x^3}{3}+bd^2x\,\mathrm{acsc}\,(cx)+bdex^2\,\mathrm{acsc}\,(cx)+\frac{be^2x^3\,\mathrm{acsc}\,(cx)}{3}+\frac{bd^2\left(\begin{cases}\mathrm{acosh}\,(cx) & \text{for } \left|c^2x^2\right|>1\\ -i\,\mathrm{asin}\,(cx) & \text{otherwise}\end{cases}\right)}{c}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(a+b*acsc(c*x)),x)
```

```
[Out] a*d**2*x + a*d*e*x**2 + a*e**2*x**3/3 + b*d**2*x*acsc(c*x) + b*d*e*x**2*acs
c(c*x) + b*e**2*x**3*acsc(c*x)/3 + b*d**2*Piecewise((acosh(c*x), Abs(c**2*x
**2) > 1), (-I*asin(c*x), True))/c + b*d*e*Piecewise((sqrt(c**2*x**2 - 1)/c
, Abs(c**2*x**2) > 1), (I*sqrt(-c**2*x**2 + 1)/c, True))/c + b*e**2*Piecewi
se((x*sqrt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1),
 (-I*c*x**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*a
sin(c*x)/(2*c**2), True))/(3*c)
```

3.46 $\int (d+ex)\left(a+b\csc^{-1}(cx)\right)\,dx$

Optimal. Leaf size=83

$$\frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e}+\frac{bd\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}+\frac{bex\sqrt{1-\frac{1}{c^2x^2}}}{2c}-\frac{bd^2\csc^{-1}(cx)}{2e}$$

```
[Out] -1/2*b*d^2*arccsc(c*x)/e+1/2*(e*x+d)^2*(a+b*arccsc(c*x))/e+b*d*arctanh((1-1
/c^2/x^2)^(1/2))/c+1/2*b*e*x*(1-1/c^2/x^2)^(1/2)/c
```

Rubi [A] time = 0.17, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5227, 1568, 1396, 1807, 844, 216, 266, 63, 208}

$$\frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e}+\frac{bd\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}+\frac{bex\sqrt{1-\frac{1}{c^2x^2}}}{2c}-\frac{bd^2\csc^{-1}(cx)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*e*Sqrt[1 - 1/(c^2*x^2)]*x)/(2*c) - (b*d^2*ArcCsc[c*x])/(2*e) + ((d + e*x
)^2*(a + b*ArcCsc[c*x]))/(2*e) + (b*d*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/c
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1396

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol
] :> -Subst[Int[((d + e/x^n)^q*(a + c/x^(2*n))^p)/x^2, x], x, 1/x] /; FreeQ
```

```
[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rule 1568

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 5227

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCsc[c*x]))/(e*(m + 1)), x] + Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)\left(a+b\csc^{-1}(cx)\right)dx = \frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e}+\frac{b\int \frac{(d+ex)^2}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{2ce}$$
$$= \frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e}+\frac{b\int \frac{\left(e+\frac{d}{x}\right)^2}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{2ce}$$
$$= \frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e}-\frac{b\,\text{Subst}\left(\int \frac{(e+dx)^2}{x^2\sqrt{1-\frac{x^2}{c^2}}}dx,x,\frac{1}{x}\right)}{2ce}$$
$$= \frac{be\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c}+\frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e}+\frac{b\,\text{Subst}\left(\int \frac{-2de-d^2x}{x\sqrt{1-\frac{x^2}{c^2}}}dx,x,\frac{1}{x}\right)}{2ce}$$
$$= \frac{be\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c}+\frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e}-\frac{(bd)\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{c^2}}}dx,x,\frac{1}{x}\right)}{c}-$$
$$= \frac{be\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c}-\frac{bd^2\csc^{-1}(cx)}{2e}+\frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e}-\frac{(bd)\,\text{Subst}\left(\int \frac{}{x}\right.}{2c}$$
$$= \frac{be\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c}-\frac{bd^2\csc^{-1}(cx)}{2e}+\frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e}+(bcd)\,\text{Subst}\left(\int\right.$$
$$= \frac{be\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c}-\frac{bd^2\csc^{-1}(cx)}{2e}+\frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e}+\frac{bd\tanh^{-1}\left(\sqrt{1-\right.}{c}$$

Mathematica [A] time = 0.20, size = 113, normalized size = 1.36

$$adx+\frac{1}{2}aex^2+\frac{bdx\sqrt{1-\frac{1}{c^2x^2}}\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{\sqrt{c^2x^2-1}}+\frac{bex\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{2c}+bdx\csc^{-1}(cx)+\frac{1}{2}bex^2\csc^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + (b*e*x*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(2*c) + b*d*x*
ArcCsc[c*x] + (b*e*x^2*ArcCsc[c*x])/2 + (b*d*Sqrt[1 - 1/(c^2*x^2)]*x*ArcTan
h[(c*x)/Sqrt[-1 + c^2*x^2]])/Sqrt[-1 + c^2*x^2]
```

fricas [A] time = 0.46, size = 129, normalized size = 1.55

$$\frac{ac^2ex^2+2\,ac^2dx-2\,bcd\log\left(-cx+\sqrt{c^2x^2-1}\right)+\sqrt{c^2x^2-1}\,be+\left(bc^2ex^2+2\,bc^2dx-2\,bc^2d-bc^2e\right)\text{arccsc}\,(cx)-}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*c^2*e*x^2 + 2*a*c^2*d*x - 2*b*c*d*log(-c*x + sqrt(c^2*x^2 - 1)) + sq
rt(c^2*x^2 - 1)*b*e + (b*c^2*e*x^2 + 2*b*c^2*d*x - 2*b*c^2*d - b*c^2*e)*arc
csc(c*x) - 2*(2*b*c^2*d + b*c^2*e)*arctan(-c*x + sqrt(c^2*x^2 - 1)))/c^2
```

giac [B] time = 0.45, size = 349, normalized size = 4.20

$$\frac{1}{8}\left(\frac{bx^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2\arcsin\left(\frac{1}{cx}\right)e}{c}+\frac{ax^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2e}{c}+\frac{4\,bdx\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{4\,ad\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/8*(b*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))*e/c + a*x^2*(sqrt
(-1/(c^2*x^2) + 1) + 1)^2*e/c + 4*b*d*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin
(1/(c*x))/c + 4*a*d*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c + 2*b*x*(sqrt(-1/(c^2*
x^2) + 1) + 1)*e/c^2 + 8*b*d*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - 8*b*d*lo
g(1/(abs(c)*abs(x)))/c^2 + 2*b*arcsin(1/(c*x))*e/c^3 + 2*a*e/c^3 + 4*b*d*ar
csin(1/(c*x))/(c^3*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 4*a*d/(c^3*x*(sqrt(-1/
(c^2*x^2) + 1) + 1)) - 2*b*e/(c^4*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + b*arcsi
n(1/(c*x))*e/(c^5*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + a*e/(c^5*x^2*(sqrt(
-1/(c^2*x^2) + 1) + 1)^2))*c
```

maple [A] time = 0.05, size = 140, normalized size = 1.69

$$\frac{a\,x^2e}{2}+adx+\frac{b\,\mathrm{arccsc}\,(cx)\,x^2e}{2}+b\,\mathrm{arccsc}\,(cx)\,xd+\frac{b\sqrt{c^2x^2-1}\;d\ln\left(cx+\sqrt{c^2x^2-1}\right)}{c^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\;x}+\frac{bxe}{2c\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{be}{2c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}\;x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(a+b*arccsc(c*x)),x)
```

```
[Out] 1/2*a*x^2*e+a*d*x+1/2*b*arccsc(c*x)*x^2*e+b*arccsc(c*x)*x*d+1/c^2*b/((c^2*x
^2-1)/c^2/x^2)^(1/2)/x*(c^2*x^2-1)^(1/2)*d*ln(c*x+(c^2*x^2-1)^(1/2))+1/2/c*
b/((c^2*x^2-1)/c^2/x^2)^(1/2)*x*e-1/2/c^3*b/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*e
```

maxima [A] time = 0.40, size = 92, normalized size = 1.11

$$\frac{1}{2}\,aex^2+\frac{1}{2}\left(x^2\,\mathrm{arccsc}\,(cx)+\frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)be+adx+\frac{\left(2\,cx\,\mathrm{arccsc}\,(cx)+\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)-\log\left(-\sqrt{-\frac{1}{c^2x^2}\right.\right.}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*e*x^2 + 1/2*(x^2*arccsc(c*x) + x*sqrt(-1/(c^2*x^2) + 1)/c)*b*e + a*d*
x + 1/2*(2*c*x*arccsc(c*x) + log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-sqrt(-1
/(c^2*x^2) + 1) + 1))*b*d/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))*(d + e*x),x)
```

```
[Out] int((a + b*asin(1/(c*x)))*(d + e*x), x)
```

sympy [A] time = 4.74, size = 104, normalized size = 1.25

$$adx+\frac{aex^2}{2}+bdx\operatorname{acsc}(cx)+\frac{bex^2\operatorname{acsc}(cx)}{2}+\frac{bd\left(\begin{cases}\operatorname{acosh}(cx) & \text{for } |c^2x^2|>1\\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}\right)}{c}+\frac{be\left(\begin{cases}\frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2|>1\\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*acsc(c*x)),x)
```

```
[Out] a*d*x + a*e*x**2/2 + b*d*x*acsc(c*x) + b*e*x**2*acsc(c*x)/2 + b*d*Piecewise
((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin(c*x), True))/c + b*e*Piecewise(
(sqrt(c**2*x**2 - 1)/c, Abs(c**2*x**2) > 1), (I*sqrt(-c**2*x**2 + 1)/c, Tru
e))/(2*c)
```

3.47 $\int \left(a + b\csc^{-1}(cx)\right)\,dx$

Optimal. Leaf size=31

$$ax + \frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c} + bx\csc^{-1}(cx)$$

```
[Out] a*x+b*x*arccsc(c*x)+b*arctanh((1-1/c^2/x^2)^(1/2))/c
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5215, 266, 63, 208}

$$ax + \frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c} + bx\csc^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcCsc[c*x],x]
```

```
[Out] a*x + b*x*ArcCsc[c*x] + (b*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/c
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5215

```
Int[ArcCsc[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcCsc[c*x], x] + Dist[1/c, Int
[1/(x*Sqrt[1 - 1/(c^2*x^2)]), x], x] /; FreeQ[c, x]
```

Rubi steps

$$\int \left(a+b\csc^{-1}(cx)\right)dx = ax + b\int \csc^{-1}(cx)\,dx$$
$$= ax + bx\csc^{-1}(cx) + \frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\,x}\,dx}{c}$$
$$= ax + bx\csc^{-1}(cx) - \frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{2c}$$
$$= ax + bx\csc^{-1}(cx) + (bc)\,\text{Subst}\left(\int \frac{1}{c^2-c^2x^2}\,dx, x, \sqrt{1-\frac{1}{c^2x^2}}\right)$$
$$= ax + bx\csc^{-1}(cx) + \frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}$$

Mathematica [A] time = 0.05, size = 58, normalized size = 1.87

$$ax + \frac{bx\sqrt{1-\frac{1}{c^2x^2}}\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{\sqrt{c^2x^2-1}} + bx\csc^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcCsc[c*x],x]
```

```
[Out] a*x + b*x*ArcCsc[c*x] + (b*Sqrt[1 - 1/(c^2*x^2)]*x*ArcTanh[(c*x)/Sqrt[-1 +
c^2*x^2]])/Sqrt[-1 + c^2*x^2]
```

fricas [B] time = 0.47, size = 64, normalized size = 2.06

$$\frac{acx - 2\,bc\arctan\left(-cx+\sqrt{c^2x^2-1}\right) + (bcx-bc)\,\text{arccsc}\,(cx) - b\log\left(-cx+\sqrt{c^2x^2-1}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccsc(c*x),x, algorithm="fricas")
```

```
[Out] (a*c*x - 2*b*c*arctan(-c*x + sqrt(c^2*x^2 - 1)) + (b*c*x - b*c)*arccsc(c*x)
 - b*log(-c*x + sqrt(c^2*x^2 - 1)))/c
```

giac [B] time = 0.13, size = 62, normalized size = 2.00

$$\frac{1}{2}\,bc\left(\frac{2\,x\arcsin\left(\frac{1}{cx}\right)}{c} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) - \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}\right) + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccsc(c*x),x, algorithm="giac")
```

```
[Out] 1/2*b*c*(2*x*arcsin(1/(c*x))/c + (log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-sq
rt(-1/(c^2*x^2) + 1) + 1))/c^2) + a*x
```

maple [A] time = 0.04, size = 37, normalized size = 1.19

$$ax + bx\,\text{arccsc}\,(cx) + \frac{b\ln\left(cx + cx\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arccsc(c*x),x)
```

```
[Out] a*x+b*x*arccsc(c*x)+b/c*ln(c*x+c*x*(1-1/c^2/x^2)^(1/2))
```

maxima [A] time = 0.35, size = 53, normalized size = 1.71

$$ax + \frac{\left(2\,cx\,\mathrm{arccsc}\left(cx\right) + \log\left(\sqrt{-\frac{1}{c^2x^2} + 1} + 1\right) - \log\left(-\sqrt{-\frac{1}{c^2x^2} + 1} + 1\right)\right)b}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccsc(c*x),x, algorithm="maxima")
```

```
[Out] a*x + 1/2*(2*c*x*arccsc(c*x) + log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-sqrt(
-1/(c^2*x^2) + 1) + 1))*b/c
```

mupad [B] time = 0.00, size = 33, normalized size = 1.06

$$a\,x + b\,x\,\mathrm{asin}\left(\frac{1}{c\,x}\right) + \frac{b\,\mathrm{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{c^2\,x^2}}}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*asin(1/(c*x)),x)
```

```
[Out] a*x + b*x*asin(1/(c*x)) + (b*atanh(1/(1 - 1/(c^2*x^2))^(1/2)))/c
```

sympy [A] time = 2.43, size = 32, normalized size = 1.03

$$ax + b\left(x\,\mathrm{acsc}\left(cx\right) + \frac{\begin{cases}\mathrm{acosh}\left(cx\right) & \text{for}\ \left|c^2x^2\right| > 1\\ -i\,\mathrm{asin}\left(cx\right) & \text{otherwise}\end{cases}}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*acsc(c*x),x)
```

```
[Out] a*x + b*(x*acsc(c*x) + Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin
(c*x), True))/c)
```

3.48 $\int \frac{a+b \csc^{-1}(cx)}{d+ex} \, dx$

Optimal. Leaf size=257

$$\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{i\left(e-\sqrt{e^2-c^2d^2}\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{e}+\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{i\left(\sqrt{e^2-c^2d^2}+e\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{e}-\frac{\log\left(1-e^{2i\csc^{-1}\right.}}{}$$

```
[Out] -(a+b*arccsc(c*x))*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)/e+(a+b*arccsc(c*x))*
ln(1-I*(I/c/x+(1-1/c^2/x^2)^(1/2))*(e-(-c^2*d^2+e^2)^(1/2))/c/d)/e+(a+b*arc
csc(c*x))*ln(1-I*(I/c/x+(1-1/c^2/x^2)^(1/2))*(e+(-c^2*d^2+e^2)^(1/2))/c/d)/
e+1/2*I*b*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)/e-I*b*polylog(2,I*(I/c/x
+(1-1/c^2/x^2)^(1/2))*(e-(-c^2*d^2+e^2)^(1/2))/c/d)/e-I*b*polylog(2,I*(I/c/
x+(1-1/c^2/x^2)^(1/2))*(e+(-c^2*d^2+e^2)^(1/2))/c/d)/e
```

Rubi [A] time = 0.41, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5225, 2518}

$$-\frac{ib\text{PolyLog}\left(2,\frac{i\left(e-\sqrt{e^2-c^2d^2}\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{e}-\frac{ib\text{PolyLog}\left(2,\frac{i\left(\sqrt{e^2-c^2d^2}+e\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{e}+\frac{ib\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2e}+\frac{\left(a+b\,\text{cs}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(d + e*x),x]
```

```
[Out] ((a + b*ArcCsc[c*x])*Log[1 - (I*(e - Sqrt[-(c^2*d^2) + e^2])*E^(I*ArcCsc[c*
x]))/(c*d)])/e + ((a + b*ArcCsc[c*x])*Log[1 - (I*(e + Sqrt[-(c^2*d^2) + e^2
])*E^(I*ArcCsc[c*x]))/(c*d)])/e - ((a + b*ArcCsc[c*x])*Log[1 - E^((2*I)*Arc
Csc[c*x])])/e - (I*b*PolyLog[2, (I*(e - Sqrt[-(c^2*d^2) + e^2])*E^(I*ArcCsc
[c*x]))/(c*d)])/e - (I*b*PolyLog[2, (I*(e + Sqrt[-(c^2*d^2) + e^2])*E^(I*Ar
cCsc[c*x]))/(c*d)])/e + ((I/2)*b*PolyLog[2, E^((2*I)*ArcCsc[c*x])])/e
```

Rule 2518

```
Int[Log[v_]*(u_), x_Symbol] :> With[{w = DerivativeDivides[v, u*(1 - v), x]
}, Simp[w*PolyLog[2, 1 - v], x] /;  !FalseQ[w]]
```

Rule 5225

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))/((d_.) + (e_.)*(x_)), x_Symbol] :> S
imp[((a + b*ArcCsc[c*x])*Log[1 - (I*(e - Sqrt[-(c^2*d^2) + e^2])*E^(I*ArcCs
c[c*x]))/(c*d)])/e, x] + (Dist[b/(c*e), Int[Log[1 - (I*(e - Sqrt[-(c^2*d^2)
 + e^2])*E^(I*ArcCsc[c*x]))/(c*d)]/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] + Di
st[b/(c*e), Int[Log[1 - (I*(e + Sqrt[-(c^2*d^2) + e^2])*E^(I*ArcCsc[c*x]))/
(c*d)]/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] - Dist[b/(c*e), Int[Log[1 - E^(2
*I*ArcCsc[c*x])]/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] + Simp[((a + b*ArcCsc[
c*x])*Log[1 - (I*(e + Sqrt[-(c^2*d^2) + e^2])*E^(I*ArcCsc[c*x]))/(c*d)])/e,
 x] - Simp[((a + b*ArcCsc[c*x])*Log[1 - E^(2*I*ArcCsc[c*x])])/e, x]) /; Fre
eQ[{a, b, c, d, e}, x]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{d+ex}\,dx = \frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{i\left(e-\sqrt{-c^2d^2+e^2}\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{e} + \frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{i\left(e+\sqrt{-c^2d^2}\right.}{}\right.}{e}$$

$$= \frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{i\left(e-\sqrt{-c^2d^2+e^2}\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{e} + \frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{i\left(e+\sqrt{-c^2d^2}\right.}{}\right.}{e}$$

Mathematica [A] time = 0.69, size = 411, normalized size = 1.60

$$\frac{a\log(d+ex)}{e} + \frac{b\left(8i\left(\text{Li}_2\left(\frac{i\left(\sqrt{e^2-c^2d^2}-e\right)e^{-i\csc^{-1}(cx)}}{cd}\right)+\text{Li}_2\left(-\frac{i\left(e+\sqrt{e^2-c^2d^2}\right)e^{-i\csc^{-1}(cx)}}{cd}\right)\right)-4\log\left(1+\frac{i\left(e-\sqrt{e^2-c^2d^2}\right)e^{-i\csc^{-1}}}{cd}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(d + e*x),x]
```

```
[Out] (a*Log[d + e*x])/e + (b*(I*(Pi - 2*ArcCsc[c*x])^2 + (32*I)*ArcSin[Sqrt[1 +
e/(c*d)]/Sqrt[2]]*ArcTan[((c*d - e)*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[-(c^2
*d^2) + e^2]] - 4*(Pi - 2*ArcCsc[c*x] + 4*ArcSin[Sqrt[1 + e/(c*d)]/Sqrt[2]]
)*Log[1 + (I*(e - Sqrt[-(c^2*d^2) + e^2]))/(c*d*E^(I*ArcCsc[c*x]))] - 4*(Pi
 - 2*ArcCsc[c*x] - 4*ArcSin[Sqrt[1 + e/(c*d)]/Sqrt[2]])*Log[1 + (I*(e + Sqr
t[-(c^2*d^2) + e^2]))/(c*d*E^(I*ArcCsc[c*x]))] - 8*ArcCsc[c*x]*Log[1 - E^((
2*I)*ArcCsc[c*x])] + 4*(Pi - 2*ArcCsc[c*x])*Log[e + d/x] + 8*ArcCsc[c*x]*Lo
g[e + d/x] + (8*I)*(PolyLog[2, (I*(-e + Sqrt[-(c^2*d^2) + e^2]))/(c*d*E^(I*
ArcCsc[c*x]))] + PolyLog[2, ((-I)*(e + Sqrt[-(c^2*d^2) + e^2]))/(c*d*E^(I*A
rcCsc[c*x]))]) + (4*I)*(ArcCsc[c*x]^2 + PolyLog[2, E^((2*I)*ArcCsc[c*x])]))
)/(8*e)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arccsc}\,(cx)+a}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)/(e*x + d), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sy
m(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Va
lue
```

maple [B] time = 1.11, size = 869, normalized size = 3.38

$$\frac{a\ln\left(cex+dc\right)}{e}-\frac{be\,\mathrm{arccsc}\left(cx\right)\ln\left(\frac{-dc\left(\frac{i}{cx}+\sqrt{1-\frac{1}{c^{2}x^{2}}}\right)-ie+\sqrt{c^{2}d^{2}-e^{2}}}{-ie+\sqrt{c^{2}d^{2}-e^{2}}}\right)}{c^{2}d^{2}-e^{2}}-\frac{be\,\mathrm{arccsc}\left(cx\right)\ln\left(\frac{dc\left(\frac{i}{cx}+\sqrt{1-\frac{1}{c^{2}x^{2}}}\right)+ie+\sqrt{c^{2}d^{2}-e^{2}}}{ie+\sqrt{c^{2}d^{2}-e^{2}}}\right)}{c^{2}d^{2}-e^{2}}+\frac{c^{2}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/(e*x+d),x)
```

```
[Out] a*ln(c*e*x+c*d)/e-b*e*arccsc(c*x)/(c^2*d^2-e^2)*ln((-d*c*(I/c/x+(1-1/c^2/x^
2)^(1/2))-I*e+(c^2*d^2-e^2)^(1/2))/(-I*e+(c^2*d^2-e^2)^(1/2)))-b*e*arccsc(c
*x)/(c^2*d^2-e^2)*ln((d*c*(I/c/x+(1-1/c^2/x^2)^(1/2))+I*e+(c^2*d^2-e^2)^(1/
2))/(I*e+(c^2*d^2-e^2)^(1/2)))+c^2*b*d^2/e*arccsc(c*x)/(c^2*d^2-e^2)*ln((-d
*c*(I/c/x+(1-1/c^2/x^2)^(1/2))-I*e+(c^2*d^2-e^2)^(1/2))/(-I*e+(c^2*d^2-e^2)
^(1/2)))+c^2*b*d^2/e*arccsc(c*x)/(c^2*d^2-e^2)*ln((d*c*(I/c/x+(1-1/c^2/x^2)
^(1/2))+I*e+(c^2*d^2-e^2)^(1/2))/(I*e+(c^2*d^2-e^2)^(1/2)))-I*c^2*b*d^2/e/(
c^2*d^2-e^2)*dilog((-d*c*(I/c/x+(1-1/c^2/x^2)^(1/2))-I*e+(c^2*d^2-e^2)^(1/2
))/(-I*e+(c^2*d^2-e^2)^(1/2)))-I*c^2*b*d^2/e/(c^2*d^2-e^2)*dilog((d*c*(I/c/
x+(1-1/c^2/x^2)^(1/2))+I*e+(c^2*d^2-e^2)^(1/2))/(I*e+(c^2*d^2-e^2)^(1/2)))-
I*b*dilog(I/c/x+(1-1/c^2/x^2)^(1/2))/e+I*b*e/(c^2*d^2-e^2)*dilog((-d*c*(I/c
/x+(1-1/c^2/x^2)^(1/2))-I*e+(c^2*d^2-e^2)^(1/2))/(-I*e+(c^2*d^2-e^2)^(1/2))
)+I*b*e/(c^2*d^2-e^2)*dilog((d*c*(I/c/x+(1-1/c^2/x^2)^(1/2))+I*e+(c^2*d^2-e
^2)^(1/2))/(I*e+(c^2*d^2-e^2)^(1/2)))+I*b/e*dilog(1+I/c/x+(1-1/c^2/x^2)^(1/
2))-b/e*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\int\frac{\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right)}{ex+d}\,dx+\frac{a\log\left(ex+d\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d),x, algorithm="maxima")
```

```
[Out] b*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x + d), x) + a*log(e
*x + d)/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\,\mathrm{acsc}\left(cx\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/(e*x+d),x)
```

```
[Out] Integral((a + b*acsc(c*x))/(d + e*x), x)
```

3.49 $\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^2}\,dx$

Optimal. Leaf size=102

$$-\frac{a+b\csc^{-1}(cx)}{e(d+ex)}+\frac{b\tanh^{-1}\left(\frac{c^2d+\frac{e}{x}}{c\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2d^2-e^2}}\right)}{d\sqrt{c^2d^2-e^2}}+\frac{b\csc^{-1}(cx)}{de}$$

```
[Out] b*arccsc(c*x)/d/e+(-a-b*arccsc(c*x))/e/(e*x+d)+b*arctanh((c^2*d+e/x)/c/(c^2
*d^2-e^2)^(1/2)/(1-1/c^2/x^2)^(1/2))/d/(c^2*d^2-e^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5227, 1568, 1475, 844, 216, 725, 206}

$$-\frac{a+b\csc^{-1}(cx)}{e(d+ex)}+\frac{b\tanh^{-1}\left(\frac{c^2d+\frac{e}{x}}{c\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2d^2-e^2}}\right)}{d\sqrt{c^2d^2-e^2}}+\frac{b\csc^{-1}(cx)}{de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(d + e*x)^2,x]
```

```
[Out] (b*ArcCsc[c*x])/(d*e) - (a + b*ArcCsc[c*x])/(e*(d + e*x)) + (b*ArcTanh[(c^2
*d + e/x)/(c*Sqrt[c^2*d^2 - e^2]*Sqrt[1 - 1/(c^2*x^2)])])/(d*Sqrt[c^2*d^2 -
 e^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1475

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
```

```
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1568

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

Rule 5227

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCsc[c*x]))/(e*(m + 1)), x] + Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^2}\,dx &= -\frac{a+b\csc^{-1}(cx)}{e(d+ex)}-\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\,x^2(d+ex)}\,dx}{ce}\\
&= -\frac{a+b\csc^{-1}(cx)}{e(d+ex)}-\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\left(e+\frac{d}{x}\right)x^3}\,dx}{ce}\\
&= -\frac{a+b\csc^{-1}(cx)}{e(d+ex)}+\frac{b\,\text{Subst}\left(\int \frac{x}{(e+dx)\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{ce}\\
&= -\frac{a+b\csc^{-1}(cx)}{e(d+ex)}-\frac{b\,\text{Subst}\left(\int \frac{1}{(e+dx)\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{cd}+\frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{cde}\\
&= \frac{b\csc^{-1}(cx)}{de}-\frac{a+b\csc^{-1}(cx)}{e(d+ex)}+\frac{b\,\text{Subst}\left(\int \frac{1}{d^2-\frac{e^2}{c^2}-x^2}\,dx,x,\frac{d+\frac{e}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{cd}\\
&= \frac{b\csc^{-1}(cx)}{de}-\frac{a+b\csc^{-1}(cx)}{e(d+ex)}+\frac{b\tanh^{-1}\left(\frac{c^2d+\frac{e}{x}}{c\sqrt{c^2d^2-e^2}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{d\sqrt{c^2d^2-e^2}}
\end{aligned}$$

Mathematica [A] time = 0.22, size = 141, normalized size = 1.38

$$-\frac{a}{e(d+ex)}-\frac{b\log\left(cx\left(cd-\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2d^2-e^2}\right)+e\right)}{d\sqrt{c^2d^2-e^2}}+\frac{b\log(d+ex)}{d\sqrt{c^2d^2-e^2}}+\frac{b\sin^{-1}\left(\frac{1}{cx}\right)}{de}-\frac{b\csc^{-1}(cx)}{e(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(d + e*x)^2,x]
```

```
[Out] -(a/(e*(d + e*x))) - (b*ArcCsc[c*x])/(e*(d + e*x)) + (b*ArcSin[1/(c*x)])/(d
*e) + (b*Log[d + e*x])/(d*Sqrt[c^2*d^2 - e^2]) - (b*Log[e + c*(c*d - Sqrt[c
^2*d^2 - e^2]*Sqrt[1 - 1/(c^2*x^2)])*x])/(d*Sqrt[c^2*d^2 - e^2])
```

fricas [B] time = 0.50, size = 475, normalized size = 4.66

$$\left[-\frac{ac^2d^3 - ade^2 - \sqrt{c^2d^2 - e^2}\left(be^2x + bde\right)\log\left(\frac{c^3d^2x + cde + \sqrt{c^2d^2-e^2}\left(c^2dx+e\right) + \left(c^2d^2 + \sqrt{c^2d^2-e^2}\,cd - e^2\right)\sqrt{c^2x^2-1}}{ex+d}\right) + \left(bc^2d^3 - \right.}{c^2d^4e - d^2e^3 + \left(c^2d^3e^2 - de^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [-(a*c^2*d^3 - a*d*e^2 - sqrt(c^2*d^2 - e^2)*(b*e^2*x + b*d*e)*log((c^3*d^2
*x + c*d*e + sqrt(c^2*d^2 - e^2)*(c^2*d*x + e) + (c^2*d^2 + sqrt(c^2*d^2 -
e^2)*c*d - e^2)*sqrt(c^2*x^2 - 1))/(e*x + d)) + (b*c^2*d^3 - b*d*e^2)*arccs
c(c*x) + 2*(b*c^2*d^3 - b*d*e^2 + (b*c^2*d^2*e - b*e^3)*x)*arctan(-c*x + sq
rt(c^2*x^2 - 1)))/(c^2*d^4*e - d^2*e^3 + (c^2*d^3*e^2 - d*e^4)*x), -(a*c^2*
d^3 - a*d*e^2 + 2*sqrt(-c^2*d^2 + e^2)*(b*e^2*x + b*d*e)*arctan(-(sqrt(-c^2
*d^2 + e^2)*sqrt(c^2*x^2 - 1)*e - sqrt(-c^2*d^2 + e^2)*(c*e*x + c*d))/(c^2*
d^2 - e^2)) + (b*c^2*d^3 - b*d*e^2)*arccsc(c*x) + 2*(b*c^2*d^3 - b*d*e^2 +
(b*c^2*d^2*e - b*e^3)*x)*arctan(-c*x + sqrt(c^2*x^2 - 1)))/(c^2*d^4*e - d^2
*e^3 + (c^2*d^3*e^2 - d*e^4)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sy
m(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Va
lue
```

maple [B] time = 0.06, size = 214, normalized size = 2.10

$$-\frac{ca}{(cex+dc)\,e}-\frac{cb\,\mathrm{arccsc}\,(cx)}{(cex+dc)\,e}+\frac{b\sqrt{c^2x^2-1}\;\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{ce\sqrt{\frac{c^2x^2-1}{c^2x^2}}\;xd}-\frac{b\sqrt{c^2x^2-1}\;\ln\left(\frac{2\sqrt{c^2x^2-1}\,\sqrt{\frac{c^2d^2-e^2}{e^2}}\,e-2c^2dx-2e}{cex+dc}\right)}{ce\sqrt{\frac{c^2x^2-1}{c^2x^2}}\;xd\sqrt{\frac{c^2d^2-e^2}{e^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/(e*x+d)^2,x)
```

```
[Out] -c*a/(c*e*x+c*d)/e-c*b/(c*e*x+c*d)/e*arccsc(c*x)+1/c*b/e*(c^2*x^2-1)^(1/2)/
((c^2*x^2-1)/c^2/x^2)^(1/2)/x/d*arctan(1/(c^2*x^2-1)^(1/2))-1/c*b/e*(c^2*x^
2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/d/((c^2*d^2-e^2)/e^2)^(1/2)*ln(2*(
(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e-c^2*d*x-e)/(c*e*x+c*d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b\left(\frac{2\,(c^2e^2x+c^2de)c\left(\frac{\arctan\left(\frac{\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^2e+2\,cd}{2\,\sqrt{-c^2d^2+e^2}}\right)e}{\sqrt{-c^2d^2+e^2}\,cd}-\frac{\arctan\left(\frac{1}{2}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^2\right)}{cd}\right)}{c^2e}+\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right)\right)}{e^2x+de}-\frac{a}{e^2x+de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -((c^2*e^2*x + c^2*d*e)*integrate(x*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1))
/(c^2*e^2*x^3 + c^2*d*e*x^2 - e^2*x - d*e + (c^2*e^2*x^3 + c^2*d*e*x^2 - e^
2*x - d*e)*e^(log(c*x + 1) + log(c*x - 1))), x) + arctan2(1, sqrt(c*x + 1)*
sqrt(c*x - 1)))*b/(e^2*x + d*e) - a/(e^2*x + d*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x)^2,x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(d + e*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{acsc}\,(cx)}{(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/(e*x+d)**2,x)
```

```
[Out] Integral((a + b*acsc(c*x))/(d + e*x)**2, x)
```

3.50 $\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^3}\,dx$

Optimal. Leaf size=172

$$-\frac{a+b\csc^{-1}(cx)}{2e(d+ex)^2}-\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{2d\left(c^2d^2-e^2\right)\left(\frac{d}{x}+e\right)}+\frac{b\left(2c^2d^2-e^2\right)\tanh^{-1}\left(\frac{c^2d+\frac{e}{x}}{c\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2d^2-e^2}}\right)}{2d^2\left(c^2d^2-e^2\right)^{3/2}}+\frac{b\csc^{-1}(cx)}{2d^2e}$$

```
[Out] 1/2*b*arccsc(c*x)/d^2/e+1/2*(-a-b*arccsc(c*x))/e/(e*x+d)^2+1/2*b*(2*c^2*d^2
-e^2)*arctanh((c^2*d+e/x)/c/(c^2*d^2-e^2)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^2/(c
^2*d^2-e^2)^(3/2)-1/2*b*c*e*(1-1/c^2/x^2)^(1/2)/d/(c^2*d^2-e^2)/(e+d/x)
```

Rubi [A] time = 0.29, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5227, 1568, 1475, 1651, 844, 216, 725, 206}

$$-\frac{a+b\csc^{-1}(cx)}{2e(d+ex)^2}-\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{2d\left(c^2d^2-e^2\right)\left(\frac{d}{x}+e\right)}+\frac{b\left(2c^2d^2-e^2\right)\tanh^{-1}\left(\frac{c^2d+\frac{e}{x}}{c\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2d^2-e^2}}\right)}{2d^2\left(c^2d^2-e^2\right)^{3/2}}+\frac{b\csc^{-1}(cx)}{2d^2e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(d + e*x)^3,x]
```

```
[Out] -(b*c*e*Sqrt[1 - 1/(c^2*x^2)])/(2*d*(c^2*d^2 - e^2)*(e + d/x)) + (b*ArcCsc[
c*x])/(2*d^2*e) - (a + b*ArcCsc[c*x])/(2*e*(d + e*x)^2) + (b*(2*c^2*d^2 - e
^2)*ArcTanh[(c^2*d + e/x)/(c*Sqrt[c^2*d^2 - e^2]*Sqrt[1 - 1/(c^2*x^2)])])/(
2*d^2*(c^2*d^2 - e^2)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1475

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1568

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 5227

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCsc[c*x]))/(e*(m + 1)), x] + Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^3}\,dx = -\frac{a+b\csc^{-1}(cx)}{2e(d+ex)^2} - \frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\,x^2(d+ex)^2}\,dx}{2ce}$$

$$= -\frac{a+b\csc^{-1}(cx)}{2e(d+ex)^2} - \frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\left(e+\frac{d}{x}\right)^2 x^4}\,dx}{2ce}$$

$$= -\frac{a+b\csc^{-1}(cx)}{2e(d+ex)^2} + \frac{b\,\text{Subst}\left(\int \frac{x^2}{(e+dx)^2\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2ce}$$

$$= -\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{2d\left(c^2d^2-e^2\right)\left(e+\frac{d}{x}\right)} - \frac{a+b\csc^{-1}(cx)}{2e(d+ex)^2} - \frac{(bc)\,\text{Subst}\left(\int \frac{e-\left(d-\frac{e^2}{c^2d}\right)x}{(e+dx)\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2e\left(c^2d^2-e^2\right)}$$

$$= -\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{2d\left(c^2d^2-e^2\right)\left(e+\frac{d}{x}\right)} - \frac{a+b\csc^{-1}(cx)}{2e(d+ex)^2} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2cd^2e} - \frac{\left(bc\left(2-\frac{e^2}{c^2d\right.}\right.}{}$$

$$= -\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{2d\left(c^2d^2-e^2\right)\left(e+\frac{d}{x}\right)} + \frac{b\csc^{-1}(cx)}{2d^2e} - \frac{a+b\csc^{-1}(cx)}{2e(d+ex)^2} + \frac{\left(bc\left(2-\frac{e^2}{c^2d^2}\right)\right)\text{Subst}\left(\int \frac{}{d^2\ldots}\right.}{2\left(c^2d^2-\right.}$$

$$= -\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{2d\left(c^2d^2-e^2\right)\left(e+\frac{d}{x}\right)} + \frac{b\csc^{-1}(cx)}{2d^2e} - \frac{a+b\csc^{-1}(cx)}{2e(d+ex)^2} + \frac{b\left(2c^2d^2-e^2\right)\tanh^{-1}\left(\frac{}{c\sqrt{c^2\ldots}}\right.}{2d^2\left(c^2d^2-e^2\right)^{3}}$$

Mathematica [A] time = 0.51, size = 250, normalized size = 1.45

$$\frac{1}{2}\left(-\frac{a}{e(d+ex)^2} - \frac{bcex\sqrt{1-\frac{1}{c^2x^2}}}{d\left(c^2d^2-e^2\right)(d+ex)} - \frac{b\left(2c^2d^2-e^2\right)\log\left(cx\left(cd-\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2d^2-e^2}\right)+e\right)}{d^2(cd-e)(cd+e)\sqrt{c^2d^2-e^2}} + \frac{b\left(2c^2d^2\right.}{d^2(cd-e)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(d + e*x)^3,x]
```

```
[Out] (-(a/(e*(d + e*x)^2)) - (b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x)/(d*(c^2*d^2 - e^2)*
(d + e*x)) - (b*ArcCsc[c*x])/(e*(d + e*x)^2) + (b*ArcSin[1/(c*x)])/(d^2*e)
+ (b*(2*c^2*d^2 - e^2)*Log[d + e*x])/(d^2*(c*d - e)*(c*d + e)*Sqrt[c^2*d^2
- e^2]) - (b*(2*c^2*d^2 - e^2)*Log[e + c*(c*d - Sqrt[c^2*d^2 - e^2]*Sqrt[1
- 1/(c^2*x^2)])*x])/(d^2*(c*d - e)*(c*d + e)*Sqrt[c^2*d^2 - e^2]))/2
```

fricas [B] time = 0.95, size = 1111, normalized size = 6.46

$$\left[-\frac{ac^4d^6 + bc^3d^5e - 2\,ac^2d^4e^2 - bcd^3e^3 + ad^2e^4 + \left(bc^3d^3e^3 - bcde^5\right)x^2 - \left(2\,bc^2d^4e - bd^2e^3 + \left(2\,bc^2d^2e^3 - be^5\right)x^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(a*c^4*d^6 + b*c^3*d^5*e - 2*a*c^2*d^4*e^2 - b*c*d^3*e^3 + a*d^2*e^4
+ (b*c^3*d^3*e^3 - b*c*d*e^5)*x^2 - (2*b*c^2*d^4*e - b*d^2*e^3 + (2*b*c^2*d
^2*e^3 - b*e^5)*x^2 + 2*(2*b*c^2*d^3*e^2 - b*d*e^4)*x)*sqrt(c^2*d^2 - e^2)*
log((c^3*d^2*x + c*d*e + sqrt(c^2*d^2 - e^2)*(c^2*d*x + e) + (c^2*d^2 + sqr
t(c^2*d^2 - e^2)*c*d - e^2)*sqrt(c^2*x^2 - 1))/(e*x + d)) + 2*(b*c^3*d^4*e^
2 - b*c*d^2*e^4)*x + (b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4)*arccsc(c*x)
+ 2*(b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4 + (b*c^4*d^4*e^2 - 2*b*c^2*d^2
*e^4 + b*e^6)*x^2 + 2*(b*c^4*d^5*e - 2*b*c^2*d^3*e^3 + b*d*e^5)*x)*arctan(-
c*x + sqrt(c^2*x^2 - 1)) + (b*c^2*d^4*e^2 - b*d^2*e^4 + (b*c^2*d^3*e^3 - b*
d*e^5)*x)*sqrt(c^2*x^2 - 1))/(c^4*d^8*e - 2*c^2*d^6*e^3 + d^4*e^5 + (c^4*d^
6*e^3 - 2*c^2*d^4*e^5 + d^2*e^7)*x^2 + 2*(c^4*d^7*e^2 - 2*c^2*d^5*e^4 + d^3
*e^6)*x), -1/2*(a*c^4*d^6 + b*c^3*d^5*e - 2*a*c^2*d^4*e^2 - b*c*d^3*e^3 + a
*d^2*e^4 + (b*c^3*d^3*e^3 - b*c*d*e^5)*x^2 + 2*(2*b*c^2*d^4*e - b*d^2*e^3 +
 (2*b*c^2*d^2*e^3 - b*e^5)*x^2 + 2*(2*b*c^2*d^3*e^2 - b*d*e^4)*x)*sqrt(-c^2
*d^2 + e^2)*arctan(-(sqrt(-c^2*d^2 + e^2)*sqrt(c^2*x^2 - 1)*e - sqrt(-c^2*d
^2 + e^2)*(c*e*x + c*d))/(c^2*d^2 - e^2)) + 2*(b*c^3*d^4*e^2 - b*c*d^2*e^4)
*x + (b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4)*arccsc(c*x) + 2*(b*c^4*d^6 -
 2*b*c^2*d^4*e^2 + b*d^2*e^4 + (b*c^4*d^4*e^2 - 2*b*c^2*d^2*e^4 + b*e^6)*x^
2 + 2*(b*c^4*d^5*e - 2*b*c^2*d^3*e^3 + b*d*e^5)*x)*arctan(-c*x + sqrt(c^2*x
^2 - 1)) + (b*c^2*d^4*e^2 - b*d^2*e^4 + (b*c^2*d^3*e^3 - b*d*e^5)*x)*sqrt(c
^2*x^2 - 1))/(c^4*d^8*e - 2*c^2*d^6*e^3 + d^4*e^5 + (c^4*d^6*e^3 - 2*c^2*d^
4*e^5 + d^2*e^7)*x^2 + 2*(c^4*d^7*e^2 - 2*c^2*d^5*e^4 + d^3*e^6)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sy
m(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Va
lue
```

maple [B] time = 0.07, size = 1007, normalized size = 5.85

$$-\frac{c^{2} a}{2 \left(c e x +d c \right)^{2} e}-\frac{c^{2} b \,\mathrm{arccsc}\left(c x \right)}{2 \left(c e x +d c \right)^{2} e}+\frac{c^{2} b \sqrt{c^{2} x^{2}-1}\, \arctan \left(\frac{1}{\sqrt{c^{2} x^{2}-1}}\right)}{2 \sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}\, \left(c^{2} d^{2}-e^{2}\right) \left(c e x +d c \right)}+\frac{c^{2} b \sqrt{c^{2} x^{2}-1}\, d \arctan \left(\frac{1}{\sqrt{c^{2} x^{2}-1}}\right)}{2 e \sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}\, x \left(c^{2} d^{2}-e^{2}\right) \left(c e x +d c \right)}-\frac{c^{2} b \sqrt{c^{2} x^{2}}}{\sqrt{\frac{c^{2} x^{2}}{c^{2} x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/(e*x+d)^3,x)
```

```
[Out] -1/2*c^2*a/(c*e*x+c*d)^2/e-1/2*c^2*b/(c*e*x+c*d)^2/e*arccsc(c*x)+1/2*c^2*b*
(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/(c^2*d^2-e^2)/(c*e*x+c*d)*arc
tan(1/(c^2*x^2-1)^(1/2))+1/2*c^2*b/e*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2
)^(1/2)/x*d/(c^2*d^2-e^2)/(c*e*x+c*d)*arctan(1/(c^2*x^2-1)^(1/2))-c^2*b*(c^
2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/((c^2*d^2-e^2)/e^2)^(1/2)/(c^2*d
^2-e^2)/(c*e*x+c*d)*ln(2*((c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e-c^2
*d*x-e)/(c*e*x+c*d))-c^2*b/e*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/
x*d/((c^2*d^2-e^2)/e^2)^(1/2)/(c^2*d^2-e^2)/(c*e*x+c*d)*ln(2*((c^2*x^2-1)^(
1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e-c^2*d*x-e)/(c*e*x+c*d))-1/2*c^2*b*e/((c^2*
```

```
x^2-1)/c^2/x^2)^(1/2)*x/d/(c^2*d^2-e^2)/(c*e*x+c*d)+1/2*b*e/((c^2*x^2-1)/c^
2/x^2)^(1/2)/x/d/(c^2*d^2-e^2)/(c*e*x+c*d)-1/2*b*e^2*(c^2*x^2-1)^(1/2)/((c^
2*x^2-1)/c^2/x^2)^(1/2)/d^2/(c^2*d^2-e^2)/(c*e*x+c*d)*arctan(1/(c^2*x^2-1)^
(1/2))-1/2*b*e*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/d/(c^2*d^2-e
^2)/(c*e*x+c*d)*arctan(1/(c^2*x^2-1)^(1/2))+1/2*b*e^2*(c^2*x^2-1)^(1/2)/((c
^2*x^2-1)/c^2/x^2)^(1/2)/d^2/((c^2*d^2-e^2)/e^2)^(1/2)/(c^2*d^2-e^2)/(c*e*x
+c*d)*ln(2*((c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e-c^2*d*x-e)/(c*e*x
+c*d))+1/2*b*e*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/d/((c^2*d^2-
e^2)/e^2)^(1/2)/(c^2*d^2-e^2)/(c*e*x+c*d)*ln(2*((c^2*x^2-1)^(1/2)*((c^2*d^2
-e^2)/e^2)^(1/2)*e-c^2*d*x-e)/(c*e*x+c*d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(\left(c^2e^3x^2 + 2\,c^2de^2x + c^2d^2e\right)\int \frac{xe^{\left(\frac{1}{2}\,\log(cx+1)+\frac{1}{2}\,\log(cx-1)\right)}}{c^2e^3x^4+2\,c^2de^2x^3-2\,de^2x+\left(c^2e^3x^4+2\,c^2de^2x^3-2\,de^2x-d^2e+\left(c^2d^2e-e^3\right)x^2\right)(cx+1)(cx-1)-d^2e+\left(c^2d^2e-e^3\right)}}{2\left(e^3x^2 + 2\,de^2x + d^2e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(c^2*e^3*x^2 + 2*c^2*d*e^2*x + c^2*d^2*e)*integrate(1/2*x*e^(1/2*lo
g(c*x + 1) + 1/2*log(c*x - 1))/(c^2*e^3*x^4 + 2*c^2*d*e^2*x^3 - 2*d*e^2*x -
 d^2*e + (c^2*d^2*e - e^3)*x^2 + (c^2*e^3*x^4 + 2*c^2*d*e^2*x^3 - 2*d*e^2*x
 - d^2*e + (c^2*d^2*e - e^3)*x^2)*e^(log(c*x + 1) + log(c*x - 1))), x) + ar
ctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*b/(e^3*x^2 + 2*d*e^2*x + d^2*e) - 1/
2*a/(e^3*x^2 + 2*d*e^2*x + d^2*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{{\left(d + e\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x)^3,x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(d + e*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{acsc}\,(cx)}{\left(d + ex\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/(e*x+d)**3,x)
```

```
[Out] Integral((a + b*acsc(c*x))/(d + e*x)**3, x)
```

3.51 $\int x^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)dx$

Optimal. Leaf size=496

$$\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{32bd^4\sqrt{1-c^2x^2}}{105c}$$

```
[Out] 2/3*d^2*(e*x+d)^(3/2)*(a+b*arccsc(c*x))/e^3-4/5*d*(e*x+d)^(5/2)*(a+b*arccsc
(c*x))/e^3+2/7*(e*x+d)^(7/2)*(a+b*arccsc(c*x))/e^3-4/35*b*(e*x+d)^(3/2)*(-c
^2*x^2+1)/c^3/e/x/(1-1/c^2/x^2)^(1/2)+4/105*b*d*(-c^2*x^2+1)*(e*x+d)^(1/2)/
c^3/e/x/(1-1/c^2/x^2)^(1/2)+4/105*b*(5*c^2*d^2-9*e^2)*EllipticE(1/2*(-c*x+1
)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)
/c^4/e^2/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)-4/105*b*d*(9*c^2*d
^2-e^2)*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*
(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c^4/e^2/x/(1-1/c^2/x^2)^(1/2)/(e*
x+d)^(1/2)-32/105*b*d^4*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/
(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e^3/x/(1-1/c
^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] time = 2.83, antiderivative size = 693, normalized size of antiderivative = 1.40, number of steps used = 31, number of rules used = 16, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.762, Rules used = {43, 5247, 12, 6721, 6742, 743, 844, 719, 424, 419, 958, 932, 168, 538, 537, 833}

$$\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{32bd^3\sqrt{1-c^2x^2}}{105c^2e}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (4*b*d*Sqrt[d + e*x]*(1 - c^2*x^2))/(105*c^3*e*Sqrt[1 - 1/(c^2*x^2)]*x) - (
4*b*(d + e*x)^(3/2)*(1 - c^2*x^2))/(35*c^3*e*Sqrt[1 - 1/(c^2*x^2)]*x) + (2*
d^2*(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^3) - (4*d*(d + e*x)^(5/2)*(a
+ b*ArcCsc[c*x]))/(5*e^3) + (2*(d + e*x)^(7/2)*(a + b*ArcCsc[c*x]))/(7*e^3)
 + (32*b*d^2*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]
/Sqrt[2]], (2*e)/(c*d + e)])/(105*c^2*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(
d + e*x))/(c*d + e)]) - (4*b*(c^2*d^2 + 3*e^2)*Sqrt[d + e*x]*Sqrt[1 - c^2*x
^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(35*c^4*e^2*
Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) - (32*b*d^3*Sqrt[(c*
(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt
[2]], (2*e)/(c*d + e)])/(105*c^2*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
 - (4*b*d*(c*d - e)*(c*d + e)*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^
2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(105*c^4*e^2*
Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (32*b*d^4*Sqrt[(c*(d + e*x))/(c*d
+ e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/
(c*d + e)])/(105*c*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 958

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 5247

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(b^IntPart[p]*(
a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[
p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)dx = \frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} - \frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3} + \frac{2(d}{}$$

$$= \frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} - \frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3} + \frac{2(d}{}$$

$$= \frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} - \frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3} + \frac{2(d}{}$$

$$= \frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} - \frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3} + \frac{2(d}{}$$

$$= \frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} - \frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3} + \frac{2(d}{}$$

$$= \frac{16bd\sqrt{d+ex}\left(1-c^2x^2\right)}{105c^3e\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{4b(d+ex)^{3/2}\left(1-c^2x^2\right)}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{2d^2(d+ex)^{3/2}\left(a+\right.}{3e^3}$$

$$= \frac{4bd\sqrt{d+ex}\left(1-c^2x^2\right)}{105c^3e\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{4b(d+ex)^{3/2}\left(1-c^2x^2\right)}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{2d^2(d+ex)^{3/2}\left(a+b\right.}{3e^3}$$

$$= \frac{4bd\sqrt{d+ex}\left(1-c^2x^2\right)}{105c^3e\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{4b(d+ex)^{3/2}\left(1-c^2x^2\right)}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{2d^2(d+ex)^{3/2}\left(a+b\right.}{3e^3}$$

$$= \frac{4bd\sqrt{d+ex}\left(1-c^2x^2\right)}{105c^3e\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{4b(d+ex)^{3/2}\left(1-c^2x^2\right)}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{2d^2(d+ex)^{3/2}\left(a+b\right.}{3e^3}$$

$$= \frac{4bd\sqrt{d+ex}\left(1-c^2x^2\right)}{105c^3e\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{4b(d+ex)^{3/2}\left(1-c^2x^2\right)}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{2d^2(d+ex)^{3/2}\left(a+b\right.}{3e^3}$$

$$= \frac{4bd\sqrt{d+ex}\left(1-c^2x^2\right)}{105c^3e\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{4b(d+ex)^{3/2}\left(1-c^2x^2\right)}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{2d^2(d+ex)^{3/2}\left(a+b\right.}{3e^3}$$

Mathematica [C] time = 13.93, size = 870, normalized size = 1.75

$$b\left(-\frac{c\left(\frac{d}{x}+e\right)x\left(-\frac{16c^3\csc^{-1}(cx)d^3}{105e^3}-\frac{2}{7}c^3x^3\csc^{-1}(cx)-\frac{2c^2x^2\left(2\sqrt{1-\frac{1}{c^2x^2}}\,e+cd\csc^{-1}(cx)\right)}{35e}-\frac{8cx\left(cde\sqrt{1-\frac{1}{c^2x^2}}-c^2d^2\csc^{-1}(cx)\right)}{105e^2}-\frac{4\left(9e^2-5c^2d^2\right)\sqrt{1-\frac{1}{c^2x^2}}}{105e^2}\right)}{\sqrt{d+ex}}-\frac{2\sqrt{\frac{d}{x}+e}\ \cdots}{\cdots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] -((a*d^3*Sqrt[d + e*x]*Beta[-((e*x)/d), 3, 3/2])/(e^3*Sqrt[1 + (e*x)/d])) +
 (b*(-((c*(e + d/x)*x*((-4*(-5*c^2*d^2 + 9*e^2)*Sqrt[1 - 1/(c^2*x^2)])/(105
*e^2) - (16*c^3*d^3*ArcCsc[c*x])/(105*e^3) - (2*c^3*x^3*ArcCsc[c*x])/7 - (2
*c^2*x^2*(2*e*Sqrt[1 - 1/(c^2*x^2)] + c*d*ArcCsc[c*x]))/(35*e) - (8*c*x*(c*
d*e*Sqrt[1 - 1/(c^2*x^2)] - c^2*d^2*ArcCsc[c*x]))/(105*e^2)))/Sqrt[d + e*x]
) - (2*Sqrt[e + d/x]*Sqrt[c*x]*((2*(9*c^3*d^3*e - c*d*e^3)*Sqrt[(c*d + c*e*
x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2
*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*(8*c
^4*d^4 + 5*c^2*d^2*e^2 - 9*e^4)*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*
x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1
 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*(-5*c^3*d^3*e + 9*c*d*e^3)*
Cos[2*ArcCsc[c*x]]*((c*d + c*e*x)*(-1 + c^2*x^2) + c^2*d*x*Sqrt[(c*d + c*e*
x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2
*e)/(c*d + e)] - (c*x*(1 + c*x)*Sqrt[(e - c*e*x)/(c*d + e)]*Sqrt[(c*d + c*e
*x)/(c*d - e)]*((c*d + e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - e)]],
(c*d - e)/(c*d + e)] - e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - e)]], (
c*d - e)/(c*d + e)]))/Sqrt[(e*(1 + c*x))/(-(c*d) + e)] + c*e*x*Sqrt[(c*d +
c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt
[2]], (2*e)/(c*d + e)]))/(c*d*Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*Sqrt[c*x]
*(-2 + c^2*x^2))))/(105*e^3*Sqrt[d + e*x])))/c^4
```

fricas [F] time = 1.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^2\,\text{arccsc}\,(cx)+ax^2\right)\sqrt{ex+d}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccsc(c*x) + a*x^2)*sqrt(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{ex+d}\,(b\,\text{arccsc}\,(cx)+a)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(b*arccsc(c*x) + a)*x^2, x)
```

maple [B] time = 0.09, size = 1222, normalized size = 2.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x)
```

```
[Out] 2/e^3*(a*(1/7*(e*x+d)^(7/2)-2/5*(e*x+d)^(5/2)*d+1/3*(e*x+d)^(3/2)*d^2)+b*(1
/7*arccsc(c*x)*(e*x+d)^(7/2)-2/5*arccsc(c*x)*(e*x+d)^(5/2)*d+1/3*arccsc(c*x
)*(e*x+d)^(3/2)*d^2+2/105/c^4*(3*(c/(c*d-e))^(1/2)*(e*x+d)^(7/2)*c^3-7*(c/(
c*d-e))^(1/2)*(e*x+d)^(5/2)*c^3*d+5*(c/(c*d-e))^(1/2)*(e*x+d)^(3/2)*c^3*d^2
+4*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*El
lipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^3*d^3+5*
(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*Ellip
ticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^3*d^3-8*d^3
*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*Elli
pticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(
c*d-e))^(1/2))*c^3-(c/(c*d-e))^(1/2)*(e*x+d)^(1/2)*c^3*d^3-5*(-((e*x+d)*c-d
*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(
1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2*e+5*(-((e*x+d)*c-d*
c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1
/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2*e-3*(c/(c*d-e))^(1/2
)*(e*x+d)^(3/2)*c*e^2+8*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c
-e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+
e))^(1/2))*c*d*e^2-9*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)
/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))
^(1/2))*c*d*e^2+(c/(c*d-e))^(1/2)*(e*x+d)^(1/2)*c*d*e^2+9*(-((e*x+d)*c-d*c+
e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2
)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e^3-9*(-((e*x+d)*c-d*c+e)/(c*d
-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c
*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e^3)/(c/(c*d-e))^(1/2)/x/((c^2*(e*x+d
)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d-e>0)', see `assume?` for more d
etails)Is c*d-e positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,\sqrt{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asin(1/(c*x)))*(d + e*x)^(1/2),x)
```

```
[Out] int(x^2*(a + b*asin(1/(c*x)))*(d + e*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2} \left(a + b \operatorname{acsc}{\left(c x \right)}\right) \sqrt{d + e x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsc(c*x))*(e*x+d)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*acsc(c*x))*sqrt(d + e*x), x)
```

3.52 $\int x\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)dx$

Optimal. Leaf size=404

$$-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}+\frac{8bd^3\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\right.}{15ce^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}$$

```
[Out] -2/3*d*(e*x+d)^(3/2)*(a+b*arccsc(c*x))/e^2+2/5*(e*x+d)^(5/2)*(a+b*arccsc(c*
x))/e^2-4/15*b*(-c^2*x^2+1)*(e*x+d)^(1/2)/c^3/x/(1-1/c^2/x^2)^(1/2)-8/15*b*
d*EllipticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(
1/2)*(-c^2*x^2+1)^(1/2)/c^2/e/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/
2)+4/15*b*(3*c^2*d^2-e^2)*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(
c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c^4/e/x/(1-1/c^
2/x^2)^(1/2)/(e*x+d)^(1/2)+8/15*b*d^3*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2)
,2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/
c/e^2/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] time = 2.19, antiderivative size = 502, normalized size of antiderivative = 1.24, number of steps used = 24, number of rules used = 15, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.790, Rules used = {43, 5247, 12, 6721, 6742, 743, 844, 719, 424, 419, 958, 932, 168, 538, 537}

$$-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}+\frac{8bd^3\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\right.}{15ce^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (-4*b*Sqrt[d + e*x]*(1 - c^2*x^2))/(15*c^3*Sqrt[1 - 1/(c^2*x^2)]*x) - (2*d*
(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^2) + (2*(d + e*x)^(5/2)*(a + b*Ar
cCsc[c*x]))/(5*e^2) - (8*b*d*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcS
in[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c^2*e*Sqrt[1 - 1/(c^2*x^2)
]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) + (8*b*d^2*Sqrt[(c*(d + e*x))/(c*d + e)]
*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)
])/(15*c^2*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) + (4*b*(c*d - e)*(c*d +
 e)*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1
 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c^4*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[
d + e*x]) + (8*b*d^3*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*Ellipt
icPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c*e^2*Sqrt[1 -
 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
```

```
a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 958

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 5247

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(b^IntPart[p]*(
a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[
p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)dx=-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}+\frac{b\int\frac{2(d+ex}{15e^2}}{}$$

$$=-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}+\frac{(2b)\int\frac{(d+}{}}{}$$

$$=-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}+\frac{\left(2b\sqrt{1-\right.}{}$$

$$=-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}+\frac{\left(2b\sqrt{1-\right.}{}$$

$$=-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}-\frac{\left(4bd\sqrt{1-\right.}{15}$$

$$=-\frac{4b\sqrt{d+ex}\left(1-c^2x^2\right)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}\,x}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+\right.}{5e^2}$$

$$=-\frac{4b\sqrt{d+ex}\left(1-c^2x^2\right)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}\,x}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+\right.}{5e^2}$$

$$=-\frac{4b\sqrt{d+ex}\left(1-c^2x^2\right)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}\,x}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+\right.}{5e^2}$$

$$=-\frac{4b\sqrt{d+ex}\left(1-c^2x^2\right)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}\,x}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+\right.}{5e^2}$$

$$=-\frac{4b\sqrt{d+ex}\left(1-c^2x^2\right)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}\,x}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+\right.}{5e^2}$$

$$=-\frac{4b\sqrt{d+ex}\left(1-c^2x^2\right)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}\,x}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+\right.}{5e^2}$$

Mathematica [C] time = 1.66, size = 368, normalized size = 0.91

$$\frac{1}{15}\left(\frac{2a\sqrt{d+ex}\left(-2d^2+dex+3e^2x^2\right)}{e^2}+\frac{4bx\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}{c}-\frac{4ib\sqrt{\frac{e(cx+1)}{e-cd}}\sqrt{\frac{e-cex}{cd+e}}\left(\left(-c^2d^2-2cde+e^2\right)F\left(i\sin\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] ((4*b*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])/c + (2*a*Sqrt[d + e*x]*(-2*d^2
 + d*e*x + 3*e^2*x^2))/e^2 + (2*b*Sqrt[d + e*x]*(-2*d^2 + d*e*x + 3*e^2*x^2
)*ArcCsc[c*x])/e^2 - ((4*I)*b*Sqrt[(e*(1 + c*x))/(-(c*d) + e)]*Sqrt[(e - c*
e*x)/(c*d + e)]*(-2*c*d*(c*d - e)*EllipticE[I*ArcSinh[Sqrt[-(c/(c*d + e))]*
Sqrt[d + e*x]], (c*d + e)/(c*d - e)] + (-(c^2*d^2) - 2*c*d*e + e^2)*Ellipti
cF[I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)] + 2*
c^2*d^2*EllipticPi[1 + e/(c*d), I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x
]], (c*d + e)/(c*d - e)]))/(c^3*e^2*Sqrt[-(c/(c*d + e))]*Sqrt[1 - 1/(c^2*x^
2)]*x))/15
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex+d}\,(b\,\mathrm{arccsc}\,(cx)+a)x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(b*arccsc(c*x) + a)*x, x)
```

maple [B] time = 0.08, size = 836, normalized size = 2.07

$$2a\left(\frac{(ex+d)^{\frac{5}{2}}}{5}-\frac{(ex+d)^{\frac{3}{2}}d}{3}\right)+2b\left(\frac{\mathrm{arccsc}(cx)(ex+d)^{\frac{5}{2}}}{5}-\frac{\mathrm{arccsc}(cx)(ex+d)^{\frac{3}{2}}d}{3}+\frac{\frac{2\sqrt{\frac{c}{dc-e}}\,(ex+d)^{\frac{5}{2}}c^2}{15}-\frac{4\sqrt{\frac{c}{dc-e}}\,(ex+d)^{\frac{3}{2}}c^2d}{15}-\frac{2d^2\sqrt{-\frac{(ex+d)c-dc+e}{dc-e}}\ \cdots}{\cdots}}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x)
```

```
[Out] 2/e^2*(a*(1/5*(e*x+d)^(5/2)-1/3*(e*x+d)^(3/2)*d)+b*(1/5*arccsc(c*x)*(e*x+d)
^(5/2)-1/3*arccsc(c*x)*(e*x+d)^(3/2)*d+2/15/c^3*((c/(c*d-e))^(1/2)*(e*x+d)^
(5/2)*c^2-2*(c/(c*d-e))^(1/2)*(e*x+d)^(3/2)*c^2*d-d^2*(-((e*x+d)*c-d*c+e)/(
c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c
/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2-2*(-((e*x+d)*c-d*c+e)/(c*d-e))
^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e
))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2+2*d^2*(-((e*x+d)*c-d*c+e)/(c*d-e)
)^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d
-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c^2+(c/(c*d-e
))^(1/2)*(e*x+d)^(1/2)*c^2*d^2+2*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x
+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d
-e)/(c*d+e))^(1/2))*c*d*e-2*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c
-d*c-e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(
c*d+e))^(1/2))*c*d*e+(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)
/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))
^(1/2))*e^2-(c/(c*d-e))^(1/2)*(e*x+d)^(1/2)*e^2)/(c/(c*d-e))^(1/2)/x/((c^2*
(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d-e>0)', see `assume?` for more d
etails)Is c*d-e positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,\sqrt{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asin(1/(c*x)))*(d + e*x)^(1/2),x)
```

```
[Out] int(x*(a + b*asin(1/(c*x)))*(d + e*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\operatorname{acsc}\left(cx\right)\right)\sqrt{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsc(c*x))*(e*x+d)**(1/2),x)
```

```
[Out] Integral(x*(a + b*acsc(c*x))*sqrt(d + e*x), x)
```

3.53 $\int \sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx$

Optimal. Leaf size=315

$$\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}-\frac{4bd^2\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}-\frac{4bd\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,F\left(\mathrm{si}\right.}{3c^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{}}$$

```
[Out] 2/3*(e*x+d)^(3/2)*(a+b*arccsc(c*x))/e-4/3*b*EllipticE(1/2*(-c*x+1)^(1/2)*2^
(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/x/(1-
1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)-4/3*b*d*EllipticF(1/2*(-c*x+1)^(
1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2
+1)^(1/2)/c^2/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-4/3*b*d^2*EllipticPi(1/2*
(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/
2)*(-c^2*x^2+1)^(1/2)/c/e/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {5227, 1574, 958, 719, 419, 933, 168, 538, 537, 844, 424}

$$\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}-\frac{4bd^2\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}-\frac{4bd\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,F\left(\mathrm{si}\right.}{3c^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (2*(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e) - (4*b*Sqrt[d + e*x]*Sqrt[1 -
 c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c^2
*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) - (4*b*d*Sqrt[(c*(d
 + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2
]], (2*e)/(c*d + e)])/(3*c^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (4*b*
d^2*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sq
rt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt
[d + e*x])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 958

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 1574

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[(x^(2*n*FracPart[p])*(a + c/x^(2*n))^FracPart[p])/
(c + a*x^(2*n))^FracPart[p], Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 5227

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCsc[c*x]))/(e*(m + 1)), x] + Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)dx = \frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e} + \frac{(2b)\int \frac{(d+ex)^{3/2}}{\sqrt{1-\frac{1}{c^2x^2}}\,x^2}\,dx}{3ce}$$

$$= \frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e} + \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{(d+ex)^{3/2}}{x\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{3ce\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= \frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e} + \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int\left(\frac{2de}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}} + \frac{\quad}{x\sqrt{d+e\ldots}}\right.}{3ce\sqrt{1-\frac{1}{c^2x^2}}\ldots}$$

$$= \frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e} + \frac{\left(4bd\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{3c\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{(2\ldots}{\ldots}$$

$$= \frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e} + \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{3c\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{\left(2bd\sqrt{-\frac{1}{c}\ldots}\right.}{\ldots}$$

$$= \frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e} - \frac{8bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\middle|\frac{2\ldots}{cd\ldots}\right.}{3c^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

$$= \frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e} - \frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\middle|\frac{2e}{cd+\ldots}\right.}{3c^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{\frac{c(d+ex)}{cd+e}}}$$

$$= \frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e} - \frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\middle|\frac{2e}{cd+\ldots}\right.}{3c^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{\frac{c(d+ex)}{cd+e}}}$$

Mathematica [B] time = 13.20, size = 657, normalized size = 2.09

$$\frac{2a(d+ex)^{3/2}}{3e} - \frac{b(cd+cex)\left(-\frac{2\left(2e\sqrt{1-\frac{1}{c^2x^2}}+cd\csc^{-1}(cx)+cex\csc^{-1}(cx)\right)}{e} + \frac{4d\sqrt{-c^2x^2\left(1-\frac{1}{c^2x^2}\right)}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\middle|\frac{2e}{cd+e}\right)}{x\sqrt{1-\frac{1}{c^2x^2}}\,(cd+e)\sqrt{\frac{cd+cex}{cd+e}}} - \frac{4\sqrt{-c^2x^2(1\ldots}}{\ldots}\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d + e*x]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (2*a*(d + e*x)^(3/2))/(3*e) - (b*(c*d + c*e*x)*((-2*(2*e*Sqrt[1 - 1/(c^2*x^
2)] + c*d*ArcCsc[c*x] + c*e*x*ArcCsc[c*x]))/e + (4*d*Sqrt[-(c^2*(1 - 1/(c^2
*x^2))*x^2)]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/((c
*d + e)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*d + c*e*x)/(c*d + e)]) - (4*(-(c*d)
 + e)*Sqrt[-(c^2*(1 - 1/(c^2*x^2))*x^2)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]
/Sqrt[2]], (2*e)/(c*d + e)])/(c*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*d + c*e*x
)/(c*d + e)]) + ((c^2*(1 - 1/(c^2*x^2))*x^2*(c*d + c*e*x) + c^2*d*x*Sqrt[-(
c^2*(1 - 1/(c^2*x^2))*x^2)]*Sqrt[(c*d + c*e*x)/(c*d + e)]*EllipticF[ArcSin[
Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)] - (c*x*(1 + c*x)*Sqrt[(e - c*e*x)/
(c*d + e)]*Sqrt[(c*d + c*e*x)/(c*d - e)]*((c*d + e)*EllipticE[ArcSin[Sqrt[(
c*d + c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)] - e*EllipticF[ArcSin[Sqrt[(c
*d + c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)]))/Sqrt[(e*(1 + c*x))/(-(c*d)
+ e)] + c*e*x*Sqrt[-(c^2*(1 - 1/(c^2*x^2))*x^2)]*Sqrt[(c*d + c*e*x)/(c*d +
e)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])*Sec[2*Ar
cCsc[c*x]]*Sin[4*ArcCsc[c*x]])/(c^2*(1 - 1/(c^2*x^2))*(e + d/x)*x^2)))/(3*c
^2*Sqrt[d + e*x])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex+d}\,(b\,\mathrm{arccsc}\,(cx)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(b*arccsc(c*x) + a), x)
```

maple [A] time = 0.07, size = 388, normalized size = 1.23

$$\frac{\frac{2(ex+d)^{\frac{3}{2}}a}{3}+2b\left(\frac{(ex+d)^{\frac{3}{2}}\mathrm{arccsc}(cx)}{3}+\frac{2\left(2d\,\mathrm{EllipticF}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{dc-e}},\sqrt{\frac{dc-e}{dc+e}}\right)c-\mathrm{EllipticE}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{dc-e}},\sqrt{\frac{dc-e}{dc+e}}\right)cd-d\,\mathrm{EllipticPi}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{dc-e}}\right.\right.}{3c^2\sqrt{\frac{c}{dc-e}}\,x\sqrt{}}\right.}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))*(e*x+d)^(1/2),x)
```

```
[Out] 2/e*(1/3*(e*x+d)^(3/2)*a+b*(1/3*(e*x+d)^(3/2)*arccsc(c*x)+2/3/c^2*(2*d*Elli
pticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c-EllipticE(
(e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d-d*EllipticPi((
e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^
(1/2))*c+EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))
*e-EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e)*(-
((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)/(c/(c*d
-e))^(1/2)/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2
)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d-e>0)', see `assume?` for more d
etails)Is c*d-e positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a + b \operatorname{asin}\left(\frac{1}{c x}\right)\right) \sqrt{d + e x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))*(d + e*x)^(1/2),x)
```

```
[Out] int((a + b*asin(1/(c*x)))*(d + e*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{acsc}(cx)) \sqrt{d + ex} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))*(e*x+d)**(1/2),x)
```

```
[Out] Integral((a + b*acsc(c*x))*sqrt(d + e*x), x)
```

3.54 $\int \frac{\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{x},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x,x)
```

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/x,x]
[Out] Defer[Int][(Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/x, x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{x}\,dx=\int \frac{\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{x}\,dx$$

Mathematica [A] time = 18.52, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/x,x]
[Out] Integrate[(Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/x, x]
```

fricas [A] time = 1.27, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex+d}\,(b\,\text{arccsc}\,(cx)+a)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x,x, algorithm="fricas")
[Out] integral(sqrt(e*x + d)*(b*arccsc(c*x) + a)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}\,(b\,\text{arccsc}\,(cx)+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x,x, algorithm="giac")
[Out] integrate(sqrt(e*x + d)*(b*arccsc(c*x) + a)/x, x)
```

maple [A] time = 8.85, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccsc}\,(cx))\,\sqrt{ex+d}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x,x)
```

```
[Out] int((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$a\sqrt{d}\,\log\left(\frac{ex}{ex+2\,\sqrt{ex+d}\,\sqrt{d}\,+2\,d}\right)+b\int\frac{\sqrt{ex+d}\,\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\,\right)}{x}\,dx+2\,\sqrt{ex+d}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] a*sqrt(d)*log(e*x/(e*x + 2*sqrt(e*x + d)*sqrt(d) + 2*d)) + b*integrate(sqrt
(e*x + d)*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/x, x) + 2*sqrt(e*x + d)*a
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,\sqrt{d+e\,x}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asin(1/(c*x)))*(d + e*x)^(1/2))/x,x)
```

```
[Out] int(((a + b*asin(1/(c*x)))*(d + e*x)^(1/2))/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))*(e*x+d)**(1/2)/x,x)
```

```
[Out] Timed out
```

3.55 $\int \frac{\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x^2,x)
```

Rubi [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/x^2,x]
[Out] Defer[Int][(Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/x^2, x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{x^2}\,dx=\int \frac{\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{x^2}\,dx$$

Mathematica [A] time = 8.31, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/x^2,x]
[Out] Integrate[(Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/x^2, x]
```

fricas [A] time = 1.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex+d}\,(b\,\text{arccsc}\,(cx)+a)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x^2,x, algorithm="fricas")
[Out] integral(sqrt(e*x + d)*(b*arccsc(c*x) + a)/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}\,(b\,\text{arccsc}\,(cx)+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x^2,x, algorithm="giac")
[Out] integrate(sqrt(e*x + d)*(b*arccsc(c*x) + a)/x^2, x)
```

maple [**A**] time = 10.09, size = 0, normalized size = 0.00

$$\int \frac{(a + b \,\mathrm{arccsc}\,(cx)) \sqrt{ex + d}}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x^2,x)
[Out] int((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x^2,x)
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x^2,x, algorithm="maxima")
[Out] Timed out
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a + b \,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right) \sqrt{d + e\,x}}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asin(1/(c*x)))*(d + e*x)^(1/2))/x^2,x)
[Out] int(((a + b*asin(1/(c*x)))*(d + e*x)^(1/2))/x^2, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))*(e*x+d)**(1/2)/x**2,x)
[Out] Timed out
```

3.56 $\int (d+ex)^{3/2} \left(a+b\csc^{-1}(cx)\right) dx$

Optimal. Leaf size=372

$$\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}-\frac{4bd^3\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}-\frac{28bd\sqrt{1-c^2x^2}\sqrt{d+ex}\,E\left(\sin\right.}{15c^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c}}$$

```
[Out] 2/5*(e*x+d)^(5/2)*(a+b*arccsc(c*x))/e-4/15*b*e*(-c^2*x^2+1)*(e*x+d)^(1/2)/c
^3/x/(1-1/c^2/x^2)^(1/2)-28/15*b*d*EllipticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(
1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/x/(1-1/c^2/x^2
)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)-4/15*b*(2*c^2*d^2+e^2)*EllipticF(1/2*(-c*
x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c
^2*x^2+1)^(1/2)/c^4/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-4/5*b*d^3*EllipticP
i(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e
))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] time = 0.75, antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {5227, 1574, 958, 719, 419, 933, 168, 538, 537, 844, 424, 931, 1584}

$$\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}-\frac{4b\sqrt{1-c^2x^2}\left(2c^2d^2+e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15c^4x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}-\frac{4bd^3\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}}{5cex\sqrt{1-}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (-4*b*e*Sqrt[d + e*x]*(1 - c^2*x^2))/(15*c^3*Sqrt[1 - 1/(c^2*x^2)]*x) + (2*
(d + e*x)^(5/2)*(a + b*ArcCsc[c*x]))/(5*e) - (28*b*d*Sqrt[d + e*x]*Sqrt[1 -
 c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c^
2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) - (4*b*(2*c^2*d^2
+ e^2)*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqr
t[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c^4*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt
[d + e*x]) - (4*b*d^3*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*Ellip
ticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(5*c*e*Sqrt[1 - 1
/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
```

```
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 931

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[(2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g*x]*Sqrt[a +
c*x^2])/(c*g*(2*m - 1)), x] - Dist[1/(c*g*(2*m - 1)), Int[((d + e*x)^(m - 3
)*Simp[a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3*g*(2*m - 1) + e*(e*(a*e*g*(2*m -
 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x + 2*e^2*(c*e*f - 3*c*d*g)*(m - 1)*x
^2, x])/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GeQ[
m, 2]
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 958

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 1574

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[(x^(2*n*FracPart[p])*(a + c/x^(2*n))^FracPart[p])/
(c + a*x^(2*n))^FracPart[p], Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 5227

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCsc[c*x]))/(e*(m + 1)), x] + Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)dx = \frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}+\frac{(2b)\int \frac{(d+ex)^{5/2}}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{5ce}$$

$$= \frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}+\frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{(d+ex)^{5/2}}{x\sqrt{-\frac{1}{c^2}+x^2}}dx}{5ce\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= \frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}+\frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int\left(\frac{3d^2e}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}+\frac{}{x\sqrt{d+}}\right.}{5ce}$$

$$= \frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}+\frac{\left(6bd^2\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}dx}{5c\sqrt{1-\frac{1}{c^2x^2}}\,x}+$$

$$= -\frac{4be\sqrt{d+ex}\left(1-c^2x^2\right)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}\,x}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}+\frac{\left(6bd\sqrt{-\frac{1}{c^2}+x\right.}{5c\sqrt{1}}$$

$$= -\frac{4be\sqrt{d+ex}\left(1-c^2x^2\right)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}\,x}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}-\frac{12bd^2\sqrt{\frac{c(d+ex)}{cd+e}}}{5c}$$

$$= -\frac{4be\sqrt{d+ex}\left(1-c^2x^2\right)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}\,x}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}-\frac{12bd\sqrt{d+ex}}{5c}$$

$$= -\frac{4be\sqrt{d+ex}\left(1-c^2x^2\right)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}\,x}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}-\frac{12bd\sqrt{d+ex}}{5c}$$

$$= -\frac{4be\sqrt{d+ex}\left(1-c^2x^2\right)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}\,x}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}-\frac{28bd\sqrt{d+ex}}{15c}$$

Mathematica [C] time = 1.50, size = 333, normalized size = 0.90

$$\frac{1}{15}\left(\frac{6a(d+ex)^{5/2}}{e}+\frac{4bex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}{c}-\frac{4ib\sqrt{\frac{e(cx+1)}{e-cd}}\sqrt{\frac{e-cex}{cd+e}}\left(\left(9c^2d^2-7cde+e^2\right)F\left(i\sinh^{-1}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{}\right.\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] ((4*b*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])/c + (6*a*(d + e*x)^(5/2))/e
+ (6*b*(d + e*x)^(5/2)*ArcCsc[c*x])/e - ((4*I)*b*Sqrt[(e*(1 + c*x))/(-(c*d)
 + e)]*Sqrt[(e - c*e*x)/(c*d + e)]*(-7*c*d*(c*d - e)*EllipticE[I*ArcSinh[Sq
rt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)] + (9*c^2*d^2 - 7*c*
d*e + e^2)*EllipticF[I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d +
e)/(c*d - e)] - 3*c^2*d^2*EllipticPi[1 + e/(c*d), I*ArcSinh[Sqrt[-(c/(c*d +
 e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)]))/(c^3*e*Sqrt[-(c/(c*d + e))]*Sq
rt[1 - 1/(c^2*x^2)]*x))/15
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex+d)^{\frac{3}{2}}(b\operatorname{arccsc}(cx)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(3/2)*(b*arccsc(c*x) + a), x)
```

maple [B] time = 0.07, size = 810, normalized size = 2.18

$$\frac{2(ex+d)^{\frac{5}{2}}a}{5}+2b\left(\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{5}{2}}}{5}+\frac{\frac{2\sqrt{\frac{c}{dc-e}}\,(ex+d)^{\frac{5}{2}}c^2}{15}-\frac{4\sqrt{\frac{c}{dc-e}}\,(ex+d)^{\frac{3}{2}}c^2d}{15}+\frac{6d^2\sqrt{-\frac{(ex+d)c-dc+e}{dc-e}}\,\sqrt{-\frac{(ex+d)c-dc-e}{dc+e}}\,\mathrm{EllipticF}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{dc-e}},\sqrt{\frac{dc-e}{dc+e}}\right)c}{5}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

```
[Out] 2/e*(1/5*(e*x+d)^(5/2)*a+b*(1/5*arccsc(c*x)*(e*x+d)^(5/2)+2/15/c^3*((c/(c*d
-e))^(1/2)*(e*x+d)^(5/2)*c^2-2*(c/(c*d-e))^(1/2)*(e*x+d)^(3/2)*c^2*d+9*d^2*
(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*Ellip
ticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2-7*(-((e*x
+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticE((e
*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2-3*d^2*(-((e*
x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticPi(
(e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))
^(1/2))*c^2+(c/(c*d-e))^(1/2)*(e*x+d)^(1/2)*c^2*d^2+7*(-((e*x+d)*c-d*c+e)/(
c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c
/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e-7*(-((e*x+d)*c-d*c+e)/(c*d-e
))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d
-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e+(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2
)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1
/2),((c*d-e)/(c*d+e))^(1/2))*e^2-(c/(c*d-e))^(1/2)*(e*x+d)^(1/2)*e^2)/(c/(c
*d-e))^(1/2)/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1
/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d-e>0)', see `assume?` for more d
etails)Is c*d-e positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,(d + e\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))*(d + e*x)^(3/2),x)
```

```
[Out] int((a + b*asin(1/(c*x)))*(d + e*x)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(a+b*acsc(c*x)),x)
```

```
[Out] Timed out
```

3.57 $\int \frac{x^3\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=714

$$-\frac{2d^3\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}-\frac{6d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^4}+\frac{2(d+ex)^{7/2}\left(a\right.}{7\ldots}$$

```
[Out] 2*d^2*(e*x+d)^(3/2)*(a+b*arccsc(c*x))/e^4-6/5*d*(e*x+d)^(5/2)*(a+b*arccsc(c
*x))/e^4+2/7*(e*x+d)^(7/2)*(a+b*arccsc(c*x))/e^4-2*d^3*(a+b*arccsc(c*x))*(e
*x+d)^(1/2)/e^4-4/35*b*(-c^2*x^2+1)*(e*x+d)^(1/2)/c^3/e/(1-1/c^2/x^2)^(1/2)
+4/21*b*d*(-c^2*x^2+1)*(e*x+d)^(1/2)/c^3/e^2/x/(1-1/c^2/x^2)^(1/2)-24/35*b*
d^2*EllipticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)
^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e^3/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))
^(1/2)+4/105*b*(2*c^2*d^2-9*e^2)*EllipticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/
2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/c^4/e^3/x/(1-1/c^2/x
^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)+64/35*b*d^3*EllipticF(1/2*(-c*x+1)^(1/2
)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)
^(1/2)/c^2/e^3/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-32/105*b*d*(c*d-e)*(c*d+
e)*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+
d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c^4/e^3/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^
(1/2)+64/35*b*d^4*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e
))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e^4/x/(1-1/c^2/x^2
)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] time = 2.59, antiderivative size = 714, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 17, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.810$, Rules used = {43, 5247, 12, 6721, 6742, 719, 424, 944, 419, 932, 168, 538, 537, 833, 844, 942, 1654}

$$-\frac{2d^3\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}-\frac{6d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^4}+\frac{2(d+ex)^{7/2}\left(a\right.}{7\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x],x]
```

```
[Out] (-4*b*Sqrt[d + e*x]*(1 - c^2*x^2))/(35*c^3*e*Sqrt[1 - 1/(c^2*x^2)]) + (4*b*
d*Sqrt[d + e*x]*(1 - c^2*x^2))/(21*c^3*e^2*Sqrt[1 - 1/(c^2*x^2)]*x) - (2*d^
3*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/e^4 + (2*d^2*(d + e*x)^(3/2)*(a + b*Ar
cCsc[c*x]))/e^4 - (6*d*(d + e*x)^(5/2)*(a + b*ArcCsc[c*x]))/(5*e^4) + (2*(d
 + e*x)^(7/2)*(a + b*ArcCsc[c*x]))/(7*e^4) - (24*b*d^2*Sqrt[d + e*x]*Sqrt[1
 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(35*
c^2*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) + (4*b*(2*c^
2*d^2 - 9*e^2)*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*
x]/Sqrt[2]], (2*e)/(c*d + e)])/(105*c^4*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c
*(d + e*x))/(c*d + e)]) + (64*b*d^3*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 -
c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(35*c^2
*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (32*b*d*(c*d - e)*(c*d + e)*S
qrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*
x]/Sqrt[2]], (2*e)/(c*d + e)])/(105*c^4*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d
+ e*x]) + (64*b*d^4*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*Ellipti
cPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(35*c*e^4*Sqrt[1 -
1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
```

```
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 942

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_) + (c_.)*
(x_)^2], x_Symbol] :> Simp[(2*e*(d + e*x)^(m - 1)*Sqrt[f + g*x]*Sqrt[a + c*
x^2])/(c*(2*m + 1)), x] - Dist[1/(c*(2*m + 1)), Int[((d + e*x)^(m - 2)*Simp
[a*e*(d*g + 2*e*f*(m - 1)) - c*d^2*f*(2*m + 1) + (a*e^2*g*(2*m - 1) - c*d*(
4*e*f*m + d*g*(2*m + 1)))*x - c*e*(e*f + d*g*(4*m - 1))*x^2, x])/(Sqrt[f +
g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f -
d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GtQ[m, 1]
```

Rule 944

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2
]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] +
Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 5247

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(b^IntPart[p]*(
a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[
p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^3\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx = -\frac{2d^3\sqrt{d+ex}\,\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}-\frac{6d(d+ex)^{5/2}\left(a\right.}{5e}$$

$$= -\frac{2d^3\sqrt{d+ex}\,\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}-\frac{6d(d+ex)^{5/2}\left(a\right.}{5e}$$

$$= -\frac{2d^3\sqrt{d+ex}\,\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}-\frac{6d(d+ex)^{5/2}\left(a\right.}{5e}$$

$$= -\frac{2d^3\sqrt{d+ex}\,\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}-\frac{6d(d+ex)^{5/2}\left(a\right.}{5e}$$

$$= -\frac{2d^3\sqrt{d+ex}\,\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}-\frac{6d(d+ex)^{5/2}\left(a\right.}{5e}$$

$$= -\frac{4b\sqrt{d+ex}\,\left(1-c^2x^2\right)}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}}+\frac{8bd\sqrt{d+ex}\,\left(1-c^2x^2\right)}{35c^3e^2\sqrt{1-\frac{1}{c^2x^2}}\,x}-\frac{2d^3\sqrt{d+ex}\,\left(a+b\csc^{-1}(cx)\right)}{e^4}+$$

$$= -\frac{4b\sqrt{d+ex}\,\left(1-c^2x^2\right)}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}}+\frac{4bd\sqrt{d+ex}\,\left(1-c^2x^2\right)}{21c^3e^2\sqrt{1-\frac{1}{c^2x^2}}\,x}-\frac{2d^3\sqrt{d+ex}\,\left(a+b\csc^{-1}(cx)\right)}{e^4}+$$

$$= -\frac{4b\sqrt{d+ex}\,\left(1-c^2x^2\right)}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}}+\frac{4bd\sqrt{d+ex}\,\left(1-c^2x^2\right)}{21c^3e^2\sqrt{1-\frac{1}{c^2x^2}}\,x}-\frac{2d^3\sqrt{d+ex}\,\left(a+b\csc^{-1}(cx)\right)}{e^4}+$$

$$= -\frac{4b\sqrt{d+ex}\,\left(1-c^2x^2\right)}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}}+\frac{4bd\sqrt{d+ex}\,\left(1-c^2x^2\right)}{21c^3e^2\sqrt{1-\frac{1}{c^2x^2}}\,x}-\frac{2d^3\sqrt{d+ex}\,\left(a+b\csc^{-1}(cx)\right)}{e^4}+$$

$$= -\frac{4b\sqrt{d+ex}\,\left(1-c^2x^2\right)}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}}+\frac{4bd\sqrt{d+ex}\,\left(1-c^2x^2\right)}{21c^3e^2\sqrt{1-\frac{1}{c^2x^2}}\,x}-\frac{2d^3\sqrt{d+ex}\,\left(a+b\csc^{-1}(cx)\right)}{e^4}+$$

Mathematica [C] time = 13.85, size = 873, normalized size = 1.22

$$\frac{a\sqrt{\frac{ex}{d}+1}\,B_{-\frac{ex}{d}}\left(4,\frac{1}{2}\right)d^4}{e^4\sqrt{d+ex}}+\frac{b\left(\frac{2\sqrt{\frac{d}{x}+e}\sqrt{cx}\left(\frac{2\left(40c^3ed^3+8ce^3d\right)\sqrt{\frac{cd+cex}{cd+e}}\sqrt{1-c^2x^2}F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{1-\frac{1}{c^2x^2}}\sqrt{\frac{d}{x}+e}(cx)^{3/2}}+\frac{2\left(48c^4d^4+16c^2e^2d^2+9e^4\right)\sqrt{\frac{cd+cex}{cd+e}}\sqrt{1-c^2x^2}\cdots}{\sqrt{1-\frac{1}{c^2x^2}}\sqrt{\frac{d}{x}+e}(c\cdots}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x],x]
```

```
[Out] (a*d^4*Sqrt[1 + (e*x)/d]*Beta[-((e*x)/d), 4, 1/2])/(e^4*Sqrt[d + e*x]) + (b
*(-((c*(e + d/x)*x*((-4*(16*c^2*d^2 + 9*e^2)*Sqrt[1 - 1/(c^2*x^2)])/(105*e^
3) + (32*c^3*d^3*ArcCsc[c*x])/(35*e^4) - (2*c^3*x^3*ArcCsc[c*x])/(7*e) - (4
*c^2*x^2*(e*Sqrt[1 - 1/(c^2*x^2)] - 3*c*d*ArcCsc[c*x]))/(35*e^2) + (4*c*x*(
5*c*d*e*Sqrt[1 - 1/(c^2*x^2)] - 12*c^2*d^2*ArcCsc[c*x]))/(105*e^3)))/Sqrt[d
 + e*x]) + (2*Sqrt[e + d/x]*Sqrt[c*x]*((2*(40*c^3*d^3*e + 8*c*d*e^3)*Sqrt[(
c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sq
rt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2))
 + (2*(48*c^4*d^4 + 16*c^2*d^2*e^2 + 9*e^4)*Sqrt[(c*d + c*e*x)/(c*d + e)]*S
qrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d +
e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*(-16*c^3*d^3*e
- 9*c*d*e^3)*Cos[2*ArcCsc[c*x]]*((c*d + c*e*x)*(-1 + c^2*x^2) + c^2*d*x*Sqr
t[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]
/Sqrt[2]], (2*e)/(c*d + e)] - (c*x*(1 + c*x)*Sqrt[(e - c*e*x)/(c*d + e)]*Sq
rt[(c*d + c*e*x)/(c*d - e)]*((c*d + e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/
(c*d - e)]], (c*d - e)/(c*d + e)] - e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(
c*d - e)]], (c*d - e)/(c*d + e)]))/Sqrt[(e*(1 + c*x))/(-(c*d) + e)] + c*e*x
*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[
1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]))/(c*d*Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d
/x]*Sqrt[c*x]*(-2 + c^2*x^2))))/(105*e^4*Sqrt[d + e*x])))/c^4
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arccsc}(cx)+a)x^3}{\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^3/sqrt(e*x + d), x)
```

maple [A] time = 0.08, size = 1248, normalized size = 1.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x)
```

```
[Out] 2/e^4*(a*(1/7*(e*x+d)^(7/2)-3/5*(e*x+d)^(5/2)*d+(e*x+d)^(3/2)*d^2-d^3*(e*x+
d)^(1/2))+b*(1/7*arccsc(c*x)*(e*x+d)^(7/2)-3/5*arccsc(c*x)*(e*x+d)^(5/2)*d+
arccsc(c*x)*(e*x+d)^(3/2)*d^2-arccsc(c*x)*d^3*(e*x+d)^(1/2)+2/105/c^4*(3*(c
/(c*d-e))^(1/2)*(e*x+d)^(7/2)*c^3-14*(c/(c*d-e))^(1/2)*(e*x+d)^(5/2)*c^3*d+
19*(c/(c*d-e))^(1/2)*(e*x+d)^(3/2)*c^3*d^2-24*(-((e*x+d)*c-d*c+e)/(c*d-e))^
(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e)
)^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^3*d^3-16*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1
/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^
(1/2),((c*d-e)/(c*d+e))^(1/2))*c^3*d^3+48*d^3*(-((e*x+d)*c-d*c+e)/(c*d-e))^
(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e
))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c^3-8*(c/(c*d-e
))^(1/2)*(e*x+d)^(1/2)*c^3*d^3+16*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*
x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*
d-e)/(c*d+e))^(1/2))*c^2*d^2*e-16*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*
x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*
d-e)/(c*d+e))^(1/2))*c^2*d^2*e-3*(c/(c*d-e))^(1/2)*(e*x+d)^(3/2)*c*e^2+(-((
e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF
((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e^2-9*(-((e*x
+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticE((e
*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e^2+8*(c/(c*d-e)
)^(1/2)*(e*x+d)^(1/2)*c*d*e^2+9*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+
d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-
e)/(c*d+e))^(1/2))*e^3-9*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*
c-e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d
+e))^(1/2))*e^3)/(c/(c*d-e))^(1/2)/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^
2-e^2)/c^2/e^2/x^2)^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d-e>0)', see `assume?` for more d
etails)Is c*d-e positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x)^(1/2),x)
```

```
[Out] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsc(c*x))/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

3.58 $\int \frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=530

$$\frac{2d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}-\frac{4d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}-\frac{32bd^3\sqrt{1-c^2x^2}\,\sqrt{}}{15ce^3}$$

```
[Out] -4/3*d*(e*x+d)^(3/2)*(a+b*arccsc(c*x))/e^3+2/5*(e*x+d)^(5/2)*(a+b*arccsc(c*
x))/e^3+2*d^2*(a+b*arccsc(c*x))*(e*x+d)^(1/2)/e^3-4/15*b*(-c^2*x^2+1)*(e*x+
d)^(1/2)/c^3/e/x/(1-1/c^2/x^2)^(1/2)+4/5*b*d*EllipticE(1/2*(-c*x+1)^(1/2)*2
^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e^2/
x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)-32/15*b*d^2*EllipticF(1/2*(
-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*
(-c^2*x^2+1)^(1/2)/c^2/e^2/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4/15*b*(c*d-
e)*(c*d+e)*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*
(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c^4/e^2/x/(1-1/c^2/x^2)^(1/2)/
(e*x+d)^(1/2)-32/15*b*d^3*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(
e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e^3/x/(1-1
/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] time = 1.91, antiderivative size = 530, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {43, 5247, 12, 6721, 6742, 719, 424, 944, 419, 932, 168, 538, 537, 833, 844}

$$\frac{2d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}-\frac{4d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}-\frac{32bd^2\sqrt{1-c^2x^2}\,\sqrt{}}{15c^2e^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x],x]
```

```
[Out] (-4*b*Sqrt[d + e*x]*(1 - c^2*x^2))/(15*c^3*e*Sqrt[1 - 1/(c^2*x^2)]*x) + (2*
d^2*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/e^3 - (4*d*(d + e*x)^(3/2)*(a + b*Ar
cCsc[c*x]))/(3*e^3) + (2*(d + e*x)^(5/2)*(a + b*ArcCsc[c*x]))/(5*e^3) + (4*
b*d*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]]
, (2*e)/(c*d + e)])/(5*c^2*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(
c*d + e)]) - (32*b*d^2*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*Elli
pticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c^2*e^2*Sqrt[1 -
 1/(c^2*x^2)]*x*Sqrt[d + e*x]) + (4*b*(c*d - e)*(c*d + e)*Sqrt[(c*(d + e*x)
)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*
e)/(c*d + e)])/(15*c^4*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (32*b*d
^3*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqr
t[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sq
rt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 944

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2
]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] +
Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 5247

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(b^IntPart[p]*(
a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[
p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx = \frac{2d^2\sqrt{d+ex}\,\left(a+b\csc^{-1}(cx)\right)}{e^3} - \frac{4d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} + \frac{2(d+ex)^{5/2}\left(a\right.}{5e}$$

$$= \frac{2d^2\sqrt{d+ex}\,\left(a+b\csc^{-1}(cx)\right)}{e^3} - \frac{4d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} + \frac{2(d+ex)^{5/2}\left(a\right.}{5e}$$

$$= \frac{2d^2\sqrt{d+ex}\,\left(a+b\csc^{-1}(cx)\right)}{e^3} - \frac{4d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} + \frac{2(d+ex)^{5/2}\left(a\right.}{5e}$$

$$= \frac{2d^2\sqrt{d+ex}\,\left(a+b\csc^{-1}(cx)\right)}{e^3} - \frac{4d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} + \frac{2(d+ex)^{5/2}\left(a\right.}{5e}$$

$$= \frac{2d^2\sqrt{d+ex}\,\left(a+b\csc^{-1}(cx)\right)}{e^3} - \frac{4d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} + \frac{2(d+ex)^{5/2}\left(a\right.}{5e}$$

$$= -\frac{4b\sqrt{d+ex}\,\left(1-c^2x^2\right)}{15c^3e\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{2d^2\sqrt{d+ex}\,\left(a+b\csc^{-1}(cx)\right)}{e^3} - \frac{4d(d+ex)^{3/2}\left(a+b\,\mathrm{cs}\right.}{3e^3}$$

$$= -\frac{4b\sqrt{d+ex}\,\left(1-c^2x^2\right)}{15c^3e\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{2d^2\sqrt{d+ex}\,\left(a+b\csc^{-1}(cx)\right)}{e^3} - \frac{4d(d+ex)^{3/2}\left(a+b\,\mathrm{cs}\right.}{3e^3}$$

$$= -\frac{4b\sqrt{d+ex}\,\left(1-c^2x^2\right)}{15c^3e\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{2d^2\sqrt{d+ex}\,\left(a+b\csc^{-1}(cx)\right)}{e^3} - \frac{4d(d+ex)^{3/2}\left(a+b\,\mathrm{cs}\right.}{3e^3}$$

$$= -\frac{4b\sqrt{d+ex}\,\left(1-c^2x^2\right)}{15c^3e\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{2d^2\sqrt{d+ex}\,\left(a+b\csc^{-1}(cx)\right)}{e^3} - \frac{4d(d+ex)^{3/2}\left(a+b\,\mathrm{cs}\right.}{3e^3}$$

$$= -\frac{4b\sqrt{d+ex}\,\left(1-c^2x^2\right)}{15c^3e\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{2d^2\sqrt{d+ex}\,\left(a+b\csc^{-1}(cx)\right)}{e^3} - \frac{4d(d+ex)^{3/2}\left(a+b\,\mathrm{cs}\right.}{3e^3}$$

Mathematica [C] time = 13.67, size = 784, normalized size = 1.48

$$b\left(-\frac{cx\left(\frac{d}{x}+e\right)\left(-\frac{16c^2d^2\csc^{-1}(cx)}{15e^3}+\frac{4cd\sqrt{1-\frac{1}{c^2x^2}}}{5e^2}-\frac{4cx\left(e\sqrt{1-\frac{1}{c^2x^2}}-2cd\csc^{-1}(cx)\right)}{15e^2}-\frac{2c^2x^2\csc^{-1}(cx)}{5e}\right)}{\sqrt{d+ex}}-\frac{2\sqrt{cx}\sqrt{\frac{d}{x}+e}\left(\frac{2\sqrt{1-c^2x^2}\left(7c^2d^2e+e^3\right)\sqrt{\frac{cd+cex}{cd+e}}F\left(\sin^{-1}\left(\frac{\sqrt{1}\cdots}{\cdots}\right)\cdots\right)}{\sqrt{1-\frac{1}{c^2x^2}}(cx)^{3/2}\sqrt{\frac{d}{x}+e}}\cdots\right)}{\cdots}\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x],x]
```

```
[Out] -((a*d^3*Sqrt[1 + (e*x)/d]*Beta[-((e*x)/d), 3, 1/2])/(e^3*Sqrt[d + e*x])) +
 (b*(-((c*(e + d/x)*x*((4*c*d*Sqrt[1 - 1/(c^2*x^2)])/(5*e^2) - (16*c^2*d^2*
ArcCsc[c*x])/(15*e^3) - (2*c^2*x^2*ArcCsc[c*x])/(5*e) - (4*c*x*(e*Sqrt[1 -
1/(c^2*x^2)] - 2*c*d*ArcCsc[c*x]))/(15*e^2)))/Sqrt[d + e*x]) - (2*Sqrt[e +
d/x]*Sqrt[c*x]*((2*(7*c^2*d^2*e + e^3)*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1
 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqr
t[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*(8*c^3*d^3 + 3*c*d*e^2)*
Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1
 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c
*x)^(3/2)) - (6*c*d*e*Cos[2*ArcCsc[c*x]]*((c*d + c*e*x)*(-1 + c^2*x^2) + c^
2*d*x*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt
[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)] - (c*x*(1 + c*x)*Sqrt[(e - c*e*x)/(c*d
 + e)]*Sqrt[(c*d + c*e*x)/(c*d - e)]*((c*d + e)*EllipticE[ArcSin[Sqrt[(c*d
+ c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)] - e*EllipticF[ArcSin[Sqrt[(c*d +
 c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)]))/Sqrt[(e*(1 + c*x))/(-(c*d) + e)
] + c*e*x*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, Arc
Sin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]))/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[
e + d/x]*Sqrt[c*x]*(-2 + c^2*x^2))))/(15*e^3*Sqrt[d + e*x])))/c^3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^2}{\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^2/sqrt(e*x + d), x)
```

maple [A] time = 0.08, size = 862, normalized size = 1.63

$$2a\left(\frac{(ex+d)^{\frac{5}{2}}}{5}-\frac{2(ex+d)^{\frac{3}{2}}d}{3}+d^2\sqrt{ex+d}\right)+2b\left(\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{5}{2}}}{5}-\frac{2\operatorname{arccsc}(cx)(ex+d)^{\frac{3}{2}}d}{3}+\operatorname{arccsc}(cx)\,d^2\sqrt{ex+d}+\frac{2\sqrt{\frac{c}{dc-}}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x)
```

```
[Out] 2/e^3*(a*(1/5*(e*x+d)^(5/2)-2/3*(e*x+d)^(3/2)*d+d^2*(e*x+d)^(1/2))+b*(1/5*a
rccsc(c*x)*(e*x+d)^(5/2)-2/3*arccsc(c*x)*(e*x+d)^(3/2)*d+arccsc(c*x)*d^2*(e
*x+d)^(1/2)+2/15/c^3*((c/(c*d-e))^(1/2)*(e*x+d)^(5/2)*c^2+4*d^2*(-((e*x+d)*
c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF((e*x+d
)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2+3*(-((e*x+d)*c-d*c+e
)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)
*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2-8*d^2*(-((e*x+d)*c-d*c+
e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/
2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c^2
-2*(c/(c*d-e))^(1/2)*(e*x+d)^(3/2)*c^2*d-3*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/
2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(
1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e+3*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-
((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),
((c*d-e)/(c*d+e))^(1/2))*c*d*e+(c/(c*d-e))^(1/2)*(e*x+d)^(1/2)*c^2*d^2+(-((
e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF
((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e^2-(c/(c*d-e))^(
1/2)*(e*x+d)^(1/2)*e^2)/(c/(c*d-e))^(1/2)/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)
+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d-e>0)', see `assume?` for more d
etails)Is c*d-e positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x)^(1/2),x)
```

```
[Out] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsc(c*x))/(e*x+d)**(1/2),x)
[Out] Timed out
```

3.59 $\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=344

$$-\frac{2d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{8bd^2\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2}{cd\cdot}\right.}{3ce^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}$$

```
[Out] 2/3*(e*x+d)^(3/2)*(a+b*arccsc(c*x))/e^2-2*d*(a+b*arccsc(c*x))*(e*x+d)^(1/2)
/e^2-4/3*b*EllipticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*
(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*
d+e))^(1/2)+8/3*b*d*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e)
)^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e/x/(1-1/c^2/x^2)
^(1/2)/(e*x+d)^(1/2)+8/3*b*d^2*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1
/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e^2/x
/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] time = 1.60, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {43, 5247, 12, 6721, 6742, 719, 424, 944, 419, 932, 168, 538, 537}

$$-\frac{2d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{8bd^2\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2}{cd\cdot}\right.}{3ce^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x],x]
```

```
[Out] (-2*d*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/e^2 + (2*(d + e*x)^(3/2)*(a + b*Ar
cCsc[c*x]))/(3*e^2) - (4*b*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin
[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c^2*e*Sqrt[1 - 1/(c^2*x^2)]*x
*Sqrt[(c*(d + e*x))/(c*d + e)]) + (8*b*d*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt
[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3
*c^2*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) + (8*b*d^2*Sqrt[(c*(d + e*x))
/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]],
(2*e)/(c*d + e)])/(3*c*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 944

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2
]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] +
Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 5247

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(b^IntPart[p]*(
a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[
p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx = -\frac{2d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{b\int \frac{2(-2d+ex)\sqrt{d+ex}}{3e^2\sqrt{1-\frac{1}{c^2x^2}}\,x^2}\,d}{c}$$

$$= -\frac{2d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{(2b)\int \frac{(-2d+ex)\sqrt{d+ex}}{\sqrt{1-\frac{1}{c^2x^2}}\,x^2}}{3ce^2}$$

$$= -\frac{2d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{\left(2b\sqrt{1-c^2x^2}\right)\int \frac{(-}{}}{3ce^2\sqrt{1-}}$$

$$= -\frac{2d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{\left(2b\sqrt{1-c^2x^2}\right)\int\Big(}{3ce^2\sqrt{}}$$

$$= -\frac{2d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}-\frac{\left(4bd\sqrt{1-c^2x^2}\right)\int}{3ce^2\sqrt{1-\frac{1}{c^2x^2}}}$$

$$= -\frac{2d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}-\frac{\left(4bd^2\sqrt{1-c^2x^2}\right)\int}{3ce^2\sqrt{1-}}$$

$$= -\frac{2d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}-\frac{4b\sqrt{d+ex}\sqrt{1-c^2x}}{3c^2e\sqrt{1-}}$$

$$= -\frac{2d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}-\frac{4b\sqrt{d+ex}\sqrt{1-c^2x}}{3c^2e\sqrt{1-}}$$

$$= -\frac{2d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}-\frac{4b\sqrt{d+ex}\sqrt{1-c^2x}}{3c^2e\sqrt{1-}}$$

$$= -\frac{2d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}-\frac{4b\sqrt{d+ex}\sqrt{1-c^2x}}{3c^2e\sqrt{1-}}$$

Mathematica [C] time = 1.51, size = 289, normalized size = 0.84

$$\frac{2\left(a\sqrt{d+ex}\,(ex-2d)+\frac{2ib\sqrt{\frac{e(cx+1)}{e-cd}}\sqrt{\frac{e-cex}{cd+e}}\left((cd+e)F\left(i\sinh^{-1}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right)|\frac{cd+e}{cd-e}\right)+(cd-e)E\left(i\sinh^{-1}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right)|\frac{cd+e}{cd-e}\right)-2cd\Pi\left(\frac{e}{cd}+\right.\right.}{c^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{-\frac{c}{cd+e}}}\right.}{3e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x],x]
```

[Out] (2*(a*(-2*d + e*x)*Sqrt[d + e*x] + b*(-2*d + e*x)*Sqrt[d + e*x]*ArcCsc[c*x] + ((2*I)*b*Sqrt[(e*(1 + c*x))/(-(c*d) + e)]*Sqrt[(e - c*e*x)/(c*d + e)]*((c*d - e)*EllipticE[I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)] + (c*d + e)*EllipticF[I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)] - 2*c*d*EllipticPi[1 + e/(c*d), I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)]))/(c^2*Sqrt[-(c/(c*d + e))]*Sqrt[1 - 1/(c^2*x^2)]*x)))/(3*e^2)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x}{\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="giac")

[Out] integrate((b*arccsc(c*x) + a)*x/sqrt(e*x + d), x)

maple [A] time = 0.07, size = 412, normalized size = 1.20

$$2a\left(\frac{(ex+d)^{\frac{3}{2}}}{3} - d\sqrt{ex+d}\right) + 2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsc}(cx)}{3} - \operatorname{arccsc}(cx)\, d\sqrt{ex+d} - \frac{2\left(d\,\mathrm{EllipticF}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{dc-e}}, \sqrt{\frac{dc-e}{dc+e}}\right)c + \mathrm{EllipticE}\left(\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x)

[Out] 2/e^2*(a*(1/3*(e*x+d)^(3/2)-d*(e*x+d)^(1/2))+b*(1/3*(e*x+d)^(3/2)*arccsc(c*x)-arccsc(c*x)*d*(e*x+d)^(1/2)-2/3/c^2*(d*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c+EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d-2*d*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c-EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e+EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)/(c/(c*d-e))^(1/2)/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(c*d-e>0)', see `assume?` for more details)Is c*d-e positive or negative?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asin(1/(c*x))))/(d + e*x)^(1/2),x)
[Out] int((x*(a + b*asin(1/(c*x))))/(d + e*x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsc(c*x))/(e*x+d)**(1/2),x)
[Out] Timed out
```

3.60 $\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=212

$$\frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e}-\frac{4b\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}-\frac{4bd\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\Pi\left(2;\sin^{-1}\right.}{cex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}$$

```
[Out] 2*(a+b*arccsc(c*x))*(e*x+d)^(1/2)/e-4*b*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2
),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c
^2/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-4*b*d*EllipticPi(1/2*(-c*x+1)^(1/2)*
2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)
^(1/2)/c/e/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5227, 1574, 944, 719, 419, 933, 168, 538, 537}

$$\frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e}-\frac{4b\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}-\frac{4bd\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\Pi\left(2;\sin^{-1}\right.}{cex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/Sqrt[d + e*x],x]
```

```
[Out] (2*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/e - (4*b*Sqrt[(c*(d + e*x))/(c*d + e)
]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e
)])/(c^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (4*b*d*Sqrt[(c*(d + e*x))
/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]],
(2*e)/(c*d + e)])/(c*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
```

```
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] && !GtQ[a, 0]
```

Rule 944

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2
]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] +
Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 1574

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[(x^(2*n*FracPart[p])*(a + c/x^(2*n))^FracPart[p])/
(c + a*x^(2*n))^FracPart[p], Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] && !In
tegerQ[p] && !IntegerQ[q] && PosQ[n]
```

Rule 5227

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCsc[c*x]))/(e*(m + 1)), x] + Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex}}\,dx = \frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e} + \frac{(2b)\int \frac{\sqrt{d+ex}}{\sqrt{1-\frac{1}{c^2x^2}}\,x^2}\,dx}{ce}$$

$$= \frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e} + \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{\sqrt{d+ex}}{x\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{ce\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= \frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e} + \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{c\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{\left(2bd\sqrt{-\frac{1}{c^2}+x^2}\right.\cdots}{ce\sqrt{\cdots}}$$

$$= \frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e} + \frac{\left(2bd\sqrt{1-c^2x^2}\right)\int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}}\,dx}{ce\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{\left(4b\sqrt{\frac{d+ex}{d+\frac{e}{c}}}\sqrt{\cdots}\right.}{\cdots}$$

$$= \frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e} - \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{\left(4bd\sqrt{1\cdots}\right.}{\cdots}$$

$$= \frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e} - \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{\left(4bd\sqrt{\cdots}\right.}{\cdots}$$

$$= \frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e} - \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{4bd\sqrt{\frac{c(\cdots}{c\cdots}}\cdots}{\cdots}$$

Mathematica [A] time = 3.69, size = 243, normalized size = 1.15

$$\frac{2\left(\frac{a(d+ex)}{e} + \frac{b\left(\frac{2cdx\sqrt{1-\frac{1}{c^2x^2}}\sqrt{\frac{c(d+ex)}{cd+e}}\,\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{1-c^2x^2}}+\csc^{-1}(cx)(d+ex)\right)}{e} + \frac{2bcx^2\sqrt{cx+1}\sqrt{\frac{c(d+ex)}{cd+e}}\left(\cos\left(\frac{1}{2}\csc^{-1}(cx)\right)+\sin\left(\frac{1}{2}\csc^{-1}(cx)\right)\cdots\right.}{\sqrt{1-c\cdots}}\cdots\right)}{\sqrt{d+ex}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsc[c*x])/Sqrt[d + e*x],x]
```

```
[Out] (2*((a*(d + e*x))/e + (b*((d + e*x)*ArcCsc[c*x] + (2*c*d*Sqrt[1 - 1/(c^2*x^
2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt
[2]], (2*e)/(c*d + e)])/Sqrt[1 - c^2*x^2]))/e + (2*b*c*x^2*Sqrt[1 + c*x]*Sq
rt[(c*(d + e*x))/(c*d + e)]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/
(c*d + e)]*(Cos[ArcCsc[c*x]/2] - Sin[ArcCsc[c*x]/2])^3*(Cos[ArcCsc[c*x]/2]
+ Sin[ArcCsc[c*x]/2]))/(Sqrt[1 - c*x]*(-1 + c^2*x^2))))/Sqrt[d + e*x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arccsc}(cx) + a}{\sqrt{ex + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/sqrt(e*x + d), x)
```

maple [A] time = 0.07, size = 254, normalized size = 1.20

$$\frac{2a\sqrt{ex+d}+2b\left(\sqrt{ex+d}\,\operatorname{arccsc}(cx)+\frac{2\sqrt{-\frac{(ex+d)c-dc+e}{dc-e}}\,\sqrt{-\frac{(ex+d)c-dc-e}{dc+e}}\left(\operatorname{EllipticF}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{dc-e}},\sqrt{\frac{dc-e}{dc+e}}\right)-\operatorname{EllipticPi}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{dc-e}}\cdots\right.\right.}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2}{c^2e^2x^2}}\;x\sqrt{\frac{c}{dc-e}}}\cdots\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/(e*x+d)^(1/2),x)
```

```
[Out] 2/e*(a*(e*x+d)^(1/2)+b*((e*x+d)^(1/2)*arccsc(c*x)+2/c*(-((e*x+d)*c-d*c+e)/(
c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*(EllipticF((e*x+d)^(1/2)*(
c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))-EllipticPi((e*x+d)^(1/2)*(c/(c*d-
e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2)))/((c^2*(e*x+d)
^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)/x/(c/(c*d-e))^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d-e>0)', see `assume?` for more d
etails)Is c*d-e positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{asin}\left(\frac{1}{c\,x}\right)}{\sqrt{d + e\,x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x)^(1/2),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(d + e*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsc}(cx)}{\sqrt{d + ex}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/(e*x+d)**(1/2),x)
```

```
[Out] Integral((a + b*acsc(c*x))/sqrt(d + e*x), x)
```

3.61 $\int \frac{a+b \csc^{-1}(cx)}{x \sqrt{d+ex}} \, dx$

Optimal. Leaf size=24

$$\mathrm{Int}\left(\frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex}},x\right)$$

[Out] Unintegrable((a+b*arccsc(c*x))/x/(e*x+d)^(1/2),x)

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex}}\,dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*ArcCsc[c*x])/(x*Sqrt[d + e*x]),x]

[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x*Sqrt[d + e*x]), x]

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex}}\,dx = \int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex}}\,dx$$

Mathematica [A] time = 6.64, size = 0, normalized size = 0.00

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex}}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*ArcCsc[c*x])/(x*Sqrt[d + e*x]),x]

[Out] Integrate[(a + b*ArcCsc[c*x])/(x*Sqrt[d + e*x]), x]

fricas [A] time = 0.84, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{ex+d}\,(b\,\mathrm{arccsc}\,(cx)+a)}{ex^2+dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccsc(c*x))/x/(e*x+d)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(e*x + d)*(b*arccsc(c*x) + a)/(e*x^2 + d*x), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\mathrm{arccsc}\,(cx)+a}{\sqrt{ex+d}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccsc(c*x))/x/(e*x+d)^(1/2),x, algorithm="giac")

[Out] integrate((b*arccsc(c*x) + a)/(sqrt(e*x + d)*x), x)

maple [A] time = 4.56, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arccsc}(cx)}{x\sqrt{ex + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x/(e*x+d)^(1/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/x/(e*x+d)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{b\sqrt{d}\int \frac{\arctan\left(1, \sqrt{cx+1}\,\sqrt{cx-1}\right)}{\sqrt{ex+d}\,x}\, dx + a\log\left(\frac{ex}{ex+2\,\sqrt{ex+d}\,\sqrt{d}+2\,d}\right)}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] (b*sqrt(d)*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(sqrt(e*x + d)
*x), x) + a*log(e*x/(e*x + 2*sqrt(e*x + d)*sqrt(d) + 2*d)))/sqrt(d)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x\,\sqrt{d + e\,x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x*(d + e*x)^(1/2)),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(x*(d + e*x)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

3.62 $\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x^2/(e*x+d)^(1/2),x)
```

Rubi [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCsc[c*x])/(x^2*Sqrt[d + e*x]),x]
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x^2*Sqrt[d + e*x]), x]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx$$

Mathematica [A] time = 9.21, size = 0, normalized size = 0.00

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x^2*Sqrt[d + e*x]),x]
[Out] Integrate[(a + b*ArcCsc[c*x])/(x^2*Sqrt[d + e*x]), x]
```

fricas [A] time = 2.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex+d}\,(b\,\text{arccsc}\,(cx)+a)}{ex^3+dx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x + d)*(b*arccsc(c*x) + a)/(e*x^3 + d*x^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arccsc}\,(cx)+a}{\sqrt{ex+d}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)/(sqrt(e*x + d)*x^2), x)
```

maple [A] time = 7.83, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arccsc}(cx)}{x^2 \sqrt{ex + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x^2/(e*x+d)^(1/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/x^2/(e*x+d)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\, b d^{\frac{3}{2}} x \int \frac{\arctan\left(1, \sqrt{cx+1}\, \sqrt{cx-1}\right)}{\sqrt{ex+d}\, x^2}\, dx - aex \log\left(\frac{ex}{ex+2\, \sqrt{ex+d}\, \sqrt{d}+2\, d}\right) - 2\, \sqrt{ex + d}\, a \sqrt{d}}{2\, d^{\frac{3}{2}} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*d^(3/2)*x*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(sqrt(
e*x + d)*x^2), x) - a*e*x*log(e*x/(e*x + 2*sqrt(e*x + d)*sqrt(d) + 2*d)) -
2*sqrt(e*x + d)*a*sqrt(d))/(d^(3/2)*x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b \operatorname{asin}\left(\frac{1}{c\, x}\right)}{x^2\, \sqrt{d + e\, x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x)^(1/2)),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsc}(cx)}{x^2 \sqrt{d + ex}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x**2/(e*x+d)**(1/2),x)
```

```
[Out] Integral((a + b*acsc(c*x))/(x**2*sqrt(d + e*x)), x)
```

3.63 $\int \frac{x^3\left(a+b\csc^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=551

$$\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(c\right.}{5e^4}$$

```
[Out] -2*d*(e*x+d)^(3/2)*(a+b*arccsc(c*x))/e^4+2/5*(e*x+d)^(5/2)*(a+b*arccsc(c*x)
)/e^4+2*d^3*(a+b*arccsc(c*x))/e^4/(e*x+d)^(1/2)+6*d^2*(a+b*arccsc(c*x))*(e*
x+d)^(1/2)/e^4-4/15*b*(-c^2*x^2+1)*(e*x+d)^(1/2)/c^3/e^2/x/(1-1/c^2/x^2)^(1
/2)+32/15*b*d*EllipticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2
))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e^3/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d
)/(c*d+e))^(1/2)-8*b*d^2*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c
*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e^3/x/(1-1/c
^2/x^2)^(1/2)/(e*x+d)^(1/2)-4/15*b*(2*c^2*d^2+e^2)*EllipticF(1/2*(-c*x+1)^(
1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2
+1)^(1/2)/c^4/e^3/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-64/5*b*d^3*EllipticPi
(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e)
)^(1/2)*(-c^2*x^2+1)^(1/2)/c/e^4/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] time = 2.47, antiderivative size = 551, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {43, 5247, 12, 6721, 6742, 719, 419, 932, 168, 538, 537, 844, 424, 931, 1584}

$$\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(c\right.}{5e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x)^(3/2),x]
```

```
[Out] (-4*b*Sqrt[d + e*x]*(1 - c^2*x^2))/(15*c^3*e^2*Sqrt[1 - 1/(c^2*x^2)]*x) + (
2*d^3*(a + b*ArcCsc[c*x]))/(e^4*Sqrt[d + e*x]) + (6*d^2*Sqrt[d + e*x]*(a +
b*ArcCsc[c*x]))/e^4 - (2*d*(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]))/e^4 + (2*(d
 + e*x)^(5/2)*(a + b*ArcCsc[c*x]))/(5*e^4) + (32*b*d*Sqrt[d + e*x]*Sqrt[1 -
 c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c^
2*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) - (8*b*d^2*Sqr
t[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]
/Sqrt[2]], (2*e)/(c*d + e)])/(c^2*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]
) - (4*b*(2*c^2*d^2 + e^2)*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*
EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c^4*e^3*Sqrt
[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (64*b*d^3*Sqrt[(c*(d + e*x))/(c*d + e)
]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d
 + e)])/(5*c*e^4*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 931

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[(2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g*x]*Sqrt[a +
c*x^2])/(c*g*(2*m - 1)), x] - Dist[1/(c*g*(2*m - 1)), Int[((d + e*x)^(m - 3
```

```
)*Simp[a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3*g*(2*m - 1) + e*(e*(a*e*g*(2*m -
 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x + 2*e^2*(c*e*f - 3*c*d*g)*(m - 1)*x
^2, x])/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GeQ[
m, 2]
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 5247

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(b^IntPart[p]*(
a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[
p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^3\left(a+b\csc^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx = \frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4} - \frac{2d(d+ex)^{3/2}\left(a+b\csc^{-}\right.}{e^4}$$

$$= \frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4} - \frac{2d(d+ex)^{3/2}\left(a+b\csc^{-}\right.}{e^4}$$

$$= \frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4} - \frac{2d(d+ex)^{3/2}\left(a+b\csc^{-}\right.}{e^4}$$

$$= \frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4} - \frac{2d(d+ex)^{3/2}\left(a+b\csc^{-}\right.}{e^4}$$

$$= \frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4} - \frac{2d(d+ex)^{3/2}\left(a+b\csc^{-}\right.}{e^4}$$

$$= -\frac{4b\sqrt{d+ex}\left(1-c^2x^2\right)}{15c^3e^2\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}$$

$$= -\frac{4b\sqrt{d+ex}\left(1-c^2x^2\right)}{15c^3e^2\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}$$

$$= -\frac{4b\sqrt{d+ex}\left(1-c^2x^2\right)}{15c^3e^2\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}$$

$$= -\frac{4b\sqrt{d+ex}\left(1-c^2x^2\right)}{15c^3e^2\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}$$

$$= -\frac{4b\sqrt{d+ex}\left(1-c^2x^2\right)}{15c^3e^2\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}$$

Mathematica [C] time = 14.02, size = 814, normalized size = 1.48

$$\frac{a\left(\frac{ex}{d}+1\right)^{3/2} B_{-\frac{ex}{d}}\left(4,-\frac{1}{2}\right) d^4}{e^4(d+ex)^{3/2}} + \frac{b\left(-\frac{c^2\left(\frac{d}{x}+e\right)^2\left(\frac{2c^2\csc^{-1}(cx)d^2}{e^3\left(\frac{d}{x}+e\right)} - \frac{32c^2\csc^{-1}(cx)d^2}{5e^4} + \frac{32c\sqrt{1-\frac{1}{c^2x^2}}\,d}{15e^3} - \frac{2c^2x^2\csc^{-1}(cx)}{5e^2} - \frac{2cx\left(2e\sqrt{1-\frac{1}{c^2x^2}}-9cd\csc^{-1}(cx)\right)}{15e^3}\right)x^{\cdots}}{(d+ex)^{3/2}} \cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x)^(3/2),x]
```

```
[Out] (a*d^4*(1 + (e*x)/d)^(3/2)*Beta[-((e*x)/d), 4, -1/2])/(e^4*(d + e*x)^(3/2))
 + (b*(-((c^2*(e + d/x)^2*x^2*((32*c*d*Sqrt[1 - 1/(c^2*x^2)])/(15*e^3) - (3
2*c^2*d^2*ArcCsc[c*x])/(5*e^4) + (2*c^2*d^2*ArcCsc[c*x])/(e^3*(e + d/x)) -
(2*c^2*x^2*ArcCsc[c*x])/(5*e^2) - (2*c*x*(2*e*Sqrt[1 - 1/(c^2*x^2)] - 9*c*d
*ArcCsc[c*x]))/(15*e^3)))/(d + e*x)^(3/2)) - (2*(e + d/x)^(3/2)*(c*x)^(3/2)
*((2*(32*c^2*d^2*e + e^3)*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*E
llipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*
x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*(48*c^3*d^3 + 8*c*d*e^2)*Sqrt[(c*d +
c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt
[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) -
 (16*c*d*e*Cos[2*ArcCsc[c*x]]*((c*d + c*e*x)*(-1 + c^2*x^2) + c^2*d*x*Sqrt[
(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/S
qrt[2]], (2*e)/(c*d + e)] - (c*x*(1 + c*x)*Sqrt[(e - c*e*x)/(c*d + e)]*Sqrt
[(c*d + c*e*x)/(c*d - e)]*((c*d + e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/(c
*d - e)]], (c*d - e)/(c*d + e)] - e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*
d - e)]], (c*d - e)/(c*d + e)]))/Sqrt[(e*(1 + c*x))/(-(c*d) + e)] + c*e*x*S
qrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1
- c*x]/Sqrt[2]], (2*e)/(c*d + e)]))/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*Sq
rt[c*x]*(-2 + c^2*x^2))))/(15*e^4*(d + e*x)^(3/2))))/c^4
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x^3}{(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^3/(e*x + d)^(3/2), x)
```

maple [A] time = 0.08, size = 890, normalized size = 1.62

$$2a\left(\frac{(ex+d)^{\frac{5}{2}}}{5}-(ex+d)^{\frac{3}{2}}d+3d^{2}\sqrt{ex+d}+\frac{d^{3}}{\sqrt{ex+d}}\right)+2b\left(\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{5}{2}}}{5}-\operatorname{arccsc}(cx)(ex+d)^{\frac{3}{2}}d+3\operatorname{arccsc}(cx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x)
```

```
[Out] 2/e^4*(a*(1/5*(e*x+d)^(5/2)-(e*x+d)^(3/2)*d+3*d^2*(e*x+d)^(1/2)+d^3/(e*x+d)
^(1/2))+b*(1/5*arccsc(c*x)*(e*x+d)^(5/2)-arccsc(c*x)*(e*x+d)^(3/2)*d+3*arcc
sc(c*x)*d^2*(e*x+d)^(1/2)+arccsc(c*x)*d^3/(e*x+d)^(1/2)+2/15/c^3*((c/(c*d-e
))^(1/2)*(e*x+d)^(5/2)*c^2-2*(c/(c*d-e))^(1/2)*(e*x+d)^(3/2)*c^2*d+24*d^2*(
-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*Ellipt
icF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2+8*(-((e*x+
d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticE((e*
x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2-48*d^2*(-((e*
x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticPi(
(e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))
^(1/2))*c^2+(c/(c*d-e))^(1/2)*(e*x+d)^(1/2)*c^2*d^2-8*(-((e*x+d)*c-d*c+e)/(
c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c
/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e+8*(-((e*x+d)*c-d*c+e)/(c*d-e
))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d
-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e+(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2
)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1
/2),((c*d-e)/(c*d+e))^(1/2))*e^2-(c/(c*d-e))^(1/2)*(e*x+d)^(1/2)*e^2)/(c/(c
*d-e))^(1/2)/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1
/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d-e>0)', see `assume?` for more d
etails)Is c*d-e positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(d+e\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x)^(3/2),x)
```

```
[Out] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsc(c*x))/(e*x+d)**(3/2),x)
```

```
[Out] Timed out
```

3.64 $\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=369

$$-\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{32bd^2\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}}{3ce^3x\sqrt{1-}}$$

```
[Out] 2/3*(e*x+d)^(3/2)*(a+b*arccsc(c*x))/e^3-2*d^2*(a+b*arccsc(c*x))/e^3/(e*x+d)
^(1/2)-4*d*(a+b*arccsc(c*x))*(e*x+d)^(1/2)/e^3-4/3*b*EllipticE(1/2*(-c*x+1)
^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/
c^2/e^2/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)+20/3*b*d*EllipticF(
1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(
1/2)*(-c^2*x^2+1)^(1/2)/c^2/e^2/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+32/3*b*
d^2*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(
e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e^3/x/(1-1/c^2/x^2)^(1/2)/(e*x+d
)^(1/2)
```

Rubi [A] time = 1.81, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {43, 5247, 12, 6721, 6742, 719, 419, 932, 168, 538, 537, 844, 424}

$$-\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{32bd^2\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}}{3ce^3x\sqrt{1-}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x)^(3/2),x]
```

```
[Out] (-2*d^2*(a + b*ArcCsc[c*x]))/(e^3*Sqrt[d + e*x]) - (4*d*Sqrt[d + e*x]*(a +
b*ArcCsc[c*x]))/e^3 + (2*(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^3) - (4*
b*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]],
(2*e)/(c*d + e)])/(3*c^2*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*
d + e)]) + (20*b*d*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*Elliptic
F[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c^2*e^2*Sqrt[1 - 1/(c
^2*x^2)]*x*Sqrt[d + e*x]) + (32*b*d^2*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1
- c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(
3*c*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
```

```
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 5247

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
```

```
reeQ[{a, b, c}, x]
```

Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(b^IntPart[p]*(
a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[
p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx=-\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}\right.}{3e^3}$$

$$=-\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}\right.}{3e^3}$$

$$=-\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}\right.}{3e^3}$$

$$=-\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}\right.}{3e^3}$$

$$=-\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}\right.}{3e^3}$$

$$=-\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}\right.}{3e^3}$$

$$=-\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}\right.}{3e^3}$$

$$=-\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}\right.}{3e^3}$$

$$=-\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}\right.}{3e^3}$$

Mathematica [C]   time = 13.84, size = 750, normalized size = 2.03

$$b\left(\frac{2(cx)^{3/2}\left(\frac{d}{x}+e\right)^{3/2}\left(\frac{2\sqrt{1-c^2x^2}\left(8c^2d^2+e^2\right)\sqrt{\frac{cd+cex}{cd+e}}\,\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{1-\frac{1}{c^2x^2}}\,(cx)^{3/2}\sqrt{\frac{d}{x}+e}}+\frac{10cde\sqrt{1-c^2x^2}\sqrt{\frac{cd+cex}{cd+e}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{1-\frac{1}{c^2x^2}}\,(cx)^{3/2}\sqrt{\frac{d}{x}+e}}-\frac{2e\cos\left(2\csc^{-1}(cx)\right)\left(\left(c^2x^2-1\right)(cd+c\cdots\right.}{\cdots}\right.}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x)^(3/2),x]
```

```
[Out] -((a*d^3*(1 + (e*x)/d)^(3/2)*Beta[-((e*x)/d), 3, -1/2])/(e^3*(d + e*x)^(3/2
))) + (b*(-((c^2*(e + d/x)^2*x^2*((-4*Sqrt[1 - 1/(c^2*x^2)])/(3*e^2) + (16*
c*d*ArcCsc[c*x])/(3*e^3) - (2*c*d*ArcCsc[c*x])/(e^2*(e + d/x)) - (2*c*x*Arc
Csc[c*x])/(3*e^2)))/(d + e*x)^(3/2)) + (2*(e + d/x)^(3/2)*(c*x)^(3/2)*((10*
c*d*e*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt
[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*
(c*x)^(3/2)) + (2*(8*c^2*d^2 + e^2)*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 -
c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sq
rt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) - (2*e*Cos[2*ArcCsc[c*x]]*((
c*d + c*e*x)*(-1 + c^2*x^2) + c^2*d*x*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1
- c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)] - (c*x
*(1 + c*x)*Sqrt[(e - c*e*x)/(c*d + e)]*Sqrt[(c*d + c*e*x)/(c*d - e)]*((c*d
+ e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)]
- e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)]))
/Sqrt[(e*(1 + c*x))/(-(c*d) + e)] + c*e*x*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqr
t[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)
]))/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*Sqrt[c*x]*(-2 + c^2*x^2))))/(3*e^3
*(d + e*x)^(3/2))))/c^3
```

fricas [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arccsc}\,(cx) + a)x^2}{(ex + d)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^2/(e*x + d)^(3/2), x)
```

maple [A] time = 0.08, size = 441, normalized size = 1.20

$$\frac{2a\left(\frac{(ex+d)^{\frac{3}{2}}}{3}-2d\sqrt{ex+d}-\frac{d^2}{\sqrt{ex+d}}\right)+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsc}(cx)}{3}-2\operatorname{arccsc}(cx)\,d\sqrt{ex+d}-\frac{\operatorname{arccsc}(cx)d^2}{\sqrt{ex+d}}-\frac{2\left(4d\,\mathrm{EllipticF}\left(\sqrt{e\right.\right.}{}}{}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x)
```

```
[Out] 2/e^3*(a*(1/3*(e*x+d)^(3/2)-2*d*(e*x+d)^(1/2)-d^2/(e*x+d)^(1/2))+b*(1/3*(e*
x+d)^(3/2)*arccsc(c*x)-2*arccsc(c*x)*d*(e*x+d)^(1/2)-arccsc(c*x)*d^2/(e*x+d
)^(1/2)-2/3/c^2*(4*d*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*
d+e))^(1/2))*c+EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^
(1/2))*c*d-8*d*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/
(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c-EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/
2),((c*d-e)/(c*d+e))^(1/2))*e+EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c
*d-e)/(c*d+e))^(1/2))*e)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*(-((e*x+d)*c-d*
c+e)/(c*d-e))^(1/2)/(c/(c*d-e))^(1/2)/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2
*d^2-e^2)/c^2/e^2/x^2)^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d-e>0)', see `assume?` for more d
etails)Is c*d-e positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(d+e\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x)^(3/2),x)
```

```
[Out] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(a + b \operatorname{acsc}{\left(c x \right)}\right)}{\left(d + e x\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsc(c*x))/(e*x+d)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*acsc(c*x))/(d + e*x)**(3/2), x)
```

3.65 $\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=238

$$\frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{2d\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}}-\frac{8bd\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{ce^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}-\frac{4b\sqrt{1-c^2x}}{}$$

```
[Out] 2*d*(a+b*arccsc(c*x))/e^2/(e*x+d)^(1/2)+2*(a+b*arccsc(c*x))*(e*x+d)^(1/2)/e
^2-4*b*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(
e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e/x/(1-1/c^2/x^2)^(1/2)/(e*x+d
)^(1/2)-8*b*d*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(
1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e^2/x/(1-1/c^2/x^2)^(1
/2)/(e*x+d)^(1/2)
```

Rubi [A] time = 1.58, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {43, 5247, 12, 6721, 6742, 719, 419, 932, 168, 538, 537}

$$\frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{2d\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}}-\frac{8bd\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{ce^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}-\frac{4b\sqrt{1-c^2x}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCsc[c*x]))/(d + e*x)^(3/2),x]
```

```
[Out] (2*d*(a + b*ArcCsc[c*x]))/(e^2*Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*(a + b*Arc
Csc[c*x]))/e^2 - (4*b*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*Ellip
ticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c^2*e*Sqrt[1 - 1/(c^
2*x^2)]*x*Sqrt[d + e*x]) - (8*b*d*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^
2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c*e^
2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
```

```
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 5247

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(b^IntPart[p]*(
a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[
p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx = \frac{2d\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2} + \frac{b\int \frac{2(2d+ex)}{e^2\sqrt{1-\frac{1}{c^2x^2}}\,x^2\sqrt{d+ex}}\,dx}{c}$$

$$= \frac{2d\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2} + \frac{(2b)\int \frac{2d+ex}{\sqrt{1-\frac{1}{c^2x^2}}\,x^2\sqrt{d+ex}}\,dx}{ce^2}$$

$$= \frac{2d\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2} + \frac{\left(2b\sqrt{1-c^2x^2}\right)\int \frac{2d+ex}{x\sqrt{d+ex}\,\sqrt{1-c^2x^2}}}{ce^2\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= \frac{2d\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2} + \frac{\left(2b\sqrt{1-c^2x^2}\right)\int \left(\frac{e}{\sqrt{d+ex}\,\sqrt{1-c^2x^2}}\right.}{ce^2\sqrt{1-\frac{1}{c^2x^2}}}$$

$$= \frac{2d\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2} + \frac{\left(4bd\sqrt{1-c^2x^2}\right)\int \frac{1}{x\sqrt{d+ex}\,\sqrt{1-c^2x^2}}}{ce^2\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= \frac{2d\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2} + \frac{\left(4bd\sqrt{1-c^2x^2}\right)\int \frac{1}{x\sqrt{1-cx}\,\sqrt{1+cx}}}{ce^2\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= \frac{2d\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2} - \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,F\left(\sin^{-1}\left(\right.\right.}{c^2e\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+e}}$$

$$= \frac{2d\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2} - \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,F\left(\sin^{-1}\left(\right.\right.}{c^2e\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+e}}$$

$$= \frac{2d\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2} - \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,F\left(\sin^{-1}\left(\right.\right.}{c^2e\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+e}}$$

Mathematica [C] time = 1.88, size = 226, normalized size = 0.95

$$\frac{2\left(\frac{a(2d+ex)}{\sqrt{d+ex}} - \frac{2ib\sqrt{\frac{e(cx+1)}{e-cd}}\sqrt{\frac{e-cex}{cd+e}}\left(F\left(i\sinh^{-1}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right)\middle|\frac{cd+e}{cd-e}\right)-2\Pi\left(\frac{e}{cd}+1;i\sinh^{-1}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right)\middle|\frac{cd+e}{cd-e}\right)\right)}{cx\sqrt{1-\frac{1}{c^2x^2}}\sqrt{-\frac{c}{cd+e}}} + \frac{b\csc^{-1}(cx)(2d+ex)}{\sqrt{d+ex}}\right)}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCsc[c*x]))/(d + e*x)^(3/2),x]
```

```
[Out] (2*((a*(2*d + e*x))/Sqrt[d + e*x] + (b*(2*d + e*x)*ArcCsc[c*x])/Sqrt[d + e*
x] - ((2*I)*b*Sqrt[(e*(1 + c*x))/(-(c*d) + e)]*Sqrt[(e - c*e*x)/(c*d + e)]*
(EllipticF[I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d -
e)] - 2*EllipticPi[1 + e/(c*d), I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x
```

```
]], (c*d + e)/(c*d - e)]))/(c*Sqrt[-(c/(c*d + e))]*Sqrt[1 - 1/(c^2*x^2)]*x)
))/e^2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x}{(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)*x/(e*x + d)^(3/2), x)
```

maple [A] time = 0.07, size = 277, normalized size = 1.16

$$\frac{2a\left(\sqrt{ex+d} + \frac{d}{\sqrt{ex+d}}\right) + 2b\left(\sqrt{ex+d}\,\operatorname{arccsc}(cx) + \frac{\operatorname{arccsc}(cx)d}{\sqrt{ex+d}} + \frac{2\sqrt{-\frac{(ex+d)c-dc+e}{dc-e}}\sqrt{-\frac{(ex+d)c-dc-e}{dc+e}}\left(\operatorname{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{dc-e}},\right.\right.}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c}{c^2e^2x^2}}}\right.}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x)
[Out] 2/e^2*(a*((e*x+d)^(1/2)+d/(e*x+d)^(1/2))+b*((e*x+d)^(1/2)*arccsc(c*x)+arccs
c(c*x)*d/(e*x+d)^(1/2)+2/c*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-
d*c-e)/(c*d+e))^(1/2)*(EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(
c*d+e))^(1/2))-2*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(
c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2)))/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^
2-e^2)/c^2/e^2/x^2)^(1/2)/x/(c/(c*d-e))^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d-e>0)', see `assume?` for more d
etails)Is c*d-e positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{(d+e\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asin(1/(c*x))))/(d + e*x)^(3/2),x)
[Out] int((x*(a + b*asin(1/(c*x))))/(d + e*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\,(a + b\operatorname{acsc}(cx))}{(d + ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsc(c*x))/(e*x+d)**(3/2),x)
[Out] Integral(x*(a + b*acsc(c*x))/(d + e*x)**(3/2), x)
```

3.66 $\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=119

$$\frac{4b\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{cex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{e\sqrt{d+ex}}$$

```
[Out] -2*(a+b*arccsc(c*x))/e/(e*x+d)^(1/2)+4*b*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1
/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/
2)/c/e/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5227, 1574, 933, 168, 538, 537}

$$\frac{4b\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{cex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{e\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(d + e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcCsc[c*x]))/(e*Sqrt[d + e*x]) + (4*b*Sqrt[(c*(d + e*x))/(c*d +
 e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(
c*d + e)])/(c*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 1574

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[(x^(2*n*FracPart[p])*(a + c/x^(2*n))^FracPart[p])/
(c + a*x^(2*n))^FracPart[p], Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 5227

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCsc[c*x]))/(e*(m + 1)), x] + Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{3/2}}\,dx &= -\frac{2\left(a+b\csc^{-1}(cx)\right)}{e\sqrt{d+ex}}-\frac{(2b)\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\,x^2\sqrt{d+ex}}\,dx}{ce}\\
&= -\frac{2\left(a+b\csc^{-1}(cx)\right)}{e\sqrt{d+ex}}-\frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{x\sqrt{d+ex}\,\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{ce\sqrt{1-\frac{1}{c^2x^2}}\,x}\\
&= -\frac{2\left(a+b\csc^{-1}(cx)\right)}{e\sqrt{d+ex}}-\frac{\left(2b\sqrt{1-c^2x^2}\right)\int \frac{1}{x\sqrt{1-cx}\,\sqrt{1+cx}\,\sqrt{d+ex}}\,dx}{ce\sqrt{1-\frac{1}{c^2x^2}}\,x}\\
&= -\frac{2\left(a+b\csc^{-1}(cx)\right)}{e\sqrt{d+ex}}+\frac{\left(4b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{2-x^2}\,\sqrt{d+\frac{e}{c}-\frac{ex^2}{c}}}\,dx,x,\sqrt{1-cx}\right)}{ce\sqrt{1-\frac{1}{c^2x^2}}\,x}\\
&= -\frac{2\left(a+b\csc^{-1}(cx)\right)}{e\sqrt{d+ex}}+\frac{\left(4b\sqrt{\frac{c(d+ex)}{cd+e}}\,\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{2-x^2}\,\sqrt{1-\frac{ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx,x,\sqrt{1-
\end{aligned}
$$

$$
= -\frac{2\left(a+b\csc^{-1}(cx)\right)}{e\sqrt{d+ex}}+\frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\,\sqrt{1-c^2x^2}\,\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{ce\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}
$$

Mathematica [A] time = 0.22, size = 124, normalized size = 1.04

$$
\frac{4bcx\sqrt{1-\frac{1}{c^2x^2}}\,\sqrt{1-c^2x^2}\,\sqrt{\frac{c(d+ex)}{cd+e}}\,\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)-2\left(c^2x^2-1\right)\left(a+b\csc^{-1}(cx)\right)}{e\left(c^2x^2-1\right)\sqrt{d+ex}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(d + e*x)^(3/2),x]
```

```
[Out] (-2*(-1 + c^2*x^2)*(a + b*ArcCsc[c*x]) + 4*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt
[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c
*x]/Sqrt[2]], (2*e)/(c*d + e)])/(e*Sqrt[d + e*x]*(-1 + c^2*x^2))
```

fricas [F] time = 2.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex+d}\,(b\,\text{arccsc}\,(cx)+a)}{e^2x^2+2\,dex+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x + d)*(b*arccsc(c*x) + a)/(e^2*x^2 + 2*d*e*x + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arccsc}\,(cx)+a}{(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)/(e*x + d)^(3/2), x)
```

maple [A] time = 0.06, size = 217, normalized size = 1.82

$$\frac{-\frac{2a}{\sqrt{ex+d}}+2b\left(-\frac{\text{arccsc}(cx)}{\sqrt{ex+d}}+\frac{2\sqrt{-\frac{(ex+d)c-dc+e}{dc-e}}\,\sqrt{-\frac{(ex+d)c-dc-e}{dc+e}}\,\text{EllipticPi}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{dc-e}},\frac{dc-e}{cd},\frac{\sqrt{\frac{c}{dc+e}}}{\sqrt{\frac{c}{dc-e}}}\right)}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2}{c^2e^2x^2}}\,xd\sqrt{\frac{c}{dc-e}}}\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/(e*x+d)^(3/2),x)
[Out] 2/e*(-1/(e*x+d)^(1/2)*a+b*(-1/(e*x+d)^(1/2)*arccsc(c*x)+2/c/((c^2*(e*x+d)^2
-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)/x/d/(c/(c*d-e))^(1/2)*(-((
e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticP
i((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e
))^(1/2))))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d-e>0)', see `assume?` for more d
etails)Is c*d-e positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\text{asin}\left(\frac{1}{c\,x}\right)}{(d+e\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x)^(3/2),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(d + e*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsc}(cx)}{(d + ex)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/(e*x+d)**(3/2),x)
```

```
[Out] Integral((a + b*acsc(c*x))/(d + e*x)**(3/2), x)
```

3.67 $\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{a+b\csc^{-1}(cx)}{x(d+ex)^{3/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x/(e*x+d)^(3/2),x)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCsc[c*x])/(x*(d + e*x)^(3/2)),x]
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x*(d + e*x)^(3/2)), x]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{3/2}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{3/2}}\,dx$$

Mathematica [A] time = 11.78, size = 0, normalized size = 0.00

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x)^(3/2)),x]
[Out] Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x)^(3/2)), x]
```

fricas [A] time = 1.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex+d}\,(b\,\text{arccsc}\,(cx)+a)}{e^2x^3+2\,dex^2+d^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x+d)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x + d)*(b*arccsc(c*x) + a)/(e^2*x^3 + 2*d*e*x^2 + d^2*x), x
)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arccsc}\,(cx)+a}{(ex+d)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)/((e*x + d)^(3/2)*x), x)
```

maple [A] time = 7.38, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arccsc}\,(cx)}{x\,(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x/(e*x+d)^(3/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/x/(e*x+d)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(bd^{\frac{3}{2}}\int \frac{\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right)}{(ex+d)^{\frac{3}{2}}x}\,dx+a\log\left(\frac{ex}{ex+2\,\sqrt{ex+d}\,\sqrt{d}+2\,d}\right)\right)\sqrt{ex+d}\;+2\,a\sqrt{d}}{\sqrt{ex+d}\,d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] ((b*d^(3/2)*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/((e*x^2 + d*x
)*sqrt(e*x + d)), x) + a*log(e*x/(e*x + 2*sqrt(e*x + d)*sqrt(d) + 2*d)))*sq
rt(e*x + d) + 2*a*sqrt(d))/(sqrt(e*x + d)*d^(3/2))
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x\,{\left(d+e\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x*(d + e*x)^(3/2)),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(x*(d + e*x)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x/(e*x+d)**(3/2),x)
```

```
[Out] Timed out
```

3.68 $\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{3/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x^2/(e*x+d)^(3/2),x)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCsc[c*x])/(x^2*(d + e*x)^(3/2)),x]
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x^2*(d + e*x)^(3/2)), x]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx$$

Mathematica [A] time = 14.91, size = 0, normalized size = 0.00

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x^2*(d + e*x)^(3/2)),x]
[Out] Integrate[(a + b*ArcCsc[c*x])/(x^2*(d + e*x)^(3/2)), x]
```

fricas [A] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex+d}\,(b\,\text{arccsc}\,(cx)+a)}{e^2x^4+2\,dex^3+d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x+d)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x + d)*(b*arccsc(c*x) + a)/(e^2*x^4 + 2*d*e*x^3 + d^2*x^2),
 x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arccsc}\,(cx)+a}{(ex+d)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)/((e*x + d)^(3/2)*x^2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arccsc}(cx)}{x^2 (ex + d)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x^2/(e*x+d)^(3/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/x^2/(e*x+d)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2 \left(bd^2ex^2 + bd^3x\right)\sqrt{d} \int \frac{\arctan\left(1, \sqrt{cx+1}\,\sqrt{cx-1}\right)}{(ex+d)^{\frac{3}{2}} x^2} \, dx - 2\,(3\,aex + ad)\sqrt{ex+d}\,\sqrt{d} - 3\left(ae^2x^2 + adex\right)\log\left(\frac{ex}{ex+2\,\sqrt{ex+d}\,\sqrt{d}+ \ldots}\right.}{2\left(d^2ex^2 + d^3x\right)\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(b*d^2*e*x^2 + b*d^3*x)*sqrt(d)*integrate(arctan2(1, sqrt(c*x + 1)*s
qrt(c*x - 1))/((e*x^3 + d*x^2)*sqrt(e*x + d)), x) - 2*(3*a*e*x + a*d)*sqrt(
e*x + d)*sqrt(d) - 3*(a*e^2*x^2 + a*d*e*x)*log(e*x/(e*x + 2*sqrt(e*x + d)*s
qrt(d) + 2*d)))/((d^2*e*x^2 + d^3*x)*sqrt(d))
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b \operatorname{asin}\left(\frac{1}{c\,x}\right)}{x^2\,(d + e\,x)^{3/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x)^(3/2)),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsc}(cx)}{x^2 (d + ex)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x**2/(e*x+d)**(3/2),x)
```

```
[Out] Integral((a + b*acsc(c*x))/(x**2*(d + e*x)**(3/2)), x)
```

3.69 $\int \frac{x^3\left(a+b\csc^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=602

$$\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^4}+\frac{64}{}$$

```
[Out] 2/3*d^3*(a+b*arccsc(c*x))/e^4/(e*x+d)^(3/2)+2/3*(e*x+d)^(3/2)*(a+b*arccsc(c
*x))/e^4-6*d^2*(a+b*arccsc(c*x))/e^4/(e*x+d)^(1/2)-4/3*b*d^2*(-c^2*x^2+1)/c
/e^2/(c^2*d^2-e^2)/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-6*d*(a+b*arccsc(c*x)
)*(e*x+d)^(1/2)/e^4+8/3*b*d^2*EllipticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*
(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/e^3/(c^2*d^2-e^2)/x/(1-
1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)-4/3*b*(2*c^2*d^2-e^2)*EllipticE(
1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x
^2+1)^(1/2)/c^2/e^3/(c^2*d^2-e^2)/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))
^(1/2)+32/3*b*d*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1
/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e^3/x/(1-1/c^2/x^2)^(
1/2)/(e*x+d)^(1/2)+64/3*b*d^2*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/
2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e^4/x/
(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] time = 2.90, antiderivative size = 602, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 18, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {43, 5247, 12, 6721, 6742, 745, 21, 719, 424, 958, 932, 168, 538, 537, 835, 844, 419, 1651}

$$\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^4}-\frac{}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x)^(5/2),x]
```

```
[Out] (-4*b*d^2*(1 - c^2*x^2))/(3*c*e^2*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*S
qrt[d + e*x]) + (2*d^3*(a + b*ArcCsc[c*x]))/(3*e^4*(d + e*x)^(3/2)) - (6*d^
2*(a + b*ArcCsc[c*x]))/(e^4*Sqrt[d + e*x]) - (6*d*Sqrt[d + e*x]*(a + b*ArcC
sc[c*x]))/e^4 + (2*(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^4) + (8*b*d^2*
Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2
*e)/(c*d + e)])/(3*e^3*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d +
 e*x))/(c*d + e)]) - (4*b*(2*c^2*d^2 - e^2)*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]
*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c^2*e^3*(c^2
*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) + (32*b*
d*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 -
 c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c^2*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d
 + e*x]) + (64*b*d^2*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*Ellipt
icPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c*e^4*Sqrt[1 -
1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
```

```
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 958

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 5247

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(b^IntPart[p]*(
a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[
```

```
p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^3\left(a+b\csc^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx = \frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}} - \frac{6d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4} + \frac{2}{}$$

$$= \frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}} - \frac{6d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4} + \frac{2}{}$$

$$= \frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}} - \frac{6d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4} + \frac{2}{}$$

$$= \frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}} - \frac{6d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4} + \frac{2}{}$$

$$= \frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}} - \frac{6d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4} + \frac{2}{}$$

$$= -\frac{68bd^2\left(1-c^2x^2\right)}{3ce^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\csc^{-1}\right.}{e^4\sqrt{d+ex}}$$

$$= -\frac{68bd^2\left(1-c^2x^2\right)}{3ce^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\csc^{-1}\right.}{e^4\sqrt{d+ex}}$$

$$= -\frac{4bd^2\left(1-c^2x^2\right)}{3ce^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\csc^{-1}\right.}{e^4\sqrt{d+ex}}$$

$$= -\frac{4bd^2\left(1-c^2x^2\right)}{3ce^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\csc^{-1}\right.}{e^4\sqrt{d+ex}}$$

$$= -\frac{4bd^2\left(1-c^2x^2\right)}{3ce^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\csc^{-1}\right.}{e^4\sqrt{d+ex}}$$

$$= -\frac{4bd^2\left(1-c^2x^2\right)}{3ce^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\csc^{-1}\right.}{e^4\sqrt{d+ex}}$$

Mathematica [C] time = 14.10, size = 887, normalized size = 1.47

$$\frac{a\left(\frac{ex}{d}+1\right)^{5/2} B_{-\frac{ex}{d}}\left(4,-\frac{3}{2}\right) d^4}{e^4(d+ex)^{5/2}}+\frac{b\left(\frac{2\left(\frac{d}{x}+e\right)^{5/2}(cx)^{5/2}\left(\frac{2\cos\left(2\csc^{-1}(cx)\right)\left(dx\sqrt{\frac{cd+cex}{cd+e}}\sqrt{1-c^2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)c^2-\frac{x(cx+1)\sqrt{\frac{e-cex}{cd+e}}\sqrt{\frac{cd+cex}{cd-e}}\left((cd+e)\cdots\right.}{\cdots}\right)}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x)^(5/2),x]
```

```
[Out] (a*d^4*(1 + (e*x)/d)^(5/2)*Beta[-((e*x)/d), 4, -3/2])/(e^4*(d + e*x)^(5/2))
 + (b*(-((c^3*(e + d/x)^3*x^3*((-4*Sqrt[1 - 1/(c^2*x^2)]])/(3*e*(-(c^2*d^2)
+ e^2)) + (32*c*d*ArcCsc[c*x]])/(3*e^4) - (2*c*d*ArcCsc[c*x]])/(3*e^2*(e + d/
x)^2) - (2*c*x*ArcCsc[c*x]])/(3*e^3) - (2*(-2*c^2*d^2*e*Sqrt[1 - 1/(c^2*x^2)
] - 7*c^3*d^3*ArcCsc[c*x] + 7*c*d*e^2*ArcCsc[c*x]))/(3*e^3*(-(c^2*d^2) + e^
2)*(e + d/x))))/(d + e*x)^(5/2)) + (2*(e + d/x)^(5/2)*(c*x)^(5/2)*((2*(8*c^
3*d^3*e - 8*c*d*e^3)*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*Ellipt
icF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]
*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*(16*c^4*d^4 - 16*c^2*d^2*e^2 - e^4)*Sqrt[(
c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x
]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3
/2)) + (2*e^3*Cos[2*ArcCsc[c*x]]*((c*d + c*e*x)*(-1 + c^2*x^2) + c^2*d*x*Sq
rt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x
]/Sqrt[2]], (2*e)/(c*d + e)] - (c*x*(1 + c*x)*Sqrt[(e - c*e*x)/(c*d + e)]*S
qrt[(c*d + c*e*x)/(c*d - e)]*((c*d + e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)
/(c*d - e)]], (c*d - e)/(c*d + e)] - e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/
(c*d - e)]], (c*d - e)/(c*d + e)]))/Sqrt[(e*(1 + c*x))/(-(c*d) + e)] + c*e*
x*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt
[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]))/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]
*Sqrt[c*x]*(-2 + c^2*x^2))))/(3*(c*d - e)*e^4*(c*d + e)*(d + e*x)^(5/2))))/
c^4
```

fricas [F] time = 1.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^3 \operatorname{arccsc}(cx) + ax^3\right)\sqrt{ex+d}}{e^3x^3 + 3\,de^2x^2 + 3\,d^2ex + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arccsc(c*x) + a*x^3)*sqrt(e*x + d)/(e^3*x^3 + 3*d*e^2*x^2 +
 3*d^2*e*x + d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x^3}{(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^3/(e*x + d)^(5/2), x)
```

maple [A] time = 0.09, size = 1077, normalized size = 1.79

$$2a\left(\frac{(ex+d)^{\frac{3}{2}}}{3}-3d\sqrt{ex+d}+\frac{d^{3}}{3(ex+d)^{\frac{3}{2}}}-\frac{3d^{2}}{\sqrt{ex+d}}\right)+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsc}(cx)}{3}-3\operatorname{arccsc}(cx)\,d\sqrt{ex+d}+\frac{\operatorname{arccsc}(cx)d^{3}}{3(ex+d)^{\frac{3}{2}}}-\frac{3\operatorname{ar}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x)
```

```
[Out] 2/e^4*(a*(1/3*(e*x+d)^(3/2)-3*d*(e*x+d)^(1/2)+1/3*d^3/(e*x+d)^(3/2)-3*d^2/(
e*x+d)^(1/2))+b*(1/3*(e*x+d)^(3/2)*arccsc(c*x)-3*arccsc(c*x)*d*(e*x+d)^(1/2
)+1/3*arccsc(c*x)*d^3/(e*x+d)^(3/2)-3*arccsc(c*x)*d^2/(e*x+d)^(1/2)+2/3/c^2
*((c/(c*d-e))^(1/2)*(e*x+d)^2*c^3*d^2-8*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*
(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2
),((c*d-e)/(c*d+e))^(1/2))*(e*x+d)^(1/2)*c^3*d^3+16*(-((e*x+d)*c-d*c+e)/(c*
d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/
(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*(e*x+d)^(
1/2)*c^3*d^3-2*(c/(c*d-e))^(1/2)*(e*x+d)*c^3*d^3+(c/(c*d-e))^(1/2)*c^3*d^4+
7*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*Ell
ipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*(e*x+d)^(1/
2)*c*d*e^2+(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^
(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*(e
*x+d)^(1/2)*c*d*e^2-16*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-
e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,
(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*(e*x+d)^(1/2)*c*d*e^2-(-((e*x+d)*c-d*c
+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/
2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*(e*x+d)^(1/2)*e^3+(-((e*x+d)*
c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticE((e*x+d
)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*(e*x+d)^(1/2)*e^3-(c/(c*
d-e))^(1/2)*c*d^2*e^2)/(c*d-e)/(c/(c*d-e))^(1/2)/(e*x+d)^(1/2)/(c*d+e)/x/((
c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d-e>0)', see `assume?` for more d
etails)Is c*d-e positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x)^(5/2),x)
```

```
[Out] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsc(c*x))/(e*x+d)**(5/2),x)
```

```
[Out] Timed out
```

3.70 $\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=440

$$-\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{4bd\left(1-c^2x^2\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}\left(c^2d^2-e^2\right)\sqrt{d+ex}}$$

```
[Out] -2/3*d^2*(a+b*arccsc(c*x))/e^3/(e*x+d)^(3/2)+4*d*(a+b*arccsc(c*x))/e^3/(e*x
+d)^(1/2)+4/3*b*d*(-c^2*x^2+1)/c/e/(c^2*d^2-e^2)/x/(1-1/c^2/x^2)^(1/2)/(e*x
+d)^(1/2)+2*(a+b*arccsc(c*x))*(e*x+d)^(1/2)/e^3-4/3*b*d*EllipticE(1/2*(-c*x
+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/
2)/e^2/(c^2*d^2-e^2)/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)-4*b*El
lipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c
*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e^2/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
-32/3*b*d*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2)
)*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e^3/x/(1-1/c^2/x^2)^(1/2)/
(e*x+d)^(1/2)
```

Rubi [A] time = 2.18, antiderivative size = 440, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 17, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.810, Rules used = {43, 5247, 12, 6721, 6742, 745, 21, 719, 424, 958, 932, 168, 538, 537, 835, 844, 419}

$$-\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{4bd\left(1-c^2x^2\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}\left(c^2d^2-e^2\right)\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x)^(5/2),x]
```

```
[Out] (4*b*d*(1 - c^2*x^2))/(3*c*e*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d
 + e*x]) - (2*d^2*(a + b*ArcCsc[c*x]))/(3*e^3*(d + e*x)^(3/2)) + (4*d*(a +
b*ArcCsc[c*x]))/(e^3*Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))
/e^3 - (4*b*d*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x
]/Sqrt[2]], (2*e)/(c*d + e)])/(3*e^2*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*
x*Sqrt[(c*(d + e*x))/(c*d + e)]) - (4*b*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[
1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c^
2*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (32*b*d*Sqrt[(c*(d + e*x))/(
c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2
*e)/(c*d + e)])/(3*c*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 958

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 5247

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(b^IntPart[p]*(
a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[
p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx = -\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} + \frac{4d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3} + \frac{b\int -}{3}$$

$$= -\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} + \frac{4d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3} + \frac{(2b)}{}$$

$$= -\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} + \frac{4d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3} + \frac{\left(2b\sqrt{}\right.}{}$$

$$= -\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} + \frac{4d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3} + \frac{\left(2b\sqrt{}\right.}{}$$

$$= -\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} + \frac{4d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3} + \frac{\left(16b\right.}{}$$

$$= \frac{12bd\left(1-c^2x^2\right)}{ce\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} + \frac{4d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}} +$$

$$= \frac{12bd\left(1-c^2x^2\right)}{ce\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} + \frac{4d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}} +$$

$$= \frac{4bd\left(1-c^2x^2\right)}{3ce\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} + \frac{4d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}} +$$

$$= \frac{4bd\left(1-c^2x^2\right)}{3ce\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} + \frac{4d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}} +$$

$$= \frac{4bd\left(1-c^2x^2\right)}{3ce\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} + \frac{4d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}} +$$

$$= \frac{4bd\left(1-c^2x^2\right)}{3ce\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} + \frac{4d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}} +$$

Mathematica [C] time = 13.95, size = 856, normalized size = 1.95

$$b\left(-\frac{c^3\left(\frac{d}{x}+e\right)^3\left(\frac{4c\sqrt{1-\frac{1}{c^2x^2}}\,d}{3e^2(e^2-c^2d^2)}+\frac{2\csc^{-1}(cx)}{3e\left(\frac{d}{x}+e\right)^2}-\frac{16\csc^{-1}(cx)}{3e^3}+\frac{4\left(-2c^2\csc^{-1}(cx)d^2-ce\sqrt{1-\frac{1}{c^2x^2}}\,d+2e^2\csc^{-1}(cx)\right)}{3e^2(e^2-c^2d^2)\left(\frac{d}{x}+e\right)}\right)x^3}{(d+ex)^{5/2}}-\frac{2\left(\frac{d}{x}+e\right)^{5/2}(cx)^{5/2}\left(\frac{2(3c^2d^2e-3e^3)\sqrt{\frac{cd+cex}{cd+e}}\cdots}{\sqrt{1-\frac{1}{c^2\cdots}}\cdots}\cdots\right.}{\cdots}\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x)^(5/2),x]
```

```
[Out] -((a*d^3*(1 + (e*x)/d)^(5/2)*Beta[-((e*x)/d), 3, -3/2])/(e^3*(d + e*x)^(5/2
))) + (b*(-((c^3*(e + d/x)^3*x^3*((4*c*d*Sqrt[1 - 1/(c^2*x^2)])/(3*e^2*(-(c
^2*d^2) + e^2)) - (16*ArcCsc[c*x])/(3*e^3) + (2*ArcCsc[c*x])/(3*e*(e + d/x)
^2) + (4*(-(c*d*e*Sqrt[1 - 1/(c^2*x^2)]) - 2*c^2*d^2*ArcCsc[c*x] + 2*e^2*Ar
cCsc[c*x]))/(3*e^2*(-(c^2*d^2) + e^2)*(e + d/x))))/(d + e*x)^(5/2)) - (2*(e
 + d/x)^(5/2)*(c*x)^(5/2)*((2*(3*c^2*d^2*e - 3*e^3)*Sqrt[(c*d + c*e*x)/(c*d
 + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*
d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*(8*c^3*d^3
- 9*c*d*e^2)*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2,
ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqr
t[e + d/x]*(c*x)^(3/2)) + (2*c*d*e*Cos[2*ArcCsc[c*x]]*((c*d + c*e*x)*(-1 +
c^2*x^2) + c^2*d*x*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*Elliptic
F[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)] - (c*x*(1 + c*x)*Sqrt[(e
- c*e*x)/(c*d + e)]*Sqrt[(c*d + c*e*x)/(c*d - e)]*((c*d + e)*EllipticE[ArcS
in[Sqrt[(c*d + c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)] - e*EllipticF[ArcSi
n[Sqrt[(c*d + c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)]))/Sqrt[(e*(1 + c*x))
/(-(c*d) + e)] + c*e*x*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*Elli
pticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]))/(Sqrt[1 - 1/(c^
2*x^2)]*Sqrt[e + d/x]*Sqrt[c*x]*(-2 + c^2*x^2))))/(3*(c*d - e)*e^3*(c*d + e
)*(d + e*x)^(5/2))))/c^3
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2\,\text{arccsc}\,(cx)+ax^2\right)\sqrt{ex+d}}{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccsc(c*x) + a*x^2)*sqrt(e*x + d)/(e^3*x^3 + 3*d*e^2*x^2 +
 3*d^2*e*x + d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arccsc}\,(cx)+a)x^2}{(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^2/(e*x + d)^(5/2), x)
```

maple [B] time = 0.08, size = 1040, normalized size = 2.36

$$2a\left(\sqrt{ex+d}-\frac{d^2}{3(ex+d)^{\frac{3}{2}}}+\frac{2d}{\sqrt{ex+d}}\right)+2b\left(\sqrt{ex+d}\,\operatorname{arccsc}(cx)-\frac{\operatorname{arccsc}(cx)d^2}{3(ex+d)^{\frac{3}{2}}}+\frac{2\operatorname{arccsc}(cx)d}{\sqrt{ex+d}}+\frac{8\sqrt{-\frac{(ex+d)c-dc+e}{dc-e}}\sqrt{-\frac{(ex+d)c-dc-e}{dc+e}}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x)
```

```
[Out] 2/e^3*(a*((e*x+d)^(1/2)-1/3*d^2/(e*x+d)^(3/2)+2*d/(e*x+d)^(1/2))+b*((e*x+d)
^(1/2)*arccsc(c*x)-1/3*arccsc(c*x)*d^2/(e*x+d)^(3/2)+2*arccsc(c*x)*d/(e*x+d
)^(1/2)+2/3/c*(4*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*
d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/
2))*(e*x+d)^(1/2)*c^2*d^2-8*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c
-d*c-e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-
e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*(e*x+d)^(1/2)*c^2*d^2-(-((e*x+d)*
c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticE((e*x+d
)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*(e*x+d)^(1/2)*c^2*d^2-(c
/(c*d-e))^(1/2)*(e*x+d)^2*c^2*d+(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+
d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-
e)/(c*d+e))^(1/2))*(e*x+d)^(1/2)*c*d*e-(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(
-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2)
,((c*d-e)/(c*d+e))^(1/2))*(e*x+d)^(1/2)*c*d*e+2*(c/(c*d-e))^(1/2)*(e*x+d)*c
^2*d^2-3*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1
/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*(e*x
+d)^(1/2)*e^2+8*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d
+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d
+e))^(1/2)/(c/(c*d-e))^(1/2))*(e*x+d)^(1/2)*e^2-(c/(c*d-e))^(1/2)*c^2*d^3+(
c/(c*d-e))^(1/2)*d*e^2)/(c*d-e)/(c/(c*d-e))^(1/2)/(e*x+d)^(1/2)/(c*d+e)/x/(
(c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d-e>0)', see `assume?` for more d
etails)Is c*d-e positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x)^(5/2),x)
```

```
[Out] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x)^(5/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsc(c*x))/(e*x+d)**(5/2),x)
```

```
[Out] Timed out
```

3.71 $\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=314

$$-\frac{2\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}}+\frac{2d\left(a+b\csc^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}-\frac{4b\left(1-c^2x^2\right)}{3cx\sqrt{1-\frac{1}{c^2x^2}}\left(c^2d^2-e^2\right)\sqrt{d+ex}}+\frac{4b\sqrt{1-c^2x^2}\sqrt{d+ex}\,E\left(\sin^{-1}\left(\right.\right.}{3ex\sqrt{1-\frac{1}{c^2x^2}}\left(c^2d^2-e^2\right)\sqrt{}}$$

```
[Out] 2/3*d*(a+b*arccsc(c*x))/e^2/(e*x+d)^(3/2)-2*(a+b*arccsc(c*x))/e^2/(e*x+d)^(
1/2)-4/3*b*(-c^2*x^2+1)/c/(c^2*d^2-e^2)/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
+4/3*b*EllipticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x
+d)^(1/2)*(-c^2*x^2+1)^(1/2)/e/(c^2*d^2-e^2)/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+
d)/(c*d+e))^(1/2)+8/3*b*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/
(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e^2/x/(1-1/c
^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] time = 2.00, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.737, Rules used = {43, 5247, 12, 6721, 6742, 745, 21, 719, 424, 958, 932, 168, 538, 537}

$$-\frac{2\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}}+\frac{2d\left(a+b\csc^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}-\frac{4b\left(1-c^2x^2\right)}{3cx\sqrt{1-\frac{1}{c^2x^2}}\left(c^2d^2-e^2\right)\sqrt{d+ex}}+\frac{4b\sqrt{1-c^2x^2}\sqrt{d+ex}\,E\left(\sin^{-1}\left(\right.\right.}{3ex\sqrt{1-\frac{1}{c^2x^2}}\left(c^2d^2-e^2\right)\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCsc[c*x]))/(d + e*x)^(5/2),x]
```

```
[Out] (-4*b*(1 - c^2*x^2))/(3*c*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d +
e*x]) + (2*d*(a + b*ArcCsc[c*x]))/(3*e^2*(d + e*x)^(3/2)) - (2*(a + b*ArcCs
c[c*x]))/(e^2*Sqrt[d + e*x]) + (4*b*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*Ellipti
cE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*e*(c^2*d^2 - e^2)*Sq
rt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) + (8*b*Sqrt[(c*(d + e*
x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]
], (2*e)/(c*d + e)])/(3*c*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 958

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
```

```
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 5247

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(b^IntPart[p]*(
a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[
p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx = \frac{2d\left(a+b\csc^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{b\int \frac{2(-2d-3ex)}{3e^2\sqrt{1-\frac{1}{c^2x^2}}\,x^2(d+ex)^{3/2}}\,dx}{c}$$

$$= \frac{2d\left(a+b\csc^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{(2b)\int \frac{-2d-3ex}{\sqrt{1-\frac{1}{c^2x^2}}\,x^2(d+ex)^{3/2}}\,dx}{3ce^2}$$

$$= \frac{2d\left(a+b\csc^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{\left(2b\sqrt{1-c^2x^2}\right)\int \frac{-2d-3ex}{x(d+ex)^{3/2}\sqrt{1-c^2x^2}}\,dx}{3ce^2\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= \frac{2d\left(a+b\csc^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{\left(2b\sqrt{1-c^2x^2}\right)\int \left(-\frac{3e}{(d+ex)^{3/2}\sqrt{1-c^2x^2}} - \right.}{3ce^2\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= \frac{2d\left(a+b\csc^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}} - \frac{\left(4bd\sqrt{1-c^2x^2}\right)\int \frac{1}{x(d+ex)^{3/2}\sqrt{1-c^2x^2}}\,dx}{3ce^2\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= -\frac{4b\left(1-c^2x^2\right)}{c\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{2d\left(a+b\csc^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}} -$$

$$= -\frac{4b\left(1-c^2x^2\right)}{c\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{2d\left(a+b\csc^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}} -$$

$$= -\frac{4b\left(1-c^2x^2\right)}{3c\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{2d\left(a+b\csc^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}} -$$

$$= -\frac{4b\left(1-c^2x^2\right)}{3c\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{2d\left(a+b\csc^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}} +$$

$$= -\frac{4b\left(1-c^2x^2\right)}{3c\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{2d\left(a+b\csc^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}} +$$

$$= -\frac{4b\left(1-c^2x^2\right)}{3c\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{2d\left(a+b\csc^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}} +$$

Mathematica [C] time = 1.75, size = 345, normalized size = 1.10

$$-\frac{2a(2d+3ex)}{3e^2(d+ex)^{3/2}} + \frac{4bcx\sqrt{1-\frac{1}{c^2x^2}}}{3\left(c^2d^2-e^2\right)\sqrt{d+ex}} + \frac{4ib\sqrt{-\frac{c}{cd+e}}\sqrt{\frac{e(cx+1)}{e-cd}}\sqrt{\frac{e-cex}{cd+e}}\left(-cdF\left(i\sinh^{-1}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right)|\frac{cd+e}{cd-e}\right)\right.}{}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(a + b*ArcCsc[c*x]))/(d + e*x)^(5/2),x]

[Out] (4*b*c*Sqrt[1 - 1/(c^2*x^2)]*x)/(3*(c^2*d^2 - e^2)*Sqrt[d + e*x]) - (2*a*(2*d + 3*e*x))/(3*e^2*(d + e*x)^(3/2)) - (2*b*(2*d + 3*e*x)*ArcCsc[c*x])/(3*e^2*(d + e*x)^(3/2)) + (((4*I)/3)*b*Sqrt[-(c/(c*d + e))]*Sqrt[(e*(1 + c*x))/(-(c*d) + e)]*Sqrt[(e - c*e*x)/(c*d + e)]*(c*d*EllipticE[I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)] - c*d*EllipticF[I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)] + 2*(c*d + e)*EllipticPi[1 + e/(c*d), I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)]))/(c^2*d*e^2*Sqrt[1 - 1/(c^2*x^2)]*x)

fricas [F] time = 1.03, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(bx\,\text{arccsc}\,(cx) + ax)\sqrt{ex + d}}{e^3x^3 + 3\,de^2x^2 + 3\,d^2ex + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")

[Out] integral((b*x*arccsc(c*x) + a*x)*sqrt(e*x + d)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arccsc}\,(cx) + a)x}{(ex + d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="giac")

[Out] integrate((b*arccsc(c*x) + a)*x/(e*x + d)^(5/2), x)

maple [B] time = 0.08, size = 913, normalized size = 2.91

$$2a\left(\frac{d}{3(ex+d)^{\frac{3}{2}}} - \frac{1}{\sqrt{ex+d}}\right) + 2b\left(\frac{\text{arccsc}(cx)d}{3(ex+d)^{\frac{3}{2}}} - \frac{\text{arccsc}(cx)}{\sqrt{ex+d}} + \frac{\frac{2\sqrt{\frac{c}{dc-e}}\,(ex+d)^2c^2d}{3} + \frac{4\sqrt{-\frac{(ex+d)c-dc+e}{dc-e}}\,\sqrt{-\frac{(ex+d)c-dc-e}{dc+e}}\,\text{EllipticPi}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{dc-e}}, \frac{dc-e}{cd}, \cdots\right.}{3}}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x)

[Out] 2/e^2*(a*(1/3*d/(e*x+d)^(3/2)-1/(e*x+d)^(1/2))+b*(1/3*arccsc(c*x)*d/(e*x+d)^(3/2)-1/(e*x+d)^(1/2)*arccsc(c*x)+2/3/c*((c/(c*d-e))^(1/2)*(e*x+d)^2*c^2*d+2*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*(e*x+d)^(1/2)*c^2*d^2-(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*(e*x+d)^(1/2)*c^2*d^2+(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*(e*x+d)^(1/2)*c^2*d^2-2*(c/(c*d-e))^(1/2)*(e*x+d)*c^2*d^2-(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*(e*x+d)^(1/2)*c*d*e+(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*(e*x+d)^(1/2)*c*d*e+(c/(c*d-e))^(1/2)*c^2*d^3-2*(-((e*x+d)*c-d*

```
c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(
1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*(
e*x+d)^(1/2)*e^2-(c/(c*d-e))^(1/2)*d*e^2)/(c*d-e)/(e*x+d)^(1/2)/(c*d+e)/(c/
(c*d-e))^(1/2)/d/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2
)^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d-e>0)', see `assume?` for more d
etails)Is c*d-e positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asin(1/(c*x))))/(d + e*x)^(5/2),x)
```

```
[Out] int((x*(a + b*asin(1/(c*x))))/(d + e*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsc(c*x))/(e*x+d)**(5/2),x)
```

```
[Out] Timed out
```

3.72 $\int \frac{a+b \csc^{-1}(cx)}{(d+ex)^{5/2}}\, dx$

Optimal. Leaf size=298

$$-\frac{2\left(a+b\csc^{-1}(cx)\right)}{3e(d+ex)^{3/2}}+\frac{4be\left(1-c^2x^2\right)}{3cdx\sqrt{1-\frac{1}{c^2x^2}}\left(c^2d^2-e^2\right)\sqrt{d+ex}}-\frac{4b\sqrt{1-c^2x^2}\,\sqrt{d+ex}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3dx\sqrt{1-\frac{1}{c^2x^2}}\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}}+\frac{4b\sqrt{1-}}{}$$

```
[Out] -2/3*(a+b*arccsc(c*x))/e/(e*x+d)^(3/2)+4/3*b*e*(-c^2*x^2+1)/c/d/(c^2*d^2-e^
2)/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-4/3*b*EllipticE(1/2*(-c*x+1)^(1/2)*2
^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*d
^2-e^2)/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)+4/3*b*EllipticPi(1/
2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(
1/2)*(-c^2*x^2+1)^(1/2)/c/d/e/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {5227, 1574, 958, 745, 21, 719, 424, 933, 168, 538, 537}

$$-\frac{2\left(a+b\csc^{-1}(cx)\right)}{3e(d+ex)^{3/2}}+\frac{4be\left(1-c^2x^2\right)}{3cdx\sqrt{1-\frac{1}{c^2x^2}}\left(c^2d^2-e^2\right)\sqrt{d+ex}}-\frac{4b\sqrt{1-c^2x^2}\,\sqrt{d+ex}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3dx\sqrt{1-\frac{1}{c^2x^2}}\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}}+\frac{4b\sqrt{1-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(d + e*x)^(5/2),x]
```

```
[Out] (4*b*e*(1 - c^2*x^2))/(3*c*d*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d
 + e*x]) - (2*(a + b*ArcCsc[c*x]))/(3*e*(d + e*x)^(3/2)) - (4*b*Sqrt[d + e*
x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d +
e)])/(3*d*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d +
 e)]) + (4*b*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2,
ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c*d*e*Sqrt[1 - 1/(c^2*x
^2)]*x*Sqrt[d + e*x])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] && !GtQ[a, 0]
```

Rule 958

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 1574

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[(x^(2*n*FracPart[p])*(a + c/x^(2*n))^FracPart[p])/
(c + a*x^(2*n))^FracPart[p], Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] && !In
tegerQ[p] && !IntegerQ[q] && PosQ[n]
```

Rule 5227

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCsc[c*x]))/(e*(m + 1)), x] + Dist[b/
```

```
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{5/2}}\,dx = -\frac{2\left(a+b\csc^{-1}(cx)\right)}{3e(d+ex)^{3/2}} - \frac{(2b)\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\,x^2(d+ex)^{3/2}}\,dx}{3ce}$$

$$= -\frac{2\left(a+b\csc^{-1}(cx)\right)}{3e(d+ex)^{3/2}} - \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{x(d+ex)^{3/2}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{3ce\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= -\frac{2\left(a+b\csc^{-1}(cx)\right)}{3e(d+ex)^{3/2}} - \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \left(-\frac{e}{d(d+ex)^{3/2}\sqrt{-\frac{1}{c^2}+x^2}}+\frac{1}{dx\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\right)dx}{3ce\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= -\frac{2\left(a+b\csc^{-1}(cx)\right)}{3e(d+ex)^{3/2}} + \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{(d+ex)^{3/2}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{3cd\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{}{x\sqrt{d+ex}}}{3cde\sqrt{1-\frac{1}{c^2x^2}}}$$

$$= \frac{4be\left(1-c^2x^2\right)}{3cd\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2\left(a+b\csc^{-1}(cx)\right)}{3e(d+ex)^{3/2}} - \frac{\left(4b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{-\frac{d}{2}-\frac{ex}{2}}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}}}}{3cd\left(d^2-\frac{e^2}{c^2}\right)\sqrt{1-\frac{1}{c^2x^2}}}$$

$$= \frac{4be\left(1-c^2x^2\right)}{3cd\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2\left(a+b\csc^{-1}(cx)\right)}{3e(d+ex)^{3/2}} + \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{-\frac{1}{c^2}+x^2}}}{3cd\left(d^2-\frac{e^2}{c^2}\right)\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= \frac{4be\left(1-c^2x^2\right)}{3cd\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2\left(a+b\csc^{-1}(cx)\right)}{3e(d+ex)^{3/2}} + \frac{\left(4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\right)\text{Sub}}{3cde}$$

$$= \frac{4be\left(1-c^2x^2\right)}{3cd\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2\left(a+b\csc^{-1}(cx)\right)}{3e(d+ex)^{3/2}} - \frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}\,E\left(\sin^{-}\right.}{3d\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}}$$

Mathematica [B] time = 13.64, size = 725, normalized size = 2.43

$$b\left(\frac{2(cx)^{5/2}\left(\frac{d}{x}+e\right)^{5/2}\left(\frac{2cd\sqrt{1-c^2x^2}\sqrt{\frac{cd+cex}{cd+e}}\,\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{1-\frac{1}{c^2x^2}}(cx)^{3/2}\sqrt{\frac{d}{x}+e}}-\frac{2e\cos\left(2\csc^{-1}(cx)\right)\left(\left(c^2x^2-1\right)(cd+cex)+c^2dx\sqrt{1-c^2x^2}\sqrt{\frac{cd+cex}{cd+e}}F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)+cex\ldots\right.}{\ldots}\right)}{3e(cd-e)(cd+e)(d+ex\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(d + e*x)^(5/2),x]
```

```
[Out] (-2*a)/(3*e*(d + e*x)^(3/2)) + (b*(-((c^3*(e + d/x)^3*x^3*((-4*Sqrt[1 - 1/(
c^2*x^2)])/(3*c*d*(c^2*d^2 - e^2)) + (2*ArcCsc[c*x])/(3*c^2*d^2*e) + (2*e*A
rcCsc[c*x])/(3*c^2*d^2*(e + d/x)^2) - (4*(-(c*d*e*Sqrt[1 - 1/(c^2*x^2)]) +
c^2*d^2*ArcCsc[c*x] - e^2*ArcCsc[c*x]))/(3*c^2*d^2*(c^2*d^2 - e^2)*(e + d/x
))))/(d + e*x)^(5/2)) + (2*(e + d/x)^(5/2)*(c*x)^(5/2)*((2*c*d*Sqrt[(c*d +
c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt
[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) -
 (2*e*Cos[2*ArcCsc[c*x]]*((c*d + c*e*x)*(-1 + c^2*x^2) + c^2*d*x*Sqrt[(c*d
+ c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2
]], (2*e)/(c*d + e)] - (c*x*(1 + c*x)*Sqrt[(e - c*e*x)/(c*d + e)]*Sqrt[(c*d
 + c*e*x)/(c*d - e)]*((c*d + e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d -
e)]], (c*d - e)/(c*d + e)] - e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - e
)]], (c*d - e)/(c*d + e)]))/Sqrt[(e*(1 + c*x))/(-(c*d) + e)] + c*e*x*Sqrt[(
c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x
]/Sqrt[2]], (2*e)/(c*d + e)]))/(c*d*Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*Sqr
t[c*x]*(-2 + c^2*x^2))))/(3*(c*d - e)*e*(c*d + e)*(d + e*x)^(5/2))))/c
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\operatorname{arccsc}(cx)+a}{(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/(e*x + d)^(5/2), x)
```

maple [B] time = 0.08, size = 886, normalized size = 2.97

$$-\frac{2a}{3(ex+d)^{\frac{3}{2}}}+2b\left(-\frac{\operatorname{arccsc}(cx)}{3(ex+d)^{\frac{3}{2}}}-\frac{2\left(\sqrt{\frac{c}{dc-e}}(ex+d)^2c^2d-\sqrt{-\frac{(ex+d)c-dc+e}{dc-e}}\sqrt{-\frac{(ex+d)c-dc-e}{dc+e}}\,\operatorname{EllipticPi}\left(\sqrt{ex+d}\sqrt{\frac{c}{dc-e}},\frac{dc-e}{cd},\frac{\sqrt{\frac{c}{dc+e}}}{\sqrt{\frac{c}{dc-e}}}\right)\sqrt{ex+d}\,c^2d^2\ldots\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/(e*x+d)^(5/2),x)
```

```
[Out] 2/e*(-1/3*a/(e*x+d)^(3/2)+b*(-1/3/(e*x+d)^(3/2)*arccsc(c*x)-2/3/c*((c/(c*d-
e))^(1/2)*(e*x+d)^2*c^2*d-(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d
*c-e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)
/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*(e*x+d)^(1/2)*c^2*d^2-(-((e*x+d)*c-
d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^
(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*(e*x+d)^(1/2)*c^2*d^2+(-((
e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticE
((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*(e*x+d)^(1/2)*c^2
*d^2-2*(c/(c*d-e))^(1/2)*(e*x+d)*c^2*d^2-(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)
*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/
2),((c*d-e)/(c*d+e))^(1/2))*(e*x+d)^(1/2)*c*d*e+(-((e*x+d)*c-d*c+e)/(c*d-e)
)^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-
e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*(e*x+d)^(1/2)*c*d*e+(c/(c*d-e))^(1/2)*c^
2*d^3+(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)
*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)
/(c/(c*d-e))^(1/2))*(e*x+d)^(1/2)*e^2-(c/(c*d-e))^(1/2)*d*e^2)/(c*d-e)/(e*x
+d)^(1/2)/(c*d+e)/(c/(c*d-e))^(1/2)/d^2/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c
^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d-e>0)', see `assume?` for more d
etails)Is c*d-e positive or negative?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{{\left(d+e\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x)^(5/2),x)
[Out] int((a + b*asin(1/(c*x)))/(d + e*x)^(5/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\operatorname{acsc}{\left(cx \right)}}{\left(d+ex\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/(e*x+d)**(5/2),x)
[Out] Integral((a + b*acsc(c*x))/(d + e*x)**(5/2), x)
```

3.73 $\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=24

$$\operatorname{Int}\left(\frac{a+b\csc^{-1}(cx)}{x(d+ex)^{5/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x/(e*x+d)^(5/2),x)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCsc[c*x])/(x*(d + e*x)^(5/2)),x]
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x*(d + e*x)^(5/2)), x]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{5/2}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{5/2}}\,dx$$

Mathematica [A] time = 29.96, size = 0, normalized size = 0.00

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x)^(5/2)),x]
[Out] Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x)^(5/2)), x]
```

fricas [A] time = 1.46, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{ex+d}\,(b\operatorname{arccsc}(cx)+a)}{e^3x^4+3\,de^2x^3+3\,d^2ex^2+d^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x+d)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x + d)*(b*arccsc(c*x) + a)/(e^3*x^4 + 3*d*e^2*x^3 + 3*d^2*e
*x^2 + d^3*x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\operatorname{arccsc}(cx)+a}{(ex+d)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)/((e*x + d)^(5/2)*x), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\text{arccsc}\,(cx)}{x\,(ex + d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x/(e*x+d)^(5/2),x)
[Out] int((a+b*arccsc(c*x))/x/(e*x+d)^(5/2),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x+d)^(5/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b\,\text{asin}\left(\frac{1}{c\,x}\right)}{x\,(d + e\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x*(d + e*x)^(5/2)),x)
[Out] int((a + b*asin(1/(c*x)))/(x*(d + e*x)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x/(e*x+d)**(5/2),x)
[Out] Timed out
```

3.74 $\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{5/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x^2/(e*x+d)^(5/2),x)
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCsc[c*x])/(x^2*(d + e*x)^(5/2)),x]
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x^2*(d + e*x)^(5/2)), x]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx$$

Mathematica [A] time = 26.68, size = 0, normalized size = 0.00

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x^2*(d + e*x)^(5/2)),x]
[Out] Integrate[(a + b*ArcCsc[c*x])/(x^2*(d + e*x)^(5/2)), x]
```

fricas [A] time = 2.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex+d}\,(b\,\text{arccsc}\,(cx)+a)}{e^3x^5+3\,de^2x^4+3\,d^2ex^3+d^3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x+d)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x + d)*(b*arccsc(c*x) + a)/(e^3*x^5 + 3*d*e^2*x^4 + 3*d^2*e
*x^3 + d^3*x^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arccsc}\,(cx)+a}{(ex+d)^{\frac{5}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)/((e*x + d)^(5/2)*x^2), x)
```

maple [**F(-2)**] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{arccsc}\,(cx)}{x^2\,(ex + d)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x^2/(e*x+d)^(5/2),x)
[Out] int((a+b*arccsc(c*x))/x^2/(e*x+d)^(5/2),x)
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x+d)^(5/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^2\,{\left(d + e\,x\right)}^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x)^(5/2)),x)
[Out] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x)^(5/2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x**2/(e*x+d)**(5/2),x)
[Out] Timed out
```

3.75 $\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{7/2}}\,dx$

Optimal. Leaf size=540

$$-\frac{2\left(a+b\csc^{-1}(cx)\right)}{5e(d+ex)^{5/2}}+\frac{4be\left(1-c^2x^2\right)}{5cd^2x\sqrt{1-\frac{1}{c^2x^2}}\left(c^2d^2-e^2\right)\sqrt{d+ex}}+\frac{16bce\left(1-c^2x^2\right)}{15x\sqrt{1-\frac{1}{c^2x^2}}\left(c^2d^2-e^2\right)^2\sqrt{d+ex}}+\frac{}{15cdx\sqrt{1-}}$$

```
[Out] -2/5*(a+b*arccsc(c*x))/e/(e*x+d)^(5/2)+4/15*b*e*(-c^2*x^2+1)/c/d/(c^2*d^2-e
^2)/x/(e*x+d)^(3/2)/(1-1/c^2/x^2)^(1/2)+16/15*b*c*e*(-c^2*x^2+1)/(c^2*d^2-e
^2)^2/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4/5*b*e*(-c^2*x^2+1)/c/d^2/(c^2*d
^2-e^2)/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-4/15*b*(7*c^2*d^2-3*e^2)*Ellipt
icE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c
^2*x^2+1)^(1/2)/(c^2*d^3-d*e^2)^2/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))
^(1/2)+4/15*b*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2
))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*d^2-e^2)/x/(1-1/c^2/
x^2)^(1/2)/(e*x+d)^(1/2)+4/5*b*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1
/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/d^2/e
/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] time = 0.75, antiderivative size = 637, normalized size of antiderivative = 1.18, number of steps used = 19, number of rules used = 14, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {5227, 1574, 958, 745, 835, 844, 719, 424, 419, 21, 933, 168, 538, 537}

$$-\frac{2\left(a+b\csc^{-1}(cx)\right)}{5e(d+ex)^{5/2}}+\frac{16bce\left(1-c^2x^2\right)}{15x\sqrt{1-\frac{1}{c^2x^2}}\left(c^2d^2-e^2\right)^2\sqrt{d+ex}}+\frac{4be\left(1-c^2x^2\right)}{5cd^2x\sqrt{1-\frac{1}{c^2x^2}}\left(c^2d^2-e^2\right)\sqrt{d+ex}}+\frac{}{15cdx\sqrt{1-}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(d + e*x)^(7/2),x]
```

```
[Out] (4*b*e*(1 - c^2*x^2))/(15*c*d*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*(d +
e*x)^(3/2)) + (16*b*c*e*(1 - c^2*x^2))/(15*(c^2*d^2 - e^2)^2*Sqrt[1 - 1/(c^
2*x^2)]*x*Sqrt[d + e*x]) + (4*b*e*(1 - c^2*x^2))/(5*c*d^2*(c^2*d^2 - e^2)*S
qrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (2*(a + b*ArcCsc[c*x]))/(5*e*(d + e
*x)^(5/2)) - (16*b*c^2*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqr
t[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*(c^2*d^2 - e^2)^2*Sqrt[1 - 1/(c^
2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) - (4*b*Sqrt[d + e*x]*Sqrt[1 - c^2*
x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(5*d^2*(c^2
*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) + (4*b*S
qrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*
x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*d*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*
x*Sqrt[d + e*x]) + (4*b*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*Ell
ipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(5*c*d^2*e*Sqrt
[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
```

```
a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
```

```
p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 958

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 1574

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[(x^(2*n*FracPart[p])*(a + c/x^(2*n))^FracPart[p])/
(c + a*x^(2*n))^FracPart[p], Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 5227

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCsc[c*x]))/(e*(m + 1)), x] + Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{7/2}}\,dx = -\frac{2\left(a+b\csc^{-1}(cx)\right)}{5e(d+ex)^{5/2}} - \frac{(2b)\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\,x^2(d+ex)^{5/2}}\,dx}{5ce}$$

$$= -\frac{2\left(a+b\csc^{-1}(cx)\right)}{5e(d+ex)^{5/2}} - \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{x(d+ex)^{5/2}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{5ce\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= -\frac{2\left(a+b\csc^{-1}(cx)\right)}{5e(d+ex)^{5/2}} - \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \left(-\frac{e}{d(d+ex)^{5/2}\sqrt{-\frac{1}{c^2}+x^2}} - \frac{e}{d^2(d+ex)^{3/2}\sqrt{-\frac{1}{c^2}+x^2}} + \frac{}{d^2x\sqrt{d}\cdots}\right.}{5ce\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= -\frac{2\left(a+b\csc^{-1}(cx)\right)}{5e(d+ex)^{5/2}} + \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{(d+ex)^{3/2}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{5cd^2\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{}{(d+ex)^{5/2}\cdots}}{5cd\sqrt{1-\frac{1}{c^2x^2}}\cdots}$$

$$= \frac{4be\left(1-c^2x^2\right)}{15cd\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x(d+ex)^{3/2}} + \frac{4be\left(1-c^2x^2\right)}{5cd^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2\left(a+b\cdots\right.}{5e(d\cdots}$$

$$= \frac{4be\left(1-c^2x^2\right)}{15cd\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x(d+ex)^{3/2}} + \frac{16bce\left(1-c^2x^2\right)}{15\left(c^2d^2-e^2\right)^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{}{5cd^2\left(c^2d\cdots\right.}$$

$$= \frac{4be\left(1-c^2x^2\right)}{15cd\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x(d+ex)^{3/2}} + \frac{16bce\left(1-c^2x^2\right)}{15\left(c^2d^2-e^2\right)^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{}{5cd^2\left(c^2d\cdots\right.}$$

$$= \frac{4be\left(1-c^2x^2\right)}{15cd\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x(d+ex)^{3/2}} + \frac{16bce\left(1-c^2x^2\right)}{15\left(c^2d^2-e^2\right)^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{}{5cd^2\left(c^2d\cdots\right.}$$

$$= \frac{4be\left(1-c^2x^2\right)}{15cd\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x(d+ex)^{3/2}} + \frac{16bce\left(1-c^2x^2\right)}{15\left(c^2d^2-e^2\right)^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{}{5cd^2\left(c^2d\cdots\right.}$$

Mathematica [A] time = 13.99, size = 1002, normalized size = 1.86

$$b\left(\frac{2\left(\frac{d}{x}+e\right)^{7/2}(cx)^{7/2}\left(\frac{2\left(c^2d^2e-e^3\right)\sqrt{\frac{cd+cex}{cd+e}}\sqrt{1-c^2x^2}F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{1-\frac{1}{c^2x^2}}\sqrt{\frac{d}{x}+e}(cx)^{3/2}}+\frac{2\left(3c^3d^3+ce^2d\right)\sqrt{\frac{cd+cex}{cd+e}}\sqrt{1-c^2x^2}\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{1-\frac{1}{c^2x^2}}\sqrt{\frac{d}{x}+e}(cx)^{3/2}}+\frac{2\left(3e^3-7c^2d^2e\right)\cos(2\cdots}{\cdots}\right.}{\cdots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(d + e*x)^(7/2),x]
```

```
[Out] (-2*a)/(5*e*(d + e*x)^(5/2)) + (b*(-((c^4*(e + d/x)^4*x^4*((4*(-7*c^2*d^2 +
 3*e^2)*Sqrt[1 - 1/(c^2*x^2)])/(15*c^2*d^2*(-(c^2*d^2) + e^2)^2) + (2*ArcCs
c[c*x])/(5*c^3*d^3*e) - (2*e^2*ArcCsc[c*x])/(5*c^3*d^3*(e + d/x)^3) - (2*(2
*c*d*e^2*Sqrt[1 - 1/(c^2*x^2)] - 9*c^2*d^2*e*ArcCsc[c*x] + 9*e^3*ArcCsc[c*x
]))/(15*c^3*d^3*(c^2*d^2 - e^2)*(e + d/x)^2) - (2*(-16*c^3*d^3*e*Sqrt[1 - 1
/(c^2*x^2)] + 8*c*d*e^3*Sqrt[1 - 1/(c^2*x^2)] + 9*c^4*d^4*ArcCsc[c*x] - 18*
c^2*d^2*e^2*ArcCsc[c*x] + 9*e^4*ArcCsc[c*x]))/(15*c^3*d^3*(c^2*d^2 - e^2)^2
*(e + d/x))))/(d + e*x)^(7/2)) + (2*(e + d/x)^(7/2)*(c*x)^(7/2)*((2*(c^2*d^
2*e - e^3)*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin
[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e +
d/x]*(c*x)^(3/2)) + (2*(3*c^3*d^3 + c*d*e^2)*Sqrt[(c*d + c*e*x)/(c*d + e)]*
Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d +
 e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*(-7*c^2*d^2*e
+ 3*e^3)*Cos[2*ArcCsc[c*x]]*((c*d + c*e*x)*(-1 + c^2*x^2) + c^2*d*x*Sqrt[(c
*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqr
t[2]], (2*e)/(c*d + e)] - (c*x*(1 + c*x)*Sqrt[(e - c*e*x)/(c*d + e)]*Sqrt[(
c*d + c*e*x)/(c*d - e)]*((c*d + e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d
 - e)]], (c*d - e)/(c*d + e)] - e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d
- e)]], (c*d - e)/(c*d + e)]))/Sqrt[(e*(1 + c*x))/(-(c*d) + e)] + c*e*x*Sqr
t[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 -
c*x]/Sqrt[2]], (2*e)/(c*d + e)]))/(c*d*Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*
Sqrt[c*x]*(-2 + c^2*x^2))))/(15*c*d*(c*d - e)^2*e*(c*d + e)^2*(d + e*x)^(7/
2))))/c
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arccsc}(cx) + a}{(ex+d)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/(e*x + d)^(7/2), x)
```

maple [B] time = 0.08, size = 1640, normalized size = 3.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/(e*x+d)^(7/2),x)
```

```
[Out] 2/e*(-1/5*a/(e*x+d)^(5/2)+b*(-1/5/(e*x+d)^(5/2)*arccsc(c*x)-2/15/c*(7*(c/(c
*d-e))^(1/2)*(e*x+d)^3*c^4*d^3-6*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x
+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d
-e)/(c*d+e))^(1/2))*(e*x+d)^(3/2)*c^4*d^4+7*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1
/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^
(1/2),((c*d-e)/(c*d+e))^(1/2))*(e*x+d)^(3/2)*c^4*d^4-3*(-((e*x+d)*c-d*c+e)/
(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*
(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*(e*x+d
)^(3/2)*c^4*d^4-13*(c/(c*d-e))^(1/2)*(e*x+d)^2*c^4*d^4-7*(-((e*x+d)*c-d*c+e
)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)
*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*(e*x+d)^(3/2)*c^3*d^3*e+7*(-((e
*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticE(
(e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*(e*x+d)^(3/2)*c^3*
d^3*e-3*(c/(c*d-e))^(1/2)*(e*x+d)^3*c^2*d*e^2+5*(c/(c*d-e))^(1/2)*(e*x+d)*c
^4*d^5+2*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1
/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*(e*x
+d)^(3/2)*c^2*d^2*e^2-3*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c
-e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+
e))^(1/2))*(e*x+d)^(3/2)*c^2*d^2*e^2+6*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(
-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2
),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*(e*x+d)^(3/2)*c^2*d^2*
e^2+5*(c/(c*d-e))^(1/2)*(e*x+d)^2*c^2*d^2*e^2+(c/(c*d-e))^(1/2)*c^4*d^6+3*(
-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*Ellipt
icF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*(e*x+d)^(3/2)*
c*d*e^3-3*(-((e*x+d)*c-d*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(
1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*(e*
x+d)^(3/2)*c*d*e^3-8*(c/(c*d-e))^(1/2)*(e*x+d)*c^2*d^3*e^2-3*(-((e*x+d)*c-d
*c+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-d*c-e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^
(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*
(e*x+d)^(3/2)*e^4-2*(c/(c*d-e))^(1/2)*c^2*d^4*e^2+3*(c/(c*d-e))^(1/2)*(e*x+
d)*d*e^4+(c/(c*d-e))^(1/2)*d^2*e^4)/(c*d-e)/(c/(c*d-e))^(1/2)/(e*x+d)^(3/2)
/(c*d+e)/(c^2*d^2-e^2)/d^3/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c
^2/e^2/x^2)^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d-e>0)', see `assume?` for more d
etails)Is c*d-e positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{{\left(d+e\,x\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x)^(7/2),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(d + e*x)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/(e*x+d)**(7/2),x)
```

```
[Out] Timed out
```

3.76 $\int x^4 \left(d+ex^2\right) \left(a+b\csc^{-1}(cx)\right) \, dx$

Optimal. Leaf size=206

$$\frac{1}{5}dx^5\left(a+b\csc^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\csc^{-1}(cx)\right)+\frac{bex^6\sqrt{c^2x^2-1}}{42c\sqrt{c^2x^2}}+\frac{bx\left(42c^2d+25e\right)\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{560c^6\sqrt{c^2x^2}}+\frac{bx^2\sqrt{c^2x^2-}}{56}$$

```
[Out] 1/5*d*x^5*(a+b*arccsc(c*x))+1/7*e*x^7*(a+b*arccsc(c*x))+1/560*b*(42*c^2*d+2
5*e)*x*arctanh(c*x/(c^2*x^2-1)^(1/2))/c^6/(c^2*x^2)^(1/2)+1/560*b*(42*c^2*d
+25*e)*x^2*(c^2*x^2-1)^(1/2)/c^5/(c^2*x^2)^(1/2)+1/840*b*(42*c^2*d+25*e)*x^
4*(c^2*x^2-1)^(1/2)/c^3/(c^2*x^2)^(1/2)+1/42*b*e*x^6*(c^2*x^2-1)^(1/2)/c/(c
^2*x^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {14, 5239, 12, 459, 321, 217, 206}

$$\frac{1}{5}dx^5\left(a+b\csc^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\csc^{-1}(cx)\right)+\frac{bx^4\sqrt{c^2x^2-1}\left(42c^2d+25e\right)}{840c^3\sqrt{c^2x^2}}+\frac{bx^2\sqrt{c^2x^2-1}\left(42c^2d+25e\right)}{560c^5\sqrt{c^2x^2}}+\frac{bx\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*(42*c^2*d + 25*e)*x^2*Sqrt[-1 + c^2*x^2])/(560*c^5*Sqrt[c^2*x^2]) + (b*(
42*c^2*d + 25*e)*x^4*Sqrt[-1 + c^2*x^2])/(840*c^3*Sqrt[c^2*x^2]) + (b*e*x^6
*Sqrt[-1 + c^2*x^2])/(42*c*Sqrt[c^2*x^2]) + (d*x^5*(a + b*ArcCsc[c*x]))/5 +
 (e*x^7*(a + b*ArcCsc[c*x]))/7 + (b*(42*c^2*d + 25*e)*x*ArcTanh[(c*x)/Sqrt[
-1 + c^2*x^2]])/(560*c^6*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int x^4\left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)dx=\frac{1}{5}dx^5\left(a+b\csc^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{x^4\left(7d+5ex^2\right)}{35\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}$$

$$=\frac{1}{5}dx^5\left(a+b\csc^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{x^4\left(7d+5ex^2\right)}{\sqrt{-1+c^2x^2}}\,dx}{35\sqrt{c^2x^2}}$$

$$=\frac{bex^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}}+\frac{1}{5}dx^5\left(a+b\csc^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\csc^{-1}(cx)\right)-$$

$$=\frac{b\left(42c^2d+25e\right)x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}}+\frac{bex^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}}+\frac{1}{5}dx^5\left(a+b\csc^{-1}(\right.$$

$$=\frac{b\left(42c^2d+25e\right)x^2\sqrt{-1+c^2x^2}}{560c^5\sqrt{c^2x^2}}+\frac{b\left(42c^2d+25e\right)x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}}+\frac{bex^6}{4}$$

$$=\frac{b\left(42c^2d+25e\right)x^2\sqrt{-1+c^2x^2}}{560c^5\sqrt{c^2x^2}}+\frac{b\left(42c^2d+25e\right)x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}}+\frac{bex^6}{4}$$

$$=\frac{b\left(42c^2d+25e\right)x^2\sqrt{-1+c^2x^2}}{560c^5\sqrt{c^2x^2}}+\frac{b\left(42c^2d+25e\right)x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}}+\frac{bex^6}{4}$$

Mathematica [A] time = 0.23, size = 140, normalized size = 0.68

$$\frac{48ac^7x^5\left(7d+5ex^2\right)+48bc^7x^5\csc^{-1}(cx)\left(7d+5ex^2\right)+3b\left(42c^2d+25e\right)\log\left(x\left(\sqrt{1-\frac{1}{c^2x^2}}+1\right)\right)+bc^2x^2\sqrt{1-}}{1680c^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (48*a*c^7*x^5*(7*d + 5*e*x^2) + b*c^2*Sqrt[1 - 1/(c^2*x^2)]*x^2*(75*e + 2*c
^2*(63*d + 25*e*x^2) + c^4*(84*d*x^2 + 40*e*x^4)) + 48*b*c^7*x^5*(7*d + 5*e
```

```
*x^2)*ArcCsc[c*x] + 3*b*(42*c^2*d + 25*e)*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x
])/(1680*c^7)
```

fricas [A] time = 1.77, size = 191, normalized size = 0.93

$$\frac{240\,ac^7ex^7+336\,ac^7dx^5+48\left(5\,bc^7ex^7+7\,bc^7dx^5-7\,bc^7d-5\,bc^7e\right)\operatorname{arccsc}(cx)-96\left(7\,bc^7d+5\,bc^7e\right)\arctan\left(-c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/1680*(240*a*c^7*e*x^7 + 336*a*c^7*d*x^5 + 48*(5*b*c^7*e*x^7 + 7*b*c^7*d*x
^5 - 7*b*c^7*d - 5*b*c^7*e)*arccsc(c*x) - 96*(7*b*c^7*d + 5*b*c^7*e)*arctan
(-c*x + sqrt(c^2*x^2 - 1)) - 3*(42*b*c^2*d + 25*b*e)*log(-c*x + sqrt(c^2*x^
2 - 1)) + (40*b*c^5*e*x^5 + 2*(42*b*c^5*d + 25*b*c^3*e)*x^3 + 3*(42*b*c^3*d
 + 25*b*c*e)*x)*sqrt(c^2*x^2 - 1))/c^7
```

giac [B] time = 2.12, size = 1190, normalized size = 5.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/13440*(15*b*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7*arcsin(1/(c*x))*e/c + 15*a
*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7*e/c + 84*b*d*x^5*(sqrt(-1/(c^2*x^2) + 1
) + 1)^5*arcsin(1/(c*x))/c + 5*b*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*e/c^2 +
 84*a*d*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5/c + 105*b*x^5*(sqrt(-1/(c^2*x^2)
 + 1) + 1)^5*arcsin(1/(c*x))*e/c^3 + 105*a*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)
^5*e/c^3 + 42*b*d*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c^2 + 420*b*d*x^3*(sqr
t(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x))/c^3 + 45*b*x^4*(sqrt(-1/(c^2*x^2
) + 1) + 1)^4*e/c^4 + 420*a*d*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^3 + 315*
b*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x))*e/c^5 + 315*a*x^3*(sqr
t(-1/(c^2*x^2) + 1) + 1)^3*e/c^5 + 336*b*d*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)
^2/c^4 + 840*b*d*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^5 + 225*b
*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*e/c^6 + 840*a*d*x*(sqrt(-1/(c^2*x^2) +
1) + 1)/c^5 + 525*b*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))*e/c^7 +
525*a*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*e/c^7 + 1008*b*d*log(sqrt(-1/(c^2*x^2)
 + 1) + 1)/c^6 - 1008*b*d*log(1/(abs(c)*abs(x)))/c^6 + 600*b*e*log(sqrt(-1/
(c^2*x^2) + 1) + 1)/c^8 - 600*b*e*log(1/(abs(c)*abs(x)))/c^8 + 840*b*d*arcs
in(1/(c*x))/(c^7*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 840*a*d/(c^7*x*(sqrt(-1/
(c^2*x^2) + 1) + 1)) + 525*b*arcsin(1/(c*x))*e/(c^9*x*(sqrt(-1/(c^2*x^2) +
1) + 1)) + 525*a*e/(c^9*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) - 336*b*d/(c^8*x^2*
(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 420*b*d*arcsin(1/(c*x))/(c^9*x^3*(sqrt(-1
/(c^2*x^2) + 1) + 1)^3) - 225*b*e/(c^10*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2)
 + 420*a*d/(c^9*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 315*b*arcsin(1/(c*x))
*e/(c^11*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 315*a*e/(c^11*x^3*(sqrt(-1/(
c^2*x^2) + 1) + 1)^3) - 42*b*d/(c^10*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) +
84*b*d*arcsin(1/(c*x))/(c^11*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) - 45*b*e/(
c^12*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) + 84*a*d/(c^11*x^5*(sqrt(-1/(c^2*x
^2) + 1) + 1)^5) + 105*b*arcsin(1/(c*x))*e/(c^13*x^5*(sqrt(-1/(c^2*x^2) + 1
) + 1)^5) + 105*a*e/(c^13*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) - 5*b*e/(c^14
*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6) + 15*b*arcsin(1/(c*x))*e/(c^15*x^7*(sq
rt(-1/(c^2*x^2) + 1) + 1)^7) + 15*a*e/(c^15*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1
)^7))*c
```

maple [A] time = 0.06, size = 338, normalized size = 1.64

$$\frac{ae\,x^7}{7}+\frac{a\,x^5d}{5}+\frac{b\operatorname{arccsc}(cx)\,e\,x^7}{7}+\frac{b\operatorname{arccsc}(cx)\,x^5d}{5}+\frac{b\,x^6e}{42c\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{b\,x^4e}{168c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{b\,x^4d}{20c\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{b\,x^2d}{40c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^2+d)*(a+b*arccsc(c*x)),x)
```

```
[Out] 1/7*a*e*x^7+1/5*a*x^5*d+1/7*b*arccsc(c*x)*e*x^7+1/5*b*arccsc(c*x)*x^5*d+1/4
2/c*b/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^6*e+1/168/c^3*b/((c^2*x^2-1)/c^2/x^2)^(
1/2)*x^4*e+1/20/c*b/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^4*d+1/40/c^3*b/((c^2*x^2-
1)/c^2/x^2)^(1/2)*x^2*d+5/336/c^5*b/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^2*e-3/40/
c^5*b/((c^2*x^2-1)/c^2/x^2)^(1/2)*d+3/40/c^6*b/((c^2*x^2-1)/c^2/x^2)^(1/2)/
x*(c^2*x^2-1)^(1/2)*d*ln(c*x+(c^2*x^2-1)^(1/2))-5/112/c^7*b/((c^2*x^2-1)/c^
2/x^2)^(1/2)*e+5/112/c^8*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*
e*ln(c*x+(c^2*x^2-1)^(1/2))
```

maxima [A] time = 0.34, size = 296, normalized size = 1.44

$$\frac{1}{7}aex^7+\frac{1}{5}adx^5+\frac{1}{80}\left(16x^5\operatorname{arccsc}(cx)-\frac{\frac{2\left(3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2c^4\left(\frac{1}{c^2x^2}-1\right)+c^4}-\frac{3\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^4}+\frac{3\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)bd\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*e*x^7 + 1/5*a*d*x^5 + 1/80*(16*x^5*arccsc(c*x) - (2*(3*(-1/(c^2*x^2)
+ 1)^(3/2) - 5*sqrt(-1/(c^2*x^2) + 1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(1/
(c^2*x^2) - 1) + c^4) - 3*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 + 3*log(sqrt(
-1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*d + 1/672*(96*x^7*arccsc(c*x) + (2*(15*(-1
/(c^2*x^2) + 1)^(5/2) - 40*(-1/(c^2*x^2) + 1)^(3/2) + 33*sqrt(-1/(c^2*x^2)
+ 1))/(c^6*(1/(c^2*x^2) - 1)^3 + 3*c^6*(1/(c^2*x^2) - 1)^2 + 3*c^6*(1/(c^2*
x^2) - 1) + c^6) + 15*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^6 - 15*log(sqrt(-1/
(c^2*x^2) + 1) - 1)/c^6)/c)*b*e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\left(e\,x^2+d\right)\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d + e*x^2)*(a + b*asin(1/(c*x))),x)
```

```
[Out] int(x^4*(d + e*x^2)*(a + b*asin(1/(c*x))), x)
```

sympy [A] time = 12.25, size = 408, normalized size = 1.98

$$\frac{adx^5}{5}+\frac{aex^7}{7}+\frac{bdx^5\operatorname{acsc}(cx)}{5}+\frac{bex^7\operatorname{acsc}(cx)}{7}+\frac{bd\left(\left\{\begin{array}{ll}\frac{cx^5}{4\sqrt{c^2x^2-1}}+\frac{x^3}{8c\sqrt{c^2x^2-1}}-\frac{3x}{8c^3\sqrt{c^2x^2-1}}+\frac{3\operatorname{acosh}(cx)}{8c^4} & \text{for } |c^2\cdots \\ -\frac{icx^5}{4\sqrt{-c^2x^2+1}}-\frac{ix^3}{8c\sqrt{-c^2x^2+1}}+\frac{3ix}{8c^3\sqrt{-c^2x^2+1}}-\frac{3i\operatorname{asin}(cx)}{8c^4} & \text{otherw}\cdots\end{array}\right.\right.}{5c}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**2+d)*(a+b*acsc(c*x)),x)
```

```
[Out] a*d*x**5/5 + a*e*x**7/7 + b*d*x**5*acsc(c*x)/5 + b*e*x**7*acsc(c*x)/7 + b*d
*Piecewise((c*x**5/(4*sqrt(c**2*x**2 - 1)) + x**3/(8*c*sqrt(c**2*x**2 - 1))
```

```
 - 3*x/(8*c**3*sqrt(c**2*x**2 - 1)) + 3*acosh(c*x)/(8*c**4), Abs(c**2*x**2)
 > 1), (-I*c*x**5/(4*sqrt(-c**2*x**2 + 1)) - I*x**3/(8*c*sqrt(-c**2*x**2 +
1)) + 3*I*x/(8*c**3*sqrt(-c**2*x**2 + 1)) - 3*I*asin(c*x)/(8*c**4), True))/
(5*c) + b*e*Piecewise((c*x**7/(6*sqrt(c**2*x**2 - 1)) + x**5/(24*c*sqrt(c**
2*x**2 - 1)) + 5*x**3/(48*c**3*sqrt(c**2*x**2 - 1)) - 5*x/(16*c**5*sqrt(c**
2*x**2 - 1)) + 5*acosh(c*x)/(16*c**6), Abs(c**2*x**2) > 1), (-I*c*x**7/(6*s
qrt(-c**2*x**2 + 1)) - I*x**5/(24*c*sqrt(-c**2*x**2 + 1)) - 5*I*x**3/(48*c*
*3*sqrt(-c**2*x**2 + 1)) + 5*I*x/(16*c**5*sqrt(-c**2*x**2 + 1)) - 5*I*asin(
c*x)/(16*c**6), True))/(7*c)
```

3.77 $\int x^2 \left(d+ex^2\right) \left(a+b \csc^{-1}(cx)\right) \, dx$

Optimal. Leaf size=161

$$\frac{1}{3}dx^3 \left(a+b \csc^{-1}(cx)\right)+\frac{1}{5}ex^5 \left(a+b \csc^{-1}(cx)\right)+\frac{bex^4\sqrt{c^2x^2-1}}{20c\sqrt{c^2x^2}}+\frac{bx \left(20c^2d+9e\right) \tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{120c^4\sqrt{c^2x^2}}+\frac{bx^2\sqrt{c^2x^2}}{12}$$

```
[Out] 1/3*d*x^3*(a+b*arccsc(c*x))+1/5*e*x^5*(a+b*arccsc(c*x))+1/120*b*(20*c^2*d+9
*e)*x*arctanh(c*x/(c^2*x^2-1)^(1/2))/c^4/(c^2*x^2)^(1/2)+1/120*b*(20*c^2*d+
9*e)*x^2*(c^2*x^2-1)^(1/2)/c^3/(c^2*x^2)^(1/2)+1/20*b*e*x^4*(c^2*x^2-1)^(1/
2)/c/(c^2*x^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {14, 5239, 12, 459, 321, 217, 206}

$$\frac{1}{3}dx^3 \left(a+b \csc^{-1}(cx)\right)+\frac{1}{5}ex^5 \left(a+b \csc^{-1}(cx)\right)+\frac{bx^2\sqrt{c^2x^2-1} \left(20c^2d+9e\right)}{120c^3\sqrt{c^2x^2}}+\frac{bx \left(20c^2d+9e\right) \tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{120c^4\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*(20*c^2*d + 9*e)*x^2*Sqrt[-1 + c^2*x^2])/(120*c^3*Sqrt[c^2*x^2]) + (b*e*
x^4*Sqrt[-1 + c^2*x^2])/(20*c*Sqrt[c^2*x^2]) + (d*x^3*(a + b*ArcCsc[c*x]))/
3 + (e*x^5*(a + b*ArcCsc[c*x]))/5 + (b*(20*c^2*d + 9*e)*x*ArcTanh[(c*x)/Sqr
t[-1 + c^2*x^2]])/(120*c^4*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int x^2\left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)dx = \frac{1}{3}dx^3\left(a+b\csc^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{x^2\left(5d+3ex^2\right)}{15\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}$$

$$= \frac{1}{3}dx^3\left(a+b\csc^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{x^2\left(5d+3ex^2\right)}{\sqrt{-1+c^2x^2}}\,dx}{15\sqrt{c^2x^2}}$$

$$= \frac{bex^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}}+\frac{1}{3}dx^3\left(a+b\csc^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\csc^{-1}(cx)\right)-\frac{(bc(}{}$$

$$= \frac{b\left(20c^2d+9e\right)x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}}+\frac{bex^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}}+\frac{1}{3}dx^3\left(a+b\csc^{-1}(cx)\right)$$

$$= \frac{b\left(20c^2d+9e\right)x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}}+\frac{bex^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}}+\frac{1}{3}dx^3\left(a+b\csc^{-1}(cx)\right)$$

$$= \frac{b\left(20c^2d+9e\right)x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}}+\frac{bex^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}}+\frac{1}{3}dx^3\left(a+b\csc^{-1}(cx)\right)$$

Mathematica [A] time = 0.18, size = 121, normalized size = 0.75

$$\frac{c^2x^2\left(8ac^3x\left(5d+3ex^2\right)+b\sqrt{1-\frac{1}{c^2x^2}}\left(c^2\left(20d+6ex^2\right)+9e\right)\right)+8bc^5x^3\csc^{-1}(cx)\left(5d+3ex^2\right)+b\left(20c^2d+9e\right)\mathrm{lo}}{120c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (c^2*x^2*(8*a*c^3*x*(5*d + 3*e*x^2) + b*Sqrt[1 - 1/(c^2*x^2)]*(9*e + c^2*(2
0*d + 6*e*x^2))) + 8*b*c^5*x^3*(5*d + 3*e*x^2)*ArcCsc[c*x] + b*(20*c^2*d +
9*e)*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x])/(120*c^5)
```

fricas [A] time = 1.61, size = 170, normalized size = 1.06

$$\frac{24\,ac^5ex^5+40\,ac^5dx^3+8\left(3\,bc^5ex^5+5\,bc^5dx^3-5\,bc^5d-3\,bc^5e\right)\operatorname{arccsc}(cx)-16\left(5\,bc^5d+3\,bc^5e\right)\arctan\left(-cx+\right.}{120\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/120*(24*a*c^5*e*x^5 + 40*a*c^5*d*x^3 + 8*(3*b*c^5*e*x^5 + 5*b*c^5*d*x^3 -
 5*b*c^5*d - 3*b*c^5*e)*arccsc(c*x) - 16*(5*b*c^5*d + 3*b*c^5*e)*arctan(-c*
x + sqrt(c^2*x^2 - 1)) - (20*b*c^2*d + 9*b*e)*log(-c*x + sqrt(c^2*x^2 - 1))
 + (6*b*c^3*e*x^3 + (20*b*c^3*d + 9*b*c*e)*x)*sqrt(c^2*x^2 - 1))/c^5
```

giac [B] time = 1.44, size = 840, normalized size = 5.22

$$\frac{1}{960}\left(\frac{6\,bx^5\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^5\arcsin\left(\frac{1}{cx}\right)e}{c}+\frac{6\,ax^5\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^5 e}{c}+\frac{40\,bdx^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3\arcsin\left(\right.}{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/960*(6*b*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*arcsin(1/(c*x))*e/c + 6*a*x^5
*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*e/c + 40*b*d*x^3*(sqrt(-1/(c^2*x^2) + 1) +
1)^3*arcsin(1/(c*x))/c + 3*b*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*e/c^2 + 40*
a*d*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c + 30*b*x^3*(sqrt(-1/(c^2*x^2) + 1)
 + 1)^3*arcsin(1/(c*x))*e/c^3 + 30*a*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*e/c
^3 + 40*b*d*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^2 + 120*b*d*x*(sqrt(-1/(c^
2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^3 + 24*b*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1
)^2*e/c^4 + 120*a*d*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^3 + 60*b*x*(sqrt(-1/(c
^2*x^2) + 1) + 1)*arcsin(1/(c*x))*e/c^5 + 60*a*x*(sqrt(-1/(c^2*x^2) + 1) +
1)*e/c^5 + 160*b*d*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 - 160*b*d*log(1/(abs
(c)*abs(x)))/c^4 + 72*b*e*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^6 - 72*b*e*log(
1/(abs(c)*abs(x)))/c^6 + 120*b*d*arcsin(1/(c*x))/(c^5*x*(sqrt(-1/(c^2*x^2)
+ 1) + 1)) + 120*a*d/(c^5*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 60*b*arcsin(1/(
c*x))*e/(c^7*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 60*a*e/(c^7*x*(sqrt(-1/(c^2*
x^2) + 1) + 1)) - 40*b*d/(c^6*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 40*b*d*
arcsin(1/(c*x))/(c^7*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) - 24*b*e/(c^8*x^2*
(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 40*a*d/(c^7*x^3*(sqrt(-1/(c^2*x^2) + 1) +
 1)^3) + 30*b*arcsin(1/(c*x))*e/(c^9*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) +
30*a*e/(c^9*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) - 3*b*e/(c^10*x^4*(sqrt(-1/
(c^2*x^2) + 1) + 1)^4) + 6*b*arcsin(1/(c*x))*e/(c^11*x^5*(sqrt(-1/(c^2*x^2)
 + 1) + 1)^5) + 6*a*e/(c^11*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))*c
```

maple [B] time = 0.06, size = 282, normalized size = 1.75

$$\frac{ae\,x^5}{5}+\frac{ad\,x^3}{3}+\frac{b\,\mathrm{arccsc}\,(cx)\,e\,x^5}{5}+\frac{b\,\mathrm{arccsc}\,(cx)\,d\,x^3}{3}+\frac{b\,x^4 e}{20c\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{b\,x^2 e}{40c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{bd\,x^2}{6c\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{bd}{6c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)*(a+b*arccsc(c*x)),x)
```

```
[Out] 1/5*a*e*x^5+1/3*a*d*x^3+1/5*b*arccsc(c*x)*e*x^5+1/3*b*arccsc(c*x)*d*x^3+1/2
0/c*b/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^4*e+1/40/c^3*b/((c^2*x^2-1)/c^2/x^2)^(1
/2)*x^2*e+1/6/c*b/((c^2*x^2-1)/c^2/x^2)^(1/2)*d*x^2-1/6/c^3*b/((c^2*x^2-1)/
c^2/x^2)^(1/2)*d+1/6/c^4*b/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*(c^2*x^2-1)^(1/2)*
d*ln(c*x+(c^2*x^2-1)^(1/2))-3/40/c^5*b/((c^2*x^2-1)/c^2/x^2)^(1/2)*e+3/40/c
^6*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*e*ln(c*x+(c^2*x^2-1)^(
1/2))
```

maxima [A] time = 0.33, size = 232, normalized size = 1.44

$$\frac{1}{5}aex^5+\frac{1}{3}adx^3+\frac{1}{12}\left(4x^3\operatorname{arccsc}(cx)+\frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}-\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bd+\frac{1}{80}\left(16x^5\operatorname{arccsc}(cx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*e*x^5 + 1/3*a*d*x^3 + 1/12*(4*x^3*arccsc(c*x) + (2*sqrt(-1/(c^2*x^2)
+ 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 -
log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d + 1/80*(16*x^5*arccsc(c*x) - (2
*(3*(-1/(c^2*x^2) + 1)^(3/2) - 5*sqrt(-1/(c^2*x^2) + 1))/(c^4*(1/(c^2*x^2)
- 1)^2 + 2*c^4*(1/(c^2*x^2) - 1) + c^4) - 3*log(sqrt(-1/(c^2*x^2) + 1) + 1)
/c^4 + 3*log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(e\,x^2+d\right)\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d + e*x^2)*(a + b*asin(1/(c*x))),x)
```

```
[Out] int(x^2*(d + e*x^2)*(a + b*asin(1/(c*x))), x)
```

sympy [A] time = 7.47, size = 294, normalized size = 1.83

$$\frac{adx^3}{3}+\frac{aex^5}{5}+\frac{bdx^3\operatorname{acsc}(cx)}{3}+\frac{bex^5\operatorname{acsc}(cx)}{5}+\frac{bd\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\operatorname{acosh}(cx)}{2c^2} & \text{for } \left|c^2x^2\right|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\operatorname{asin}(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c}+\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)*(a+b*acsc(c*x)),x)
```

```
[Out] a*d*x**3/3 + a*e*x**5/5 + b*d*x**3*acsc(c*x)/3 + b*e*x**5*acsc(c*x)/5 + b*d
*Piecewise((x*sqrt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**
2) > 1), (-I*c*x**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1
)) - I*asin(c*x)/(2*c**2), True))/(3*c) + b*e*Piecewise((c*x**5/(4*sqrt(c**
2*x**2 - 1)) + x**3/(8*c*sqrt(c**2*x**2 - 1)) - 3*x/(8*c**3*sqrt(c**2*x**2
- 1)) + 3*acosh(c*x)/(8*c**4), Abs(c**2*x**2) > 1), (-I*c*x**5/(4*sqrt(-c**
2*x**2 + 1)) - I*x**3/(8*c*sqrt(-c**2*x**2 + 1)) + 3*I*x/(8*c**3*sqrt(-c**2
*x**2 + 1)) - 3*I*asin(c*x)/(8*c**4), True))/(5*c)
```

3.78 $\int \left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)\,dx$

Optimal. Leaf size=109

$$dx\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\csc^{-1}(cx)\right)+\frac{bx\left(6c^2d+e\right)\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{6c^2\sqrt{c^2x^2}}+\frac{bex^2\sqrt{c^2x^2-1}}{6c\sqrt{c^2x^2}}$$

```
[Out] d*x*(a+b*arccsc(c*x))+1/3*e*x^3*(a+b*arccsc(c*x))+1/6*b*(6*c^2*d+e)*x*arcta
nh(c*x/(c^2*x^2-1)^(1/2))/c^2/(c^2*x^2)^(1/2)+1/6*b*e*x^2*(c^2*x^2-1)^(1/2)
/c/(c^2*x^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5229, 12, 388, 217, 206}

$$dx\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\csc^{-1}(cx)\right)+\frac{bx\left(6c^2d+e\right)\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{6c^2\sqrt{c^2x^2}}+\frac{bex^2\sqrt{c^2x^2-1}}{6c\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*e*x^2*Sqrt[-1 + c^2*x^2])/(6*c*Sqrt[c^2*x^2]) + d*x*(a + b*ArcCsc[c*x])
+ (e*x^3*(a + b*ArcCsc[c*x]))/3 + (b*(6*c^2*d + e)*x*ArcTanh[(c*x)/Sqrt[-1
+ c^2*x^2]])/(6*c^2*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 5229

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCsc[c*x], u, x]
+ Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1])
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1/2,
0])
```

Rubi steps

$$\int \left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)\,dx = dx\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int \frac{3d+ex^2}{3\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}$$

$$= dx\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int \frac{3d+ex^2}{\sqrt{-1+c^2x^2}}\,dx}{3\sqrt{c^2x^2}}$$

$$= \frac{bex^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}}+dx\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\csc^{-1}(cx)\right)-\frac{\left(b\left(-6c^2d\right.\right.}{}$$

$$= \frac{bex^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}}+dx\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\csc^{-1}(cx)\right)-\frac{\left(b\left(-6c^2d\right.\right.}{}$$

$$= \frac{bex^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}}+dx\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\csc^{-1}(cx)\right)+\frac{b\left(6c^2d+\right.}{}$$

Mathematica [A] time = 0.28, size = 149, normalized size = 1.37

$$adx+\frac{1}{3}aex^3+\frac{bdx\sqrt{1-\frac{1}{c^2x^2}}\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{\sqrt{c^2x^2-1}}+\frac{bex^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{6c}+\frac{be\log\left(x\left(\sqrt{\frac{c^2x^2-1}{c^2x^2}}+1\right)\right)}{6c^3}+bdx\csc^{-1}(cx)+\frac{1}{3}bex^3\csc$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] a*d*x + (a*e*x^3)/3 + (b*e*x^2*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(6*c) + b*d*
x*ArcCsc[c*x] + (b*e*x^3*ArcCsc[c*x])/3 + (b*d*Sqrt[1 - 1/(c^2*x^2)]*x*ArcT
anh[(c*x)/Sqrt[-1 + c^2*x^2]])/Sqrt[-1 + c^2*x^2] + (b*e*Log[x*(1 + Sqrt[(-
1 + c^2*x^2)/(c^2*x^2)])])/(6*c^3)
```

fricas [A] time = 0.71, size = 141, normalized size = 1.29

$$\frac{2\,ac^3ex^3+6\,ac^3dx+\sqrt{c^2x^2-1}\,bcex+2\left(bc^3ex^3+3\,bc^3dx-3\,bc^3d-bc^3e\right)\operatorname{arccsc}(cx)-4\left(3\,bc^3d+bc^3e\right)\arctan}{6\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*e*x^3 + 6*a*c^3*d*x + sqrt(c^2*x^2 - 1)*b*c*e*x + 2*(b*c^3*e*x
^3 + 3*b*c^3*d*x - 3*b*c^3*d - b*c^3*e)*arccsc(c*x) - 4*(3*b*c^3*d + b*c^3*
e)*arctan(-c*x + sqrt(c^2*x^2 - 1)) - (6*b*c^2*d + b*e)*log(-c*x + sqrt(c^2
*x^2 - 1)))/c^3
```

giac [B] time = 1.23, size = 485, normalized size = 4.45

$$\frac{1}{24}\left(\frac{bx^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3\arcsin\left(\frac{1}{cx}\right)e}{c}+\frac{ax^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3e}{c}+\frac{12\,bdx\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{bx^2(}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/24*(b*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x))*e/c + a*x^3*(sqr
t(-1/(c^2*x^2) + 1) + 1)^3*e/c + 12*b*d*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcs
```

```
in(1/(c*x))/c + b*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*e/c^2 + 12*a*d*x*(sqrt
(-1/(c^2*x^2) + 1) + 1)/c + 3*b*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*
x))*e/c^3 + 3*a*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*e/c^3 + 24*b*d*log(sqrt(-1/(
c^2*x^2) + 1) + 1)/c^2 - 24*b*d*log(1/(abs(c)*abs(x)))/c^2 + 4*b*e*log(sqrt
(-1/(c^2*x^2) + 1) + 1)/c^4 - 4*b*e*log(1/(abs(c)*abs(x)))/c^4 + 12*b*d*arc
sin(1/(c*x))/(c^3*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 12*a*d/(c^3*x*(sqrt(-1/
(c^2*x^2) + 1) + 1)) + 3*b*arcsin(1/(c*x))*e/(c^5*x*(sqrt(-1/(c^2*x^2) + 1)
 + 1)) + 3*a*e/(c^5*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) - b*e/(c^6*x^2*(sqrt(-1
/(c^2*x^2) + 1) + 1)^2) + b*arcsin(1/(c*x))*e/(c^7*x^3*(sqrt(-1/(c^2*x^2) +
 1) + 1)^3) + a*e/(c^7*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3))*c
```

maple [B] time = 0.06, size = 194, normalized size = 1.78

$$\frac{ae\,x^3}{3}+adx+\frac{b\,\mathrm{arccsc}\,(cx)\,x^3e}{3}+b\,\mathrm{arccsc}\,(cx)\,xd+\frac{b\sqrt{c^2x^2-1}\,\,d\ln\left(cx+\sqrt{c^2x^2-1}\right)}{c^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,\,x}+\frac{be\,x^2}{6c\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{be}{6c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccsc(c*x)),x)
```

```
[Out] 1/3*a*e*x^3+a*d*x+1/3*b*arccsc(c*x)*x^3*e+b*arccsc(c*x)*x*d+1/c^2*b/((c^2*x
^2-1)/c^2/x^2)^(1/2)/x*(c^2*x^2-1)^(1/2)*d*ln(c*x+(c^2*x^2-1)^(1/2))+1/6/c*
b/((c^2*x^2-1)/c^2/x^2)^(1/2)*e*x^2-1/6/c^3*b/((c^2*x^2-1)/c^2/x^2)^(1/2)*e
+1/6/c^4*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*e*ln(c*x+(c^2*x^
2-1)^(1/2))
```

maxima [A] time = 0.32, size = 153, normalized size = 1.40

$$\frac{1}{3}\,aex^3+\frac{1}{12}\left(4\,x^3\,\mathrm{arccsc}\,(cx)+\frac{\frac{2\,\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}-\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)be+adx+\frac{\left(2\,cx\,\mathrm{arccsc}\,(cx)+1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*e*x^3 + 1/12*(4*x^3*arccsc(c*x) + (2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(
c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c
^2*x^2) + 1) - 1)/c^2)/c)*b*e + a*d*x + 1/2*(2*c*x*arccsc(c*x) + log(sqrt(-
1/(c^2*x^2) + 1) + 1) - log(-sqrt(-1/(c^2*x^2) + 1) + 1))*b*d/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(e\,x^2+d\right)\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)*(a + b*asin(1/(c*x))),x)
```

```
[Out] int((d + e*x^2)*(a + b*asin(1/(c*x))), x)
```

sympy [A] time = 6.03, size = 153, normalized size = 1.40

$$adx+\frac{aex^3}{3}+bdx\,\mathrm{acsc}\,(cx)+\frac{bex^3\,\mathrm{acsc}\,(cx)}{3}+\frac{bd\left(\begin{cases}\mathrm{acosh}\,(cx) & \text{for}\ \left|c^2x^2\right|>1\\ -i\,\mathrm{asin}\,(cx) & \text{otherwise}\end{cases}\right)}{c}+\frac{be\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\mathrm{acosh}\,(cx)}{2c^2}\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}\end{cases}\right.}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acsc(c*x)),x)
```

```
[Out] a*d*x + a*e*x**3/3 + b*d*x*acsc(c*x) + b*e*x**3*acsc(c*x)/3 + b*d*Piecewise
((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin(c*x), True))/c + b*e*Piecewise(
(x*sqrt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1), (-
I*c*x**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*asin
(c*x)/(2*c**2), True))/(3*c)
```

3.79 $\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=87

$$-\frac{d\left(a+b\csc^{-1}(cx)\right)}{x}+ex\left(a+b\csc^{-1}(cx)\right)-\frac{bcd\sqrt{c^2x^2-1}}{\sqrt{c^2x^2}}+\frac{bex\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{\sqrt{c^2x^2}}$$

```
[Out] -d*(a+b*arccsc(c*x))/x+e*x*(a+b*arccsc(c*x))+b*e*x*arctanh(c*x/(c^2*x^2-1)^
(1/2))/(c^2*x^2)^(1/2)-b*c*d*(c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {14, 5239, 451, 217, 206}

$$-\frac{d\left(a+b\csc^{-1}(cx)\right)}{x}+ex\left(a+b\csc^{-1}(cx)\right)-\frac{bcd\sqrt{c^2x^2-1}}{\sqrt{c^2x^2}}+\frac{bex\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^2,x]
```

```
[Out] -((b*c*d*Sqrt[-1 + c^2*x^2])/Sqrt[c^2*x^2]) - (d*(a + b*ArcCsc[c*x]))/x + e
*x*(a + b*ArcCsc[c*x]) + (b*e*x*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/Sqrt[c^2
*x^2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
```

```
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^2}\,dx = -\frac{d(a+b\csc^{-1}(cx))}{x} + ex(a+b\csc^{-1}(cx)) + \frac{(bcx)\int \frac{-d+ex^2}{x^2\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}$$

$$= -\frac{bcd\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}} - \frac{d(a+b\csc^{-1}(cx))}{x} + ex(a+b\csc^{-1}(cx)) + \frac{(bcex)\int \frac{}{\sqrt{-1\cdots}}}{\sqrt{c^2x^2}}$$

$$= -\frac{bcd\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}} - \frac{d(a+b\csc^{-1}(cx))}{x} + ex(a+b\csc^{-1}(cx)) + \frac{(bcex)\,\mathrm{Subst}\cdots}{\cdots}$$

$$= -\frac{bcd\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}} - \frac{d(a+b\csc^{-1}(cx))}{x} + ex(a+b\csc^{-1}(cx)) + \frac{bex\tanh^{-1}(\cdots}{\sqrt{c^2x\cdots}}$$

Mathematica [A] time = 0.12, size = 104, normalized size = 1.20

$$-\frac{ad}{x} + aex - bcd\sqrt{\frac{c^2x^2-1}{c^2x^2}} + \frac{bex\sqrt{1-\frac{1}{c^2x^2}}\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{\sqrt{c^2x^2-1}} - \frac{bd\csc^{-1}(cx)}{x} + bex\csc^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^2,x]
```

```
[Out] -((a*d)/x) + a*e*x - b*c*d*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)] - (b*d*ArcCsc[c*x
])/x + b*e*x*ArcCsc[c*x] + (b*e*Sqrt[1 - 1/(c^2*x^2)]*x*ArcTanh[(c*x)/Sqrt[
-1 + c^2*x^2]])/Sqrt[-1 + c^2*x^2]
```

fricas [A] time = 0.55, size = 125, normalized size = 1.44

$$-\frac{bc^2dx - acex^2 + bex\log\left(-cx+\sqrt{c^2x^2-1}\right) + \sqrt{c^2x^2-1}\,bcd + acd - 2(bcd-bce)x\arctan\left(-cx+\sqrt{c^2x^2-1}\right) - \cdots}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] -(b*c^2*d*x - a*c*e*x^2 + b*e*x*log(-c*x + sqrt(c^2*x^2 - 1)) + sqrt(c^2*x^
2 - 1)*b*c*d + a*c*d - 2*(b*c*d - b*c*e)*x*arctan(-c*x + sqrt(c^2*x^2 - 1))
 - (b*c*e*x^2 - b*c*d + (b*c*d - b*c*e)*x)*arccsc(c*x))/(c*x)
```

giac [B] time = 0.46, size = 1062, normalized size = 12.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*arcsin(1/(c*x))*e/(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sq
rt(-1/(c^2*x^2) + 1) + 1)^3)) + a*e/(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1
/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3)) - 2*b*c*d/(x*(sqrt(-1/(c^2*x^2) +
1) + 1)*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sqrt(-1/(c^2*x^2) +
 1) + 1)^3))) + 2*b*e*log(sqrt(-1/(c^2*x^2) + 1) + 1)/(c*x*(sqrt(-1/(c^2*x^
```

```
2) + 1) + 1)*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sqrt(-1/(c^2*x
^2) + 1) + 1)^3))) - 2*b*e*log(1/(abs(c)*abs(x)))/(c*x*(sqrt(-1/(c^2*x^2) +
 1) + 1)*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sqrt(-1/(c^2*x^2)
+ 1) + 1)^3))) - 4*b*d*arcsin(1/(c*x))/(x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*
(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)
^3))) - 4*a*d/(x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*(c/(x*(sqrt(-1/(c^2*x^2)
+ 1) + 1)) + 1/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3))) + 2*b*arcsin(1/(c*x
))*e/(c^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*(c/(x*(sqrt(-1/(c^2*x^2) + 1)
+ 1)) + 1/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3))) + 2*a*e/(c^2*x^2*(sqrt(-
1/(c^2*x^2) + 1) + 1)^2*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sqr
t(-1/(c^2*x^2) + 1) + 1)^3))) + 2*b*d/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3
*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1
)^3))) + 2*b*e*log(sqrt(-1/(c^2*x^2) + 1) + 1)/(c^3*x^3*(sqrt(-1/(c^2*x^2)
+ 1) + 1)^3*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sqrt(-1/(c^2*x^
2) + 1) + 1)^3))) - 2*b*e*log(1/(abs(c)*abs(x)))/(c^3*x^3*(sqrt(-1/(c^2*x^2
) + 1) + 1)^3*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sqrt(-1/(c^2*
x^2) + 1) + 1)^3))) + b*arcsin(1/(c*x))*e/(c^4*x^4*(sqrt(-1/(c^2*x^2) + 1)
+ 1)^4*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sqrt(-1/(c^2*x^2) +
1) + 1)^3))) + a*e/(c^4*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*(c/(x*(sqrt(-1/(
c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3))))*c
```

maple [A] time = 0.06, size = 136, normalized size = 1.56

$$aex-\frac{ad}{x}+b\,\mathrm{arccsc}\,(cx)\,ex-\frac{b\,\mathrm{arccsc}\,(cx)\,d}{x}-\frac{cbd}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{bd}{c\,x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{b\sqrt{c^2x^2-1}\;e\ln\left(cx+\sqrt{c^2x^2-1}\right)}{c^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\;x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccsc(c*x))/x^2,x)
```

```
[Out] a*e*x-a*d/x+b*arccsc(c*x)*e*x-b*arccsc(c*x)*d/x-c*b/((c^2*x^2-1)/c^2/x^2)^(
1/2)*d+b/c/x^2/((c^2*x^2-1)/c^2/x^2)^(1/2)*d+b/c^2*(c^2*x^2-1)^(1/2)/((c^2*
x^2-1)/c^2/x^2)^(1/2)/x*e*ln(c*x+(c^2*x^2-1)^(1/2))
```

maxima [A] time = 0.32, size = 89, normalized size = 1.02

$$-\left(c\sqrt{-\frac{1}{c^2x^2}+1}+\frac{\mathrm{arccsc}\,(cx)}{x}\right)bd+aex+\frac{\left(2\,cx\,\mathrm{arccsc}\,(cx)+\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)-\log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)b}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] -(c*sqrt(-1/(c^2*x^2) + 1) + arccsc(c*x)/x)*b*d + a*e*x + 1/2*(2*c*x*arccsc
(c*x) + log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-sqrt(-1/(c^2*x^2) + 1) + 1))
*b*e/c - a*d/x
```

mupad [B] time = 0.83, size = 75, normalized size = 0.86

$$a\,e\,x-\frac{a\,d}{x}+\frac{b\,e\,\mathrm{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{c^2\,x^2}}}\right)}{c}-b\,c\,d\,\sqrt{1-\frac{1}{c^2\,x^2}}-\frac{b\,d\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x}+b\,e\,x\,\mathrm{asin}\left(\frac{1}{c\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(a + b*asin(1/(c*x))))/x^2,x)
```

```
[Out] a*e*x - (a*d)/x + (b*e*atanh(1/(1 - 1/(c^2*x^2))^(1/2)))/c - b*c*d*(1 - 1/(
c^2*x^2))^(1/2) - (b*d*asin(1/(c*x)))/x + b*e*x*asin(1/(c*x))
```

sympy [A] time = 5.43, size = 73, normalized size = 0.84

$$-\frac{ad}{x}+aex-bcd\sqrt{1-\frac{1}{c^2x^2}}-\frac{bd\operatorname{acsc}(cx)}{x}+bex\operatorname{acsc}(cx)+\frac{be\left(\begin{cases}\operatorname{acosh}(cx) & \text{for } \left|c^2x^2\right|>1\\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acsc(c*x))/x**2,x)
```

```
[Out] -a*d/x + a*e*x - b*c*d*sqrt(1 - 1/(c**2*x**2)) - b*d*acsc(c*x)/x + b*e*x*ac
sc(c*x) + b*e*Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin(c*x), Tr
ue))/c
```

3.80 $\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^4}\,dx$

Optimal. Leaf size=105

$$-\frac{d\left(a+b\csc^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\csc^{-1}(cx)\right)}{x}-\frac{bc\sqrt{c^2x^2-1}\,\left(2c^2d+9e\right)}{9\sqrt{c^2x^2}}-\frac{bcd\sqrt{c^2x^2-1}}{9x^2\sqrt{c^2x^2}}$$

```
[Out] -1/3*d*(a+b*arccsc(c*x))/x^3-e*(a+b*arccsc(c*x))/x-1/9*b*c*(2*c^2*d+9*e)*(c
^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)-1/9*b*c*d*(c^2*x^2-1)^(1/2)/x^2/(c^2*x^2)^(
1/2)
```

Rubi [A] time = 0.08, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {14, 5239, 12, 453, 264}

$$-\frac{d\left(a+b\csc^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\csc^{-1}(cx)\right)}{x}-\frac{bc\sqrt{c^2x^2-1}\,\left(2c^2d+9e\right)}{9\sqrt{c^2x^2}}-\frac{bcd\sqrt{c^2x^2-1}}{9x^2\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^4,x]
```

```
[Out] -(b*c*(2*c^2*d + 9*e)*Sqrt[-1 + c^2*x^2])/(9*Sqrt[c^2*x^2]) - (b*c*d*Sqrt[-
1 + c^2*x^2])/(9*x^2*Sqrt[c^2*x^2]) - (d*(a + b*ArcCsc[c*x]))/(3*x^3) - (e*
(a + b*ArcCsc[c*x]))/x
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
```

```
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)}{x^4}\,dx = -\frac{d\left(a+b\csc^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\csc^{-1}(cx)\right)}{x}+\frac{(bcx)\int \frac{-d-3ex^2}{3x^4\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}$$

$$= -\frac{d\left(a+b\csc^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\csc^{-1}(cx)\right)}{x}+\frac{(bcx)\int \frac{-d-3ex^2}{x^4\sqrt{-1+c^2x^2}}\,dx}{3\sqrt{c^2x^2}}$$

$$= -\frac{bcd\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}}-\frac{d\left(a+b\csc^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\csc^{-1}(cx)\right)}{x}+\frac{\left(bc\left(-2c^2d-\right.\right.}{}$$

$$= -\frac{bc\left(2c^2d+9e\right)\sqrt{-1+c^2x^2}}{9\sqrt{c^2x^2}}-\frac{bcd\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}}-\frac{d\left(a+b\csc^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+\right.}{}$$

Mathematica [A] time = 0.11, size = 69, normalized size = 0.66

$$-\frac{3a\left(d+3ex^2\right)+bcx\sqrt{1-\frac{1}{c^2x^2}}\left(2c^2dx^2+d+9ex^2\right)+3b\csc^{-1}(cx)\left(d+3ex^2\right)}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^4,x]
```

```
[Out] -1/9*(3*a*(d + 3*e*x^2) + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(d + 2*c^2*d*x^2 + 9*
e*x^2) + 3*b*(d + 3*e*x^2)*ArcCsc[c*x])/x^3
```

fricas [A] time = 0.94, size = 66, normalized size = 0.63

$$-\frac{9\,aex^2+3\,ad+3\left(3\,bex^2+bd\right)\operatorname{arccsc}(cx)+\sqrt{c^2x^2-1}\left(\left(2\,bc^2d+9\,be\right)x^2+bd\right)}{9\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/9*(9*a*e*x^2 + 3*a*d + 3*(3*b*e*x^2 + b*d)*arccsc(c*x) + sqrt(c^2*x^2 -
1)*((2*b*c^2*d + 9*b*e)*x^2 + b*d))/x^3
```

giac [A] time = 0.14, size = 139, normalized size = 1.32

$$\frac{1}{9}\left(bc^2d\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,bc^2d\sqrt{-\frac{1}{c^2x^2}+1}-\frac{3\,bcd\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)}{x}-\frac{3\,bcd\arcsin\left(\frac{1}{cx}\right)}{x}-9\,b\sqrt{-\frac{1}{c^2x^2}+1}\,e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^4,x, algorithm="giac")
```

```
[Out] 1/9*(b*c^2*d*(-1/(c^2*x^2) + 1)^(3/2) - 3*b*c^2*d*sqrt(-1/(c^2*x^2) + 1) -
3*b*c*d*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))/x - 3*b*c*d*arcsin(1/(c*x))/x - 9
*b*sqrt(-1/(c^2*x^2) + 1)*e - 9*b*arcsin(1/(c*x))*e/(c*x) - 9*a*e/(c*x) - 3
*a*d/(c*x^3))*c
```

maple [A] time = 0.06, size = 121, normalized size = 1.15

$$c^{3}\left(\frac{a\left(-\frac{d}{3c\,x^{3}}-\frac{e}{cx}\right)}{c^{2}}+\frac{b\left(-\frac{\operatorname{arccsc}(cx)d}{3c\,x^{3}}-\frac{\operatorname{arccsc}(cx)e}{cx}-\frac{\left(c^{2}x^{2}-1\right)\left(2c^{4}d\,x^{2}+9c^{2}e\,x^{2}+c^{2}d\right)}{9\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,c^{4}x^{4}}\right)}{c^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccsc(c*x))/x^4,x)
```

```
[Out] c^3*(a/c^2*(-1/3/c*d/x^3-e/c/x)+b/c^2*(-1/3*arccsc(c*x)/c*d/x^3-arccsc(c*x)
*e/c/x-1/9*(c^2*x^2-1)*(2*c^4*d*x^2+9*c^2*e*x^2+c^2*d)/((c^2*x^2-1)/c^2/x^2
)^(1/2)/c^4/x^4))
```

maxima [A] time = 0.33, size = 94, normalized size = 0.90

$$-\left(c\sqrt{-\frac{1}{c^{2}x^{2}}+1}+\frac{\operatorname{arccsc}(cx)}{x}\right)be+\frac{1}{9}\,bd\left(\frac{c^{4}\left(-\frac{1}{c^{2}x^{2}}+1\right)^{\frac{3}{2}}-3\,c^{4}\sqrt{-\frac{1}{c^{2}x^{2}}+1}}{c}-\frac{3\operatorname{arccsc}(cx)}{x^{3}}\right)-\frac{ae}{x}-\frac{ad}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] -(c*sqrt(-1/(c^2*x^2) + 1) + arccsc(c*x)/x)*b*e + 1/9*b*d*((c^4*(-1/(c^2*x^
2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c - 3*arccsc(c*x)/x^3) - a*e/
x - 1/3*a*d/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(e\,x^{2}+d\right)\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(a + b*asin(1/(c*x))))/x^4,x)
```

```
[Out] int(((d + e*x^2)*(a + b*asin(1/(c*x))))/x^4, x)
```

sympy [A] time = 4.73, size = 151, normalized size = 1.44

$$-\frac{ad}{3x^{3}}-\frac{ae}{x}-bce\sqrt{1-\frac{1}{c^{2}x^{2}}}-\frac{bd\operatorname{acsc}(cx)}{3x^{3}}-\frac{be\operatorname{acsc}(cx)}{x}-\frac{bd\left(\begin{cases}\frac{2c^{3}\sqrt{c^{2}x^{2}-1}}{3x}+\frac{c\sqrt{c^{2}x^{2}-1}}{3x^{3}} & \text{for } \left|c^{2}x^{2}\right|>1\\ \frac{2ic^{3}\sqrt{-c^{2}x^{2}+1}}{3x}+\frac{ic\sqrt{-c^{2}x^{2}+1}}{3x^{3}} & \text{otherwise}\end{cases}\right)}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acsc(c*x))/x**4,x)
```

```
[Out] -a*d/(3*x**3) - a*e/x - b*c*e*sqrt(1 - 1/(c**2*x**2)) - b*d*acsc(c*x)/(3*x*
*3) - b*e*acsc(c*x)/x - b*d*Piecewise((2*c**3*sqrt(c**2*x**2 - 1)/(3*x) + c
*sqrt(c**2*x**2 - 1)/(3*x**3), Abs(c**2*x**2) > 1), (2*I*c**3*sqrt(-c**2*x*
*2 + 1)/(3*x) + I*c*sqrt(-c**2*x**2 + 1)/(3*x**3), True))/(3*c)
```

3.81 $\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^6}\,dx$

Optimal. Leaf size=152

$$-\frac{d\left(a+b\csc^{-1}(cx)\right)}{5x^5}-\frac{e\left(a+b\csc^{-1}(cx)\right)}{3x^3}-\frac{bc\sqrt{c^2x^2-1}\left(12c^2d+25e\right)}{225x^2\sqrt{c^2x^2}}-\frac{bcd\sqrt{c^2x^2-1}}{25x^4\sqrt{c^2x^2}}-\frac{2bc^3\sqrt{c^2x^2-1}\left(12c^2d+2\right.}{225\sqrt{c^2x^2}}$$

```
[Out] -1/5*d*(a+b*arccsc(c*x))/x^5-1/3*e*(a+b*arccsc(c*x))/x^3-2/225*b*c^3*(12*c^
2*d+25*e)*(c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)-1/25*b*c*d*(c^2*x^2-1)^(1/2)/x^
4/(c^2*x^2)^(1/2)-1/225*b*c*(12*c^2*d+25*e)*(c^2*x^2-1)^(1/2)/x^2/(c^2*x^2)
^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {14, 5239, 12, 453, 271, 264}

$$-\frac{d\left(a+b\csc^{-1}(cx)\right)}{5x^5}-\frac{e\left(a+b\csc^{-1}(cx)\right)}{3x^3}-\frac{2bc^3\sqrt{c^2x^2-1}\left(12c^2d+25e\right)}{225\sqrt{c^2x^2}}-\frac{bc\sqrt{c^2x^2-1}\left(12c^2d+25e\right)}{225x^2\sqrt{c^2x^2}}-\frac{bcd\sqrt{c^2x^2}}{25x^4\sqrt{c^2x}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^6,x]
```

```
[Out] (-2*b*c^3*(12*c^2*d + 25*e)*Sqrt[-1 + c^2*x^2])/(225*Sqrt[c^2*x^2]) - (b*c*
d*Sqrt[-1 + c^2*x^2])/(25*x^4*Sqrt[c^2*x^2]) - (b*c*(12*c^2*d + 25*e)*Sqrt[
-1 + c^2*x^2])/(225*x^2*Sqrt[c^2*x^2]) - (d*(a + b*ArcCsc[c*x]))/(5*x^5) -
(e*(a + b*ArcCsc[c*x]))/(3*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^6}\,dx = -\frac{d(a+b\csc^{-1}(cx))}{5x^5} - \frac{e(a+b\csc^{-1}(cx))}{3x^3} + \frac{(bcx)\int \frac{-3d-5ex^2}{15x^6\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}$$

$$= -\frac{d(a+b\csc^{-1}(cx))}{5x^5} - \frac{e(a+b\csc^{-1}(cx))}{3x^3} + \frac{(bcx)\int \frac{-3d-5ex^2}{x^6\sqrt{-1+c^2x^2}}\,dx}{15\sqrt{c^2x^2}}$$

$$= -\frac{bcd\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}} - \frac{d(a+b\csc^{-1}(cx))}{5x^5} - \frac{e(a+b\csc^{-1}(cx))}{3x^3} + \frac{(bc(-12c^2 \ldots}{\ldots}$$

$$= -\frac{bcd\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}} - \frac{bc(12c^2d+25e)\sqrt{-1+c^2x^2}}{225x^2\sqrt{c^2x^2}} - \frac{d(a+b\csc^{-1}(cx))}{5x^5} - \ldots$$

$$= -\frac{2bc^3(12c^2d+25e)\sqrt{-1+c^2x^2}}{225\sqrt{c^2x^2}} - \frac{bcd\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}} - \frac{bc(12c^2d+25e)\sqrt{\ldots}}{225x^2\sqrt{c^2x^2}} \ldots$$

Mathematica [A] time = 0.17, size = 94, normalized size = 0.62

$$-\frac{15a(3d+5ex^2)+bcx\sqrt{1-\frac{1}{c^2x^2}}\left(25ex^2(2c^2x^2+1)+3d(8c^4x^4+4c^2x^2+3)\right)+15b\csc^{-1}(cx)(3d+5ex^2)}{225x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^6,x]
```

```
[Out] -1/225*(15*a*(3*d + 5*e*x^2) + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(25*e*x^2*(1 + 2
*c^2*x^2) + 3*d*(3 + 4*c^2*x^2 + 8*c^4*x^4)) + 15*b*(3*d + 5*e*x^2)*ArcCsc[
c*x])/x^5
```

fricas [A] time = 0.53, size = 88, normalized size = 0.58

$$-\frac{75\,aex^2+45\,ad+15\left(5\,bex^2+3\,bd\right)\operatorname{arccsc}(cx)+\left(2\left(12\,bc^4d+25\,bc^2e\right)x^4+\left(12\,bc^2d+25\,be\right)x^2+9\,bd\right)\sqrt{c^2 \ldots}}{225\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] -1/225*(75*a*e*x^2 + 45*a*d + 15*(5*b*e*x^2 + 3*b*d)*arccsc(c*x) + (2*(12*b
*c^4*d + 25*b*c^2*e)*x^4 + (12*b*c^2*d + 25*b*e)*x^2 + 9*b*d)*sqrt(c^2*x^2
- 1))/x^5
```

giac [A] time = 0.17, size = 250, normalized size = 1.64

$$-\frac{1}{225}\left(9\,bc^4d\left(\frac{1}{c^2x^2}-1\right)^2\sqrt{-\frac{1}{c^2x^2}+1}-30\,bc^4d\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+\frac{45\,bc^3d\left(\frac{1}{c^2x^2}-1\right)^2\arcsin\left(\frac{1}{cx}\right)}{x}+45\,bc^4d\sqrt{-\frac{1}{c^2x^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^6,x, algorithm="giac")
```

```
[Out] -1/225*(9*b*c^4*d*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1) - 30*b*c^4*d*(
-1/(c^2*x^2) + 1)^(3/2) + 45*b*c^3*d*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*x))/x
+ 45*b*c^4*d*sqrt(-1/(c^2*x^2) + 1) + 90*b*c^3*d*(1/(c^2*x^2) - 1)*arcsin(1
/(c*x))/x - 25*b*c^2*(-1/(c^2*x^2) + 1)^(3/2)*e + 45*b*c^3*d*arcsin(1/(c*x)
)/x + 75*b*c^2*sqrt(-1/(c^2*x^2) + 1)*e + 75*b*c*(1/(c^2*x^2) - 1)*arcsin(1
/(c*x))*e/x + 75*b*c*arcsin(1/(c*x))*e/x + 75*a*e/(c*x^3) + 45*a*d/(c*x^5))
*c
```

maple [A] time = 0.06, size = 140, normalized size = 0.92

$$c^5\left(\frac{a\left(-\frac{e}{3c^3x^3}-\frac{d}{5c^3x^5}\right)}{c^2}+\frac{b\left(-\frac{\mathrm{arccsc}(cx)e}{3c^3x^3}-\frac{\mathrm{arccsc}(cx)d}{5c^3x^5}-\frac{(c^2x^2-1)(24x^4c^6d+50c^4e\,x^4+12c^4d\,x^2+25c^2e\,x^2+9c^2d)}{225\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^6x^6}\right)}{c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccsc(c*x))/x^6,x)
```

```
[Out] c^5*(a/c^2*(-1/3*e/c^3/x^3-1/5/c^3*d/x^5)+b/c^2*(-1/3*arccsc(c*x)*e/c^3/x^3
-1/5*arccsc(c*x)/c^3*d/x^5-1/225*(c^2*x^2-1)*(24*c^6*d*x^4+50*c^4*e*x^4+12*
c^4*d*x^2+25*c^2*e*x^2+9*c^2*d)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^6/x^6))
```

maxima [A] time = 0.34, size = 137, normalized size = 0.90

$$-\frac{1}{75}\,bd\left(\frac{3\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,c^6\sqrt{-\frac{1}{c^2x^2}+1}}{c}+\frac{15\,\mathrm{arccsc}\,(cx)}{x^5}\right)+\frac{1}{9}\,be\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4}{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] -1/75*b*d*((3*c^6*(-1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(-1/(c^2*x^2) + 1)^(3/2
) + 15*c^6*sqrt(-1/(c^2*x^2) + 1))/c + 15*arccsc(c*x)/x^5) + 1/9*b*e*((c^4*
(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c - 3*arccsc(c*x)/
x^3) - 1/3*a*e/x^3 - 1/5*a*d/x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(a + b*asin(1/(c*x))))/x^6,x)
```

[Out] int(((d + e*x^2)*(a + b*asin(1/(c*x))))/x^6, x)

sympy [A] time = 9.75, size = 280, normalized size = 1.84

$$-\frac{ad}{5x^5}-\frac{ae}{3x^3}-\frac{bd\operatorname{acsc}(cx)}{5x^5}-\frac{be\operatorname{acsc}(cx)}{3x^3}-\frac{bd\left(\begin{cases}\frac{8c^5\sqrt{c^2x^2-1}}{15x}+\frac{4c^3\sqrt{c^2x^2-1}}{15x^3}+\frac{c\sqrt{c^2x^2-1}}{5x^5} & \text{for } \left|c^2x^2\right|>1\\ \frac{8ic^5\sqrt{-c^2x^2+1}}{15x}+\frac{4ic^3\sqrt{-c^2x^2+1}}{15x^3}+\frac{ic\sqrt{-c^2x^2+1}}{5x^5} & \text{otherwise}\end{cases}\right)}{5c}-\frac{be\left(\left\{\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)*(a+b*acsc(c*x))/x**6,x)

[Out] -a*d/(5*x**5) - a*e/(3*x**3) - b*d*acsc(c*x)/(5*x**5) - b*e*acsc(c*x)/(3*x**3) - b*d*Piecewise((8*c**5*sqrt(c**2*x**2 - 1)/(15*x) + 4*c**3*sqrt(c**2*x**2 - 1)/(15*x**3) + c*sqrt(c**2*x**2 - 1)/(5*x**5), Abs(c**2*x**2) > 1), (8*I*c**5*sqrt(-c**2*x**2 + 1)/(15*x) + 4*I*c**3*sqrt(-c**2*x**2 + 1)/(15*x**3) + I*c*sqrt(-c**2*x**2 + 1)/(5*x**5), True))/(5*c) - b*e*Piecewise((2*c**3*sqrt(c**2*x**2 - 1)/(3*x) + c*sqrt(c**2*x**2 - 1)/(3*x**3), Abs(c**2*x**2) > 1), (2*I*c**3*sqrt(-c**2*x**2 + 1)/(3*x) + I*c*sqrt(-c**2*x**2 + 1)/(3*x**3), True))/(3*c)

3.82 $\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^8}\,dx$

Optimal. Leaf size=197

$$-\frac{d\left(a+b\csc^{-1}(cx)\right)}{7x^7}-\frac{e\left(a+b\csc^{-1}(cx)\right)}{5x^5}-\frac{bc\sqrt{c^2x^2-1}\left(30c^2d+49e\right)}{1225x^4\sqrt{c^2x^2}}-\frac{bcd\sqrt{c^2x^2-1}}{49x^6\sqrt{c^2x^2}}-\frac{8bc^5\sqrt{c^2x^2-1}\left(30c^2d+4\right.}{3675\sqrt{c^2x^2}}$$

```
[Out] -1/7*d*(a+b*arccsc(c*x))/x^7-1/5*e*(a+b*arccsc(c*x))/x^5-8/3675*b*c^5*(30*c
^2*d+49*e)*(c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)-1/49*b*c*d*(c^2*x^2-1)^(1/2)/x
^6/(c^2*x^2)^(1/2)-1/1225*b*c*(30*c^2*d+49*e)*(c^2*x^2-1)^(1/2)/x^4/(c^2*x^
2)^(1/2)-4/3675*b*c^3*(30*c^2*d+49*e)*(c^2*x^2-1)^(1/2)/x^2/(c^2*x^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {14, 5239, 12, 453, 271, 264}

$$-\frac{d\left(a+b\csc^{-1}(cx)\right)}{7x^7}-\frac{e\left(a+b\csc^{-1}(cx)\right)}{5x^5}-\frac{8bc^5\sqrt{c^2x^2-1}\left(30c^2d+49e\right)}{3675\sqrt{c^2x^2}}-\frac{4bc^3\sqrt{c^2x^2-1}\left(30c^2d+49e\right)}{3675x^2\sqrt{c^2x^2}}-\frac{bc\sqrt{c^2x^2}}{1}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^8,x]
```

```
[Out] (-8*b*c^5*(30*c^2*d + 49*e)*Sqrt[-1 + c^2*x^2])/(3675*Sqrt[c^2*x^2]) - (b*c
*d*Sqrt[-1 + c^2*x^2])/(49*x^6*Sqrt[c^2*x^2]) - (b*c*(30*c^2*d + 49*e)*Sqrt
[-1 + c^2*x^2])/(1225*x^4*Sqrt[c^2*x^2]) - (4*b*c^3*(30*c^2*d + 49*e)*Sqrt[
-1 + c^2*x^2])/(3675*x^2*Sqrt[c^2*x^2]) - (d*(a + b*ArcCsc[c*x]))/(7*x^7) -
 (e*(a + b*ArcCsc[c*x]))/(5*x^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
```

```
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^8}\,dx = -\frac{d(a+b\csc^{-1}(cx))}{7x^7} - \frac{e(a+b\csc^{-1}(cx))}{5x^5} + \frac{(bcx)\int \frac{-5d-7ex^2}{35x^8\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}$$

$$= -\frac{d(a+b\csc^{-1}(cx))}{7x^7} - \frac{e(a+b\csc^{-1}(cx))}{5x^5} + \frac{(bcx)\int \frac{-5d-7ex^2}{x^8\sqrt{-1+c^2x^2}}\,dx}{35\sqrt{c^2x^2}}$$

$$= -\frac{bcd\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}} - \frac{d(a+b\csc^{-1}(cx))}{7x^7} - \frac{e(a+b\csc^{-1}(cx))}{5x^5} + \frac{(bc(-30c^2 \ldots}{\ldots}$$

$$= -\frac{bcd\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}} - \frac{bc(30c^2d+49e)\sqrt{-1+c^2x^2}}{1225x^4\sqrt{c^2x^2}} - \frac{d(a+b\csc^{-1}(cx))}{7x^7} - \ldots$$

$$= -\frac{bcd\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}} - \frac{bc(30c^2d+49e)\sqrt{-1+c^2x^2}}{1225x^4\sqrt{c^2x^2}} - \frac{4bc^3(30c^2d+49e)\sqrt{\ldots}}{3675x^2\sqrt{c^2x^2}} \ldots$$

$$= -\frac{8bc^5(30c^2d+49e)\sqrt{-1+c^2x^2}}{3675\sqrt{c^2x^2}} - \frac{bcd\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}} - \frac{bc(30c^2d+49e)\sqrt{\ldots}}{1225x^4\sqrt{c^2 \ldots}} \ldots$$

Mathematica [A] time = 0.17, size = 110, normalized size = 0.56

$$-\frac{105a(5d+7ex^2)+bcx\sqrt{1-\frac{1}{c^2x^2}}\left(49ex^2(8c^4x^4+4c^2x^2+3)+15d(16c^6x^6+8c^4x^4+6c^2x^2+5)\right)+105b\,\mathrm{cs}\ldots}{3675x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^8,x]
```

```
[Out] -1/3675*(105*a*(5*d + 7*e*x^2) + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(49*e*x^2*(3 +
 4*c^2*x^2 + 8*c^4*x^4) + 15*d*(5 + 6*c^2*x^2 + 8*c^4*x^4 + 16*c^6*x^6)) +
105*b*(5*d + 7*e*x^2)*ArcCsc[c*x])/x^7
```

fricas [A] time = 0.60, size = 109, normalized size = 0.55

$$-\frac{735\,aex^2+525\,ad+105\left(7\,bex^2+5\,bd\right)\mathrm{arccsc}\left(cx\right)+\left(8\left(30\,bc^6d+49\,bc^4e\right)x^6+4\left(30\,bc^4d+49\,bc^2e\right)x^4+3\ldots}{3675\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^8,x, algorithm="fricas")
```

```
[Out] -1/3675*(735*a*e*x^2 + 525*a*d + 105*(7*b*e*x^2 + 5*b*d)*arccsc(c*x) + (8*(
30*b*c^6*d + 49*b*c^4*e)*x^6 + 4*(30*b*c^4*d + 49*b*c^2*e)*x^4 + 3*(30*b*c^
2*d + 49*b*e)*x^2 + 75*b*d)*sqrt(c^2*x^2 - 1))/x^7
```

giac [B] time = 0.16, size = 374, normalized size = 1.90

$$-\frac{1}{3675}\left(75\,bc^6d\left(\frac{1}{c^2x^2}-1\right)^3\sqrt{-\frac{1}{c^2x^2}+1}+315\,bc^6d\left(\frac{1}{c^2x^2}-1\right)^2\sqrt{-\frac{1}{c^2x^2}+1}+\frac{525\,bc^5d\left(\frac{1}{c^2x^2}-1\right)^3\arcsin\left(\frac{1}{cx}\right)}{x}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^8,x, algorithm="giac")
```

```
[Out] -1/3675*(75*b*c^6*d*(1/(c^2*x^2) - 1)^3*sqrt(-1/(c^2*x^2) + 1) + 315*b*c^6*
d*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1) + 525*b*c^5*d*(1/(c^2*x^2) - 1
)^3*arcsin(1/(c*x))/x - 525*b*c^6*d*(-1/(c^2*x^2) + 1)^(3/2) + 1575*b*c^5*d
*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*x))/x + 525*b*c^6*d*sqrt(-1/(c^2*x^2) + 1)
 + 147*b*c^4*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1)*e + 1575*b*c^5*d*(1
/(c^2*x^2) - 1)*arcsin(1/(c*x))/x - 490*b*c^4*(-1/(c^2*x^2) + 1)^(3/2)*e +
525*b*c^5*d*arcsin(1/(c*x))/x + 735*b*c^3*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*x
))*e/x + 735*b*c^4*sqrt(-1/(c^2*x^2) + 1)*e + 1470*b*c^3*(1/(c^2*x^2) - 1)*
arcsin(1/(c*x))*e/x + 735*b*c^3*arcsin(1/(c*x))*e/x + 735*a*e/(c*x^5) + 525
*a*d/(c*x^7))*c
```

maple [A] time = 0.06, size = 158, normalized size = 0.80

$$c^7\left(\frac{a\left(-\frac{d}{7c^5x^7}-\frac{e}{5c^5x^5}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arccsc}(cx)d}{7c^5x^7}-\frac{\operatorname{arccsc}(cx)e}{5c^5x^5}-\frac{(c^2x^2-1)(240c^8d\,x^6+392e\,c^6x^6+120x^4c^6d+196c^4e\,x^4+90c^4d\,x^2+147c^2e\,x^2+75c^2d)}{3675\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^8x^8}\right.}{c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccsc(c*x))/x^8,x)
```

```
[Out] c^7*(a/c^2*(-1/7/c^5*d/x^7-1/5*e/c^5/x^5)+b/c^2*(-1/7*arccsc(c*x)/c^5*d/x^7
-1/5*arccsc(c*x)*e/c^5/x^5-1/3675*(c^2*x^2-1)*(240*c^8*d*x^6+392*c^6*e*x^6+
120*c^6*d*x^4+196*c^4*e*x^4+90*c^4*d*x^2+147*c^2*e*x^2+75*c^2*d)/((c^2*x^2-
1)/c^2/x^2)^(1/2)/c^8/x^8))
```

maxima [A] time = 0.34, size = 172, normalized size = 0.87

$$\frac{1}{245}\,bd\left(\frac{5\,c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{7}{2}}-21\,c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+35\,c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-35\,c^8\sqrt{-\frac{1}{c^2x^2}+1}}{c}-\frac{35\,\operatorname{arccsc}(cx)}{x^7}\right)-\frac{1}{75}\,be\left(3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^8,x, algorithm="maxima")
```

```
[Out] 1/245*b*d*((5*c^8*(-1/(c^2*x^2) + 1)^(7/2) - 21*c^8*(-1/(c^2*x^2) + 1)^(5/2
) + 35*c^8*(-1/(c^2*x^2) + 1)^(3/2) - 35*c^8*sqrt(-1/(c^2*x^2) + 1))/c - 35
*arccsc(c*x)/x^7) - 1/75*b*e*((3*c^6*(-1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(-1/
(c^2*x^2) + 1)^(3/2) + 15*c^6*sqrt(-1/(c^2*x^2) + 1))/c + 15*arccsc(c*x)/x^
5) - 1/5*a*e/x^5 - 1/7*a*d/x^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(a + b*asin(1/(c*x))))/x^8,x)
```

```
[Out] int(((d + e*x^2)*(a + b*asin(1/(c*x))))/x^8, x)
```

sympy [A] time = 56.15, size = 372, normalized size = 1.89

$$-\frac{ad}{7x^7}-\frac{ae}{5x^5}-\frac{bd\operatorname{acsc}(cx)}{7x^7}-\frac{be\operatorname{acsc}(cx)}{5x^5}-\frac{bd\left(\begin{cases}\frac{16c^7\sqrt{c^2x^2-1}}{35x}+\frac{8c^5\sqrt{c^2x^2-1}}{35x^3}+\frac{6c^3\sqrt{c^2x^2-1}}{35x^5}+\frac{c\sqrt{c^2x^2-1}}{7x^7} & \text{for } \left|c^2x^2\right|>1\\ \frac{16ic^7\sqrt{-c^2x^2+1}}{35x}+\frac{8ic^5\sqrt{-c^2x^2+1}}{35x^3}+\frac{6ic^3\sqrt{-c^2x^2+1}}{35x^5}+\frac{ic\sqrt{-c^2x^2+1}}{7x^7} & \text{otherwise}\end{cases}\right)}{7c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acsc(c*x))/x**8,x)
```

```
[Out] -a*d/(7*x**7) - a*e/(5*x**5) - b*d*acsc(c*x)/(7*x**7) - b*e*acsc(c*x)/(5*x*
*5) - b*d*Piecewise((16*c**7*sqrt(c**2*x**2 - 1)/(35*x) + 8*c**5*sqrt(c**2*
x**2 - 1)/(35*x**3) + 6*c**3*sqrt(c**2*x**2 - 1)/(35*x**5) + c*sqrt(c**2*x*
*2 - 1)/(7*x**7), Abs(c**2*x**2) > 1), (16*I*c**7*sqrt(-c**2*x**2 + 1)/(35*
x) + 8*I*c**5*sqrt(-c**2*x**2 + 1)/(35*x**3) + 6*I*c**3*sqrt(-c**2*x**2 + 1
)/(35*x**5) + I*c*sqrt(-c**2*x**2 + 1)/(7*x**7), True))/(7*c) - b*e*Piecewi
se((8*c**5*sqrt(c**2*x**2 - 1)/(15*x) + 4*c**3*sqrt(c**2*x**2 - 1)/(15*x**3
) + c*sqrt(c**2*x**2 - 1)/(5*x**5), Abs(c**2*x**2) > 1), (8*I*c**5*sqrt(-c*
*2*x**2 + 1)/(15*x) + 4*I*c**3*sqrt(-c**2*x**2 + 1)/(15*x**3) + I*c*sqrt(-c
**2*x**2 + 1)/(5*x**5), True))/(5*c)
```

3.83 $\int x^5 \left(d+ex^2\right) \left(a+b\csc^{-1}(cx)\right) dx$

Optimal. Leaf size=196

$$\frac{1}{6}dx^6\left(a+b\csc^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\csc^{-1}(cx)\right)+\frac{bx\left(c^2x^2-1\right)^{5/2}\left(4c^2d+9e\right)}{120c^7\sqrt{c^2x^2}}+\frac{bx\left(c^2x^2-1\right)^{3/2}\left(8c^2d+9e\right)}{72c^7\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2}}{}$$

```
[Out] 1/6*d*x^6*(a+b*arccsc(c*x))+1/8*e*x^8*(a+b*arccsc(c*x))+1/72*b*(8*c^2*d+9*e
)*x*(c^2*x^2-1)^(3/2)/c^7/(c^2*x^2)^(1/2)+1/120*b*(4*c^2*d+9*e)*x*(c^2*x^2-
1)^(5/2)/c^7/(c^2*x^2)^(1/2)+1/56*b*e*x*(c^2*x^2-1)^(7/2)/c^7/(c^2*x^2)^(1/
2)+1/24*b*(4*c^2*d+3*e)*x*(c^2*x^2-1)^(1/2)/c^7/(c^2*x^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {14, 5239, 12, 446, 77}

$$\frac{1}{6}dx^6\left(a+b\csc^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\csc^{-1}(cx)\right)+\frac{bx\left(c^2x^2-1\right)^{5/2}\left(4c^2d+9e\right)}{120c^7\sqrt{c^2x^2}}+\frac{bx\left(c^2x^2-1\right)^{3/2}\left(8c^2d+9e\right)}{72c^7\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*(4*c^2*d + 3*e)*x*Sqrt[-1 + c^2*x^2])/(24*c^7*Sqrt[c^2*x^2]) + (b*(8*c^2
*d + 9*e)*x*(-1 + c^2*x^2)^(3/2))/(72*c^7*Sqrt[c^2*x^2]) + (b*(4*c^2*d + 9*
e)*x*(-1 + c^2*x^2)^(5/2))/(120*c^7*Sqrt[c^2*x^2]) + (b*e*x*(-1 + c^2*x^2)^
(7/2))/(56*c^7*Sqrt[c^2*x^2]) + (d*x^6*(a + b*ArcCsc[c*x]))/6 + (e*x^8*(a +
 b*ArcCsc[c*x]))/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
```

```
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int x^5\left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)dx = \frac{1}{6}dx^6\left(a+b\csc^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{x^5\left(4d+3ex^2\right)}{24\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}$$

$$=\frac{1}{6}dx^6\left(a+b\csc^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{x^5\left(4d+3ex^2\right)}{\sqrt{-1+c^2x^2}}\,dx}{24\sqrt{c^2x^2}}$$

$$=\frac{1}{6}dx^6\left(a+b\csc^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\,\text{Subst}\left(\int\frac{x^2(4d+}{\sqrt{-1+}}\right.}{48\sqrt{c^2x^2}}$$

$$=\frac{1}{6}dx^6\left(a+b\csc^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\,\text{Subst}\left(\int\left(\frac{4c^2}{c^6\sqrt{-}}\right.\right.}{}$$

$$=\frac{b\left(4c^2d+3e\right)x\sqrt{-1+c^2x^2}}{24c^7\sqrt{c^2x^2}}+\frac{b\left(8c^2d+9e\right)x\left(-1+c^2x^2\right)^{3/2}}{72c^7\sqrt{c^2x^2}}+\frac{b\left(4c^2d+\right.}{}$$

Mathematica [A] time = 0.25, size = 115, normalized size = 0.59

$$\frac{x\left(105ax^5\left(4d+3ex^2\right)+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\left(c^6\left(84dx^4+45ex^6\right)+2c^4\left(56dx^2+27ex^4\right)+8c^2\left(28d+9ex^2\right)+144e\right)}{c^7}+105bx^5\csc^{-1}(cx)\left(4d+3ex\right.\right.}{2520}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (x*(105*a*x^5*(4*d + 3*e*x^2) + (b*Sqrt[1 - 1/(c^2*x^2)]*(144*e + 8*c^2*(28
*d + 9*e*x^2) + 2*c^4*(56*d*x^2 + 27*e*x^4) + c^6*(84*d*x^4 + 45*e*x^6)))/c
^7 + 105*b*x^5*(4*d + 3*e*x^2)*ArcCsc[c*x]))/2520
```

fricas [A] time = 0.71, size = 127, normalized size = 0.65

$$\frac{315\,ac^8ex^8+420\,ac^8dx^6+105\left(3\,bc^8ex^8+4\,bc^8dx^6\right)\text{arccsc}\,(cx)+\left(45\,bc^6ex^6+6\left(14\,bc^6d+9\,bc^4e\right)x^4+224\,bc\right.}{2520\,c^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/2520*(315*a*c^8*e*x^8 + 420*a*c^8*d*x^6 + 105*(3*b*c^8*e*x^8 + 4*b*c^8*d*
x^6)*arccsc(c*x) + (45*b*c^6*e*x^6 + 6*(14*b*c^6*d + 9*b*c^4*e)*x^4 + 224*b
*c^2*d + 8*(14*b*c^4*d + 9*b*c^2*e)*x^2 + 144*b*e)*sqrt(c^2*x^2 - 1))/c^8
```

giac [B] time = 0.32, size = 1270, normalized size = 6.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/645120*(315*b*x^8*(sqrt(-1/(c^2*x^2) + 1) + 1)^8*arcsin(1/(c*x))*e/c + 31
5*a*x^8*(sqrt(-1/(c^2*x^2) + 1) + 1)^8*e/c + 1680*b*d*x^6*(sqrt(-1/(c^2*x^2
) + 1) + 1)^6*arcsin(1/(c*x))/c + 90*b*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7*e
/c^2 + 1680*a*d*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6/c + 2520*b*x^6*(sqrt(-1/
(c^2*x^2) + 1) + 1)^6*arcsin(1/(c*x))*e/c^3 + 2520*a*x^6*(sqrt(-1/(c^2*x^2)
 + 1) + 1)^6*e/c^3 + 672*b*d*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5/c^2 + 10080
*b*d*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))/c^3 + 882*b*x^5*(sq
rt(-1/(c^2*x^2) + 1) + 1)^5*e/c^4 + 10080*a*d*x^4*(sqrt(-1/(c^2*x^2) + 1) +
 1)^4/c^3 + 8820*b*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))*e/c^5
 + 8820*a*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*e/c^5 + 5600*b*d*x^3*(sqrt(-1/
(c^2*x^2) + 1) + 1)^3/c^4 + 25200*b*d*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*ar
csin(1/(c*x))/c^5 + 4410*b*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*e/c^6 + 25200
*a*d*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^5 + 17640*b*x^2*(sqrt(-1/(c^2*x^2
) + 1) + 1)^2*arcsin(1/(c*x))*e/c^7 + 17640*a*x^2*(sqrt(-1/(c^2*x^2) + 1) +
 1)^2*e/c^7 + 33600*b*d*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^6 + 33600*b*d*arcs
in(1/(c*x))/c^7 + 22050*b*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*e/c^8 + 33600*a*d/
c^7 + 22050*b*arcsin(1/(c*x))*e/c^9 + 22050*a*e/c^9 - 33600*b*d/(c^8*x*(sqr
t(-1/(c^2*x^2) + 1) + 1)) + 25200*b*d*arcsin(1/(c*x))/(c^9*x^2*(sqrt(-1/(c^
2*x^2) + 1) + 1)^2) - 22050*b*e/(c^10*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 252
00*a*d/(c^9*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 17640*b*arcsin(1/(c*x))*e
/(c^11*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 17640*a*e/(c^11*x^2*(sqrt(-1/(
c^2*x^2) + 1) + 1)^2) - 5600*b*d/(c^10*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3)
+ 10080*b*d*arcsin(1/(c*x))/(c^11*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) - 441
0*b*e/(c^12*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 10080*a*d/(c^11*x^4*(sqrt
(-1/(c^2*x^2) + 1) + 1)^4) + 8820*b*arcsin(1/(c*x))*e/(c^13*x^4*(sqrt(-1/(c
^2*x^2) + 1) + 1)^4) + 8820*a*e/(c^13*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) -
 672*b*d/(c^12*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1680*b*d*arcsin(1/(c*x
))/(c^13*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6) - 882*b*e/(c^14*x^5*(sqrt(-1/(
c^2*x^2) + 1) + 1)^5) + 1680*a*d/(c^13*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6)
+ 2520*b*arcsin(1/(c*x))*e/(c^15*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6) + 2520
*a*e/(c^15*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6) - 90*b*e/(c^16*x^7*(sqrt(-1/
(c^2*x^2) + 1) + 1)^7) + 315*b*arcsin(1/(c*x))*e/(c^17*x^8*(sqrt(-1/(c^2*x^
2) + 1) + 1)^8) + 315*a*e/(c^17*x^8*(sqrt(-1/(c^2*x^2) + 1) + 1)^8))*c
```

maple [A] time = 0.05, size = 152, normalized size = 0.78

$$\frac{\frac{a\left(\frac{1}{8}e\,c^{8}x^{8}+\frac{1}{6}c^{8}d\,x^{6}\right)}{c^{2}}+\frac{b\left(\frac{\mathrm{arccsc}(cx)e\,c^{8}x^{8}}{8}+\frac{\mathrm{arccsc}(cx)c^{8}x^{6}d}{6}+\frac{(c^{2}x^{2}-1)\left(45e\,c^{6}x^{6}+84x^{4}c^{6}d+54c^{4}e\,x^{4}+112c^{4}d\,x^{2}+72c^{2}e\,x^{2}+224c^{2}d+144e\right)}{2520\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,cx}\right)}{c^{2}}}{c^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x^2+d)*(a+b*arccsc(c*x)),x)
```

```
[Out] 1/c^6*(a/c^2*(1/8*e*c^8*x^8+1/6*c^8*d*x^6)+b/c^2*(1/8*arccsc(c*x)*e*c^8*x^8
+1/6*arccsc(c*x)*c^8*x^6*d+1/2520*(c^2*x^2-1)*(45*c^6*e*x^6+84*c^6*d*x^4+54
*c^4*e*x^4+112*c^4*d*x^2+72*c^2*e*x^2+224*c^2*d+144*e)/((c^2*x^2-1)/c^2/x^2
)^(1/2)/c/x))
```

maxima [A] time = 0.34, size = 183, normalized size = 0.93

$$\frac{1}{8}aex^{8}+\frac{1}{6}adx^{6}+\frac{1}{90}\left(15\,x^{6}\,\mathrm{arccsc}\,(cx)+\frac{3\,c^{4}x^{5}\left(-\frac{1}{c^{2}x^{2}}+1\right)^{\frac{5}{2}}+10\,c^{2}x^{3}\left(-\frac{1}{c^{2}x^{2}}+1\right)^{\frac{3}{2}}+15\,x\sqrt{-\frac{1}{c^{2}x^{2}}+1}}{c^{5}}\right)bd+\frac{1}{280}\Bigg(3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*e*x^8 + 1/6*a*d*x^6 + 1/90*(15*x^6*arccsc(c*x) + (3*c^4*x^5*(-1/(c^2*
x^2) + 1)^(5/2) + 10*c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 15*x*sqrt(-1/(c^2*x
^2) + 1))/c^5)*b*d + 1/280*(35*x^8*arccsc(c*x) + (5*c^6*x^7*(-1/(c^2*x^2) +
 1)^(7/2) + 21*c^4*x^5*(-1/(c^2*x^2) + 1)^(5/2) + 35*c^2*x^3*(-1/(c^2*x^2)
+ 1)^(3/2) + 35*x*sqrt(-1/(c^2*x^2) + 1))/c^7)*b*e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\,\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d + e*x^2)*(a + b*asin(1/(c*x))),x)
```

```
[Out] int(x^5*(d + e*x^2)*(a + b*asin(1/(c*x))), x)
```

sympy [A] time = 8.36, size = 364, normalized size = 1.86

$$\frac{adx^6}{6}+\frac{aex^8}{8}+\frac{bdx^6\operatorname{acsc}(cx)}{6}+\frac{bex^8\operatorname{acsc}(cx)}{8}+\frac{bd\left(\begin{cases}\frac{x^4\sqrt{c^2x^2-1}}{5c}+\frac{4x^2\sqrt{c^2x^2-1}}{15c^3}+\frac{8\sqrt{c^2x^2-1}}{15c^5} & \text{for } \left|c^2x^2\right|>1\\ \frac{ix^4\sqrt{-c^2x^2+1}}{5c}+\frac{4ix^2\sqrt{-c^2x^2+1}}{15c^3}+\frac{8i\sqrt{-c^2x^2+1}}{15c^5} & \text{otherwise}\end{cases}\right)}{6c}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x**2+d)*(a+b*acsc(c*x)),x)
```

```
[Out] a*d*x**6/6 + a*e*x**8/8 + b*d*x**6*acsc(c*x)/6 + b*e*x**8*acsc(c*x)/8 + b*d
*Piecewise((x**4*sqrt(c**2*x**2 - 1)/(5*c) + 4*x**2*sqrt(c**2*x**2 - 1)/(15
*c**3) + 8*sqrt(c**2*x**2 - 1)/(15*c**5), Abs(c**2*x**2) > 1), (I*x**4*sqrt
(-c**2*x**2 + 1)/(5*c) + 4*I*x**2*sqrt(-c**2*x**2 + 1)/(15*c**3) + 8*I*sqrt
(-c**2*x**2 + 1)/(15*c**5), True))/(6*c) + b*e*Piecewise((x**6*sqrt(c**2*x*
*2 - 1)/(7*c) + 6*x**4*sqrt(c**2*x**2 - 1)/(35*c**3) + 8*x**2*sqrt(c**2*x**
2 - 1)/(35*c**5) + 16*sqrt(c**2*x**2 - 1)/(35*c**7), Abs(c**2*x**2) > 1), (
I*x**6*sqrt(-c**2*x**2 + 1)/(7*c) + 6*I*x**4*sqrt(-c**2*x**2 + 1)/(35*c**3)
 + 8*I*x**2*sqrt(-c**2*x**2 + 1)/(35*c**5) + 16*I*sqrt(-c**2*x**2 + 1)/(35*
c**7), True))/(8*c)
```

3.84 $\int x^3 \left(d+ex^2\right) \left(a+b\csc^{-1}(cx)\right)\,dx$

Optimal. Leaf size=153

$$\frac{1}{4}dx^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\csc^{-1}(cx)\right)+\frac{bx\left(c^2x^2-1\right)^{3/2}\left(3c^2d+4e\right)}{36c^5\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2x^2-1}\left(3c^2d+2e\right)}{12c^5\sqrt{c^2x^2}}+\frac{bex\left(c^2x^2\right.}{30c^5\sqrt{}}$$

```
[Out] 1/4*d*x^4*(a+b*arccsc(c*x))+1/6*e*x^6*(a+b*arccsc(c*x))+1/36*b*(3*c^2*d+4*e
)*x*(c^2*x^2-1)^(3/2)/c^5/(c^2*x^2)^(1/2)+1/30*b*e*x*(c^2*x^2-1)^(5/2)/c^5/
(c^2*x^2)^(1/2)+1/12*b*(3*c^2*d+2*e)*x*(c^2*x^2-1)^(1/2)/c^5/(c^2*x^2)^(1/2
)
```

Rubi [A] time = 0.12, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {14, 5239, 12, 446, 77}

$$\frac{1}{4}dx^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\csc^{-1}(cx)\right)+\frac{bx\left(c^2x^2-1\right)^{3/2}\left(3c^2d+4e\right)}{36c^5\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2x^2-1}\left(3c^2d+2e\right)}{12c^5\sqrt{c^2x^2}}+\frac{bex\left(c^2x^2\right.}{30c^5\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*(3*c^2*d + 2*e)*x*Sqrt[-1 + c^2*x^2])/(12*c^5*Sqrt[c^2*x^2]) + (b*(3*c^2
*d + 4*e)*x*(-1 + c^2*x^2)^(3/2))/(36*c^5*Sqrt[c^2*x^2]) + (b*e*x*(-1 + c^2
*x^2)^(5/2))/(30*c^5*Sqrt[c^2*x^2]) + (d*x^4*(a + b*ArcCsc[c*x]))/4 + (e*x^
6*(a + b*ArcCsc[c*x]))/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
```

```
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int x^3\left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)dx=\frac{1}{4}dx^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{x^3\left(3d+2ex^2\right)}{12\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}$$

$$=\frac{1}{4}dx^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{x^3\left(3d+2ex^2\right)}{\sqrt{-1+c^2x^2}}\,dx}{12\sqrt{c^2x^2}}$$

$$=\frac{1}{4}dx^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\,\mathrm{Subst}\left(\int\frac{x(3d+2}{\sqrt{-1+c}}\right.}{24\sqrt{c^2x^2}}$$

$$=\frac{1}{4}dx^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\,\mathrm{Subst}\left(\int\left(\frac{3c^2}{c^4\sqrt{-}}\right.\right.}{}$$

$$=\frac{b\left(3c^2d+2e\right)x\sqrt{-1+c^2x^2}}{12c^5\sqrt{c^2x^2}}+\frac{b\left(3c^2d+4e\right)x\left(-1+c^2x^2\right)^{3/2}}{36c^5\sqrt{c^2x^2}}+\frac{bex\left(-1\right.}{30c^5}$$

Mathematica [A] time = 0.22, size = 97, normalized size = 0.63

$$\frac{1}{180}x\left(15ax^3\left(3d+2ex^2\right)+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\left(3c^4\left(5dx^2+2ex^4\right)+c^2\left(30d+8ex^2\right)+16e\right)}{c^5}+15bx^3\csc^{-1}(cx)\left(3d+2\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (x*(15*a*x^3*(3*d + 2*e*x^2) + (b*Sqrt[1 - 1/(c^2*x^2)]*(16*e + c^2*(30*d +
 8*e*x^2) + 3*c^4*(5*d*x^2 + 2*e*x^4)))/c^5 + 15*b*x^3*(3*d + 2*e*x^2)*ArcC
sc[c*x]))/180
```

fricas [A] time = 0.60, size = 106, normalized size = 0.69

$$\frac{30\,ac^6ex^6+45\,ac^6dx^4+15\left(2\,bc^6ex^6+3\,bc^6dx^4\right)\operatorname{arccsc}(cx)+\left(6\,bc^4ex^4+30\,bc^2d+\left(15\,bc^4d+8\,bc^2e\right)x^2+16\right.}{180\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/180*(30*a*c^6*e*x^6 + 45*a*c^6*d*x^4 + 15*(2*b*c^6*e*x^6 + 3*b*c^6*d*x^4)
*arccsc(c*x) + (6*b*c^4*e*x^4 + 30*b*c^2*d + (15*b*c^4*d + 8*b*c^2*e)*x^2 +
 16*b*e)*sqrt(c^2*x^2 - 1))/c^6
```

giac [B] time = 0.26, size = 920, normalized size = 6.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/5760*(15*b*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*arcsin(1/(c*x))*e/c + 15*a*
x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*e/c + 90*b*d*x^4*(sqrt(-1/(c^2*x^2) + 1)
 + 1)^4*arcsin(1/(c*x))/c + 6*b*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*e/c^2 +
90*a*d*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c + 90*b*x^4*(sqrt(-1/(c^2*x^2) +
 1) + 1)^4*arcsin(1/(c*x))*e/c^3 + 90*a*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*
e/c^3 + 60*b*d*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^2 + 360*b*d*x^2*(sqrt(-
1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c^3 + 50*b*x^3*(sqrt(-1/(c^2*x^2) +
 1) + 1)^3*e/c^4 + 360*a*d*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^3 + 225*b*x
^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))*e/c^5 + 225*a*x^2*(sqrt(-
1/(c^2*x^2) + 1) + 1)^2*e/c^5 + 540*b*d*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4
+ 540*b*d*arcsin(1/(c*x))/c^5 + 300*b*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*e/c^6
+ 540*a*d/c^5 + 300*b*arcsin(1/(c*x))*e/c^7 + 300*a*e/c^7 - 540*b*d/(c^6*x*
(sqrt(-1/(c^2*x^2) + 1) + 1)) + 360*b*d*arcsin(1/(c*x))/(c^7*x^2*(sqrt(-1/(
c^2*x^2) + 1) + 1)^2) - 300*b*e/(c^8*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 360*
a*d/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 225*b*arcsin(1/(c*x))*e/(c^9
*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 225*a*e/(c^9*x^2*(sqrt(-1/(c^2*x^2)
+ 1) + 1)^2) - 60*b*d/(c^8*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 90*b*d*arc
sin(1/(c*x))/(c^9*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) - 50*b*e/(c^10*x^3*(s
qrt(-1/(c^2*x^2) + 1) + 1)^3) + 90*a*d/(c^9*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1
)^4) + 90*b*arcsin(1/(c*x))*e/(c^11*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) + 9
0*a*e/(c^11*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) - 6*b*e/(c^12*x^5*(sqrt(-1/
(c^2*x^2) + 1) + 1)^5) + 15*b*arcsin(1/(c*x))*e/(c^13*x^6*(sqrt(-1/(c^2*x^2
) + 1) + 1)^6) + 15*a*e/(c^13*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6))*c
```

maple [A] time = 0.05, size = 134, normalized size = 0.88

$$\frac{\frac{a\left(\frac{1}{6}ec^6x^6+\frac{1}{4}x^4c^6d\right)}{c^2}+\frac{b\left(\frac{\operatorname{arccsc}(cx)ec^6x^6}{6}+\frac{\operatorname{arccsc}(cx)c^6x^4d}{4}+\frac{(c^2x^2-1)(6c^4ex^4+15c^4dx^2+8c^2ex^2+30c^2d+16e)}{180\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^2}}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)*(a+b*arccsc(c*x)),x)
```

```
[Out] 1/c^4*(a/c^2*(1/6*e*c^6*x^6+1/4*x^4*c^6*d)+b/c^2*(1/6*arccsc(c*x)*e*c^6*x^6
+1/4*arccsc(c*x)*c^6*x^4*d+1/180*(c^2*x^2-1)*(6*c^4*e*x^4+15*c^4*d*x^2+8*c^
2*e*x^2+30*c^2*d+16*e)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x))
```

maxima [A] time = 0.33, size = 142, normalized size = 0.93

$$\frac{1}{6}aex^6+\frac{1}{4}adx^4+\frac{1}{12}\left(3x^4\operatorname{arccsc}(cx)+\frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+3x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)bd+\frac{1}{90}\left(15x^6\operatorname{arccsc}(cx)+\frac{3c^4x^5\left(-\cdots\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*e*x^6 + 1/4*a*d*x^4 + 1/12*(3*x^4*arccsc(c*x) + (c^2*x^3*(-1/(c^2*x^2
) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*b*d + 1/90*(15*x^6*arccsc(c
*x) + (3*c^4*x^5*(-1/(c^2*x^2) + 1)^(5/2) + 10*c^2*x^3*(-1/(c^2*x^2) + 1)^(
3/2) + 15*x*sqrt(-1/(c^2*x^2) + 1))/c^5)*b*e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d + e*x^2)*(a + b*asin(1/(c*x))),x)
```

```
[Out] int(x^3*(d + e*x^2)*(a + b*asin(1/(c*x))), x)
```

sympy [A] time = 5.55, size = 272, normalized size = 1.78

$$\frac{adx^4}{4}+\frac{aex^6}{6}+\frac{bdx^4\operatorname{acsc}(cx)}{4}+\frac{bex^6\operatorname{acsc}(cx)}{6}+\frac{bd\left(\begin{cases}\frac{x^2\sqrt{c^2x^2-1}}{3c}+\frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for } \left|c^2x^2\right|>1\\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c}+\frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise}\end{cases}\right)}{4c}+\frac{be\left(\begin{cases}\frac{x^4\sqrt{c^2x^2-1}}{5c} \\ \frac{ix^4\sqrt{-c^2x^2+1}}{5c}\end{cases}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)*(a+b*acsc(c*x)),x)
```

```
[Out] a*d*x**4/4 + a*e*x**6/6 + b*d*x**4*acsc(c*x)/4 + b*e*x**6*acsc(c*x)/6 + b*d
*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3*c) + 2*sqrt(c**2*x**2 - 1)/(3*c**3)
, Abs(c**2*x**2) > 1), (I*x**2*sqrt(-c**2*x**2 + 1)/(3*c) + 2*I*sqrt(-c**2*
x**2 + 1)/(3*c**3), True))/(4*c) + b*e*Piecewise((x**4*sqrt(c**2*x**2 - 1)/
(5*c) + 4*x**2*sqrt(c**2*x**2 - 1)/(15*c**3) + 8*sqrt(c**2*x**2 - 1)/(15*c*
*5), Abs(c**2*x**2) > 1), (I*x**4*sqrt(-c**2*x**2 + 1)/(5*c) + 4*I*x**2*sqr
t(-c**2*x**2 + 1)/(15*c**3) + 8*I*sqrt(-c**2*x**2 + 1)/(15*c**5), True))/(6
*c)
```

3.85 $\int x\left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)\,dx$

Optimal. Leaf size=138

$$\frac{\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)}{4e}+\frac{bcd^2x\tan^{-1}\left(\sqrt{c^2x^2-1}\right)}{4e\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2x^2-1}\left(2c^2d+e\right)}{4c^3\sqrt{c^2x^2}}+\frac{bex\left(c^2x^2-1\right)^{3/2}}{12c^3\sqrt{c^2x^2}}$$

```
[Out] 1/4*(e*x^2+d)^2*(a+b*arccsc(c*x))/e+1/12*b*e*x*(c^2*x^2-1)^(3/2)/c^3/(c^2*x
^2)^(1/2)+1/4*b*c*d^2*x*arctan((c^2*x^2-1)^(1/2))/e/(c^2*x^2)^(1/2)+1/4*b*(
2*c^2*d+e)*x*(c^2*x^2-1)^(1/2)/c^3/(c^2*x^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {5237, 446, 88, 63, 205}

$$\frac{\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)}{4e}+\frac{bcd^2x\tan^{-1}\left(\sqrt{c^2x^2-1}\right)}{4e\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2x^2-1}\left(2c^2d+e\right)}{4c^3\sqrt{c^2x^2}}+\frac{bex\left(c^2x^2-1\right)^{3/2}}{12c^3\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*(2*c^2*d + e)*x*Sqrt[-1 + c^2*x^2])/(4*c^3*Sqrt[c^2*x^2]) + (b*e*x*(-1 +
 c^2*x^2)^(3/2))/(12*c^3*Sqrt[c^2*x^2]) + ((d + e*x^2)^2*(a + b*ArcCsc[c*x]
))/(4*e) + (b*c*d^2*x*ArcTan[Sqrt[-1 + c^2*x^2]])/(4*e*Sqrt[c^2*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5237

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCsc[c*x]))/(2*e*(p + 1)), x
] + Dist[(b*c*x)/(2*e*(p + 1)*Sqrt[c^2*x^2]), Int[(d + e*x^2)^(p + 1)/(x*Sq
```

```
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\int x\left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)dx = \frac{\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)}{4e} + \frac{(bcx)\int\frac{\left(d+ex^2\right)^2}{x\sqrt{-1+c^2x^2}}\,dx}{4e\sqrt{c^2x^2}}$$

$$= \frac{\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)}{4e} + \frac{(bcx)\,\text{Subst}\left(\int\frac{(d+ex)^2}{x\sqrt{-1+c^2x}}\,dx,x,x^2\right)}{8e\sqrt{c^2x^2}}$$

$$= \frac{\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)}{4e} + \frac{(bcx)\,\text{Subst}\left(\int\left(\frac{e\left(2c^2d+e\right)}{c^2\sqrt{-1+c^2x}}+\frac{d^2}{x\sqrt{-1+c^2x}}+\frac{e}{}\right.\right.}{8e\sqrt{c^2x^2}}$$

$$= \frac{b\left(2c^2d+e\right)x\sqrt{-1+c^2x^2}}{4c^3\sqrt{c^2x^2}} + \frac{bex\left(-1+c^2x^2\right)^{3/2}}{12c^3\sqrt{c^2x^2}} + \frac{\left(d+ex^2\right)^2\left(a+b\csc^{-1}\right.}{4e}$$

$$= \frac{b\left(2c^2d+e\right)x\sqrt{-1+c^2x^2}}{4c^3\sqrt{c^2x^2}} + \frac{bex\left(-1+c^2x^2\right)^{3/2}}{12c^3\sqrt{c^2x^2}} + \frac{\left(d+ex^2\right)^2\left(a+b\csc^{-1}\right.}{4e}$$

$$= \frac{b\left(2c^2d+e\right)x\sqrt{-1+c^2x^2}}{4c^3\sqrt{c^2x^2}} + \frac{bex\left(-1+c^2x^2\right)^{3/2}}{12c^3\sqrt{c^2x^2}} + \frac{\left(d+ex^2\right)^2\left(a+b\csc^{-1}\right.}{4e}$$

Mathematica [A] time = 0.10, size = 78, normalized size = 0.57

$$\frac{x\left(3ac^3x\left(2d+ex^2\right)+3bc^3x\csc^{-1}(cx)\left(2d+ex^2\right)+b\sqrt{1-\frac{1}{c^2x^2}}\left(c^2\left(6d+ex^2\right)+2e\right)\right)}{12c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (x*(3*a*c^3*x*(2*d + e*x^2) + b*Sqrt[1 - 1/(c^2*x^2)]*(2*e + c^2*(6*d + e*x
^2)) + 3*b*c^3*x*(2*d + e*x^2)*ArcCsc[c*x]))/(12*c^3)
```

fricas [A] time = 0.89, size = 85, normalized size = 0.62

$$\frac{3\,ac^4ex^4+6\,ac^4dx^2+3\left(bc^4ex^4+2\,bc^4dx^2\right)\text{arccsc}\left(cx\right)+\left(bc^2ex^2+6\,bc^2d+2\,be\right)\sqrt{c^2x^2-1}}{12\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*c^4*e*x^4 + 6*a*c^4*d*x^2 + 3*(b*c^4*e*x^4 + 2*b*c^4*d*x^2)*arccs
c(c*x) + (b*c^2*e*x^2 + 6*b*c^2*d + 2*b*e)*sqrt(c^2*x^2 - 1))/c^4
```

giac [B] time = 0.22, size = 570, normalized size = 4.13

$$\frac{1}{192}\left(\frac{3\,bx^4\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^4\arcsin\left(\frac{1}{cx}\right)e}{c}+\frac{3\,ax^4\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^4e}{c}+\frac{24\,bdx^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2\arcsin\left(\right.}{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/192*(3*b*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))*e/c + 3*a*x^4
*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*e/c + 24*b*d*x^2*(sqrt(-1/(c^2*x^2) + 1) +
1)^2*arcsin(1/(c*x))/c + 2*b*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*e/c^2 + 24*
a*d*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c + 12*b*x^2*(sqrt(-1/(c^2*x^2) + 1)
 + 1)^2*arcsin(1/(c*x))*e/c^3 + 12*a*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*e/c
^3 + 48*b*d*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 + 48*b*d*arcsin(1/(c*x))/c^3
 + 18*b*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*e/c^4 + 48*a*d/c^3 + 18*b*arcsin(1/(
c*x))*e/c^5 + 18*a*e/c^5 - 48*b*d/(c^4*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 24
*b*d*arcsin(1/(c*x))/(c^5*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) - 18*b*e/(c^6
*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 24*a*d/(c^5*x^2*(sqrt(-1/(c^2*x^2) + 1)
+ 1)^2) + 12*b*arcsin(1/(c*x))*e/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) +
 12*a*e/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) - 2*b*e/(c^8*x^3*(sqrt(-1/
(c^2*x^2) + 1) + 1)^3) + 3*b*arcsin(1/(c*x))*e/(c^9*x^4*(sqrt(-1/(c^2*x^2)
+ 1) + 1)^4) + 3*a*e/(c^9*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4))*c
```

maple [A] time = 0.05, size = 115, normalized size = 0.83

$$\frac{\frac{a\left(\frac{1}{4}c^4 e\,x^4+\frac{1}{2}c^4 d\,x^2\right)}{c^2}+\frac{b\left(\frac{\operatorname{arccsc}(cx) e\,c^4 x^4}{4}+\frac{\operatorname{arccsc}(cx) c^4 d\,x^2}{2}+\frac{\left(c^2x^2-1\right)\left(c^2 e\,x^2+6c^2 d+2e\right)}{12\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^2}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)*(a+b*arccsc(c*x)),x)
```

```
[Out] 1/c^2*(a/c^2*(1/4*c^4*e*x^4+1/2*c^4*d*x^2)+b/c^2*(1/4*arccsc(c*x)*e*c^4*x^4
+1/2*arccsc(c*x)*c^4*d*x^2+1/12*(c^2*x^2-1)*(c^2*e*x^2+6*c^2*d+2*e)/((c^2*x
^2-1)/c^2/x^2)^(1/2)/c/x))
```

maxima [A] time = 0.34, size = 98, normalized size = 0.71

$$\frac{1}{4}\,aex^4+\frac{1}{2}\,adx^2+\frac{1}{2}\left(x^2\operatorname{arccsc}\left(cx\right)+\frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)bd+\frac{1}{12}\left(3\,x^4\operatorname{arccsc}\left(cx\right)+\frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+3\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*e*x^4 + 1/2*a*d*x^2 + 1/2*(x^2*arccsc(c*x) + x*sqrt(-1/(c^2*x^2) + 1)
/c)*b*d + 1/12*(3*x^4*arccsc(c*x) + (c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 3*x
*sqrt(-1/(c^2*x^2) + 1))/c^3)*b*e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d + e*x^2)*(a + b*asin(1/(c*x))),x)
```

```
[Out] int(x*(d + e*x^2)*(a + b*asin(1/(c*x))), x)
```

sympy [A] time = 3.78, size = 177, normalized size = 1.28

$$\frac{adx^2}{2}+\frac{aex^4}{4}+\frac{bdx^2\operatorname{acsc}(cx)}{2}+\frac{bex^4\operatorname{acsc}(cx)}{4}+\frac{bd\left(\begin{cases}\frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2|>1\\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)}{2c}+\frac{be\left(\begin{cases}\frac{x^2\sqrt{c^2x^2-1}}{3c}+\frac{2\sqrt{c^2x^2-1}}{3c^3}\\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c}+\frac{2i\sqrt{-c^2x^2+1}}{3c^3}\end{cases}\right.}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)*(a+b*acsc(c*x)),x)
```

```
[Out] a*d*x**2/2 + a*e*x**4/4 + b*d*x**2*acsc(c*x)/2 + b*e*x**4*acsc(c*x)/4 + b*d
*Piecewise((sqrt(c**2*x**2 - 1)/c, Abs(c**2*x**2) > 1), (I*sqrt(-c**2*x**2
+ 1)/c, True))/(2*c) + b*e*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3*c) + 2*sq
rt(c**2*x**2 - 1)/(3*c**3), Abs(c**2*x**2) > 1), (I*x**2*sqrt(-c**2*x**2 +
1)/(3*c) + 2*I*sqrt(-c**2*x**2 + 1)/(3*c**3), True))/(4*c)
```

3.86 $\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=124

$$-d\log\left(\frac{1}{x}\right)\left(a+b\csc^{-1}(cx)\right)+\frac{1}{2}ex^2\left(a+b\csc^{-1}(cx)\right)+\frac{bex\sqrt{1-\frac{1}{c^2x^2}}}{2c}+\frac{1}{2}ibd\mathrm{Li}_2\left(e^{2i\csc^{-1}(cx)}\right)+\frac{1}{2}ibd\csc^{-1}(cx)^2-bd\,\mathrm{cs}$$

```
[Out] 1/2*I*b*d*arccsc(c*x)^2+1/2*e*x^2*(a+b*arccsc(c*x))-b*d*arccsc(c*x)*ln(1-(I
/c/x+(1-1/c^2/x^2)^(1/2))^2)+b*d*arccsc(c*x)*ln(1/x)-d*(a+b*arccsc(c*x))*ln
(1/x)+1/2*I*b*d*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)+1/2*b*e*x*(1-1/c^2
/x^2)^(1/2)/c
```

Rubi [A] time = 0.29, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {5241, 14, 4731, 6742, 264, 2326, 4625, 3717, 2190, 2279, 2391}

$$\frac{1}{2}ibd\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)-d\log\left(\frac{1}{x}\right)\left(a+b\csc^{-1}(cx)\right)+\frac{1}{2}ex^2\left(a+b\csc^{-1}(cx)\right)+\frac{bex\sqrt{1-\frac{1}{c^2x^2}}}{2c}+\frac{1}{2}ibd\csc^{-1}(cx)^2$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x,x]
```

```
[Out] (b*e*Sqrt[1 - 1/(c^2*x^2)]*x)/(2*c) + (I/2)*b*d*ArcCsc[c*x]^2 + (e*x^2*(a +
 b*ArcCsc[c*x]))/2 - b*d*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] + b*d*A
rcCsc[c*x]*Log[x^(-1)] - d*(a + b*ArcCsc[c*x])*Log[x^(-1)] + (I/2)*b*d*Poly
Log[2, E^((2*I)*ArcCsc[c*x])]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2326

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[(ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/Rt[-e, 2], x
```

```
] - Dist[(b*n)/Rt[-e, 2], Int[ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4625

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n/Tan[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 4731

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSin[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 - c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e, 0] &
& IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Rule 5241

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSin[x/c])^n)/x^
(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x}\,dx = -\text{Subst}\left(\int \frac{(e+dx^2)\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{x^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}ex^2(a+b\csc^{-1}(cx)) - d(a+b\csc^{-1}(cx))\log\left(\frac{1}{x}\right) + \frac{b\,\text{Subst}\left(\int \frac{-\frac{e}{2x^2}+d\log(x)}{\sqrt{1-\frac{x^2}{c^2}}}\right.}{c}$$

$$= \frac{1}{2}ex^2(a+b\csc^{-1}(cx)) - d(a+b\csc^{-1}(cx))\log\left(\frac{1}{x}\right) + \frac{b\,\text{Subst}\left(\int\left(-\frac{e}{2x^2\sqrt{1-\frac{x^2}{c^2}}}\right.\right.}{c}$$

$$= \frac{1}{2}ex^2(a+b\csc^{-1}(cx)) - d(a+b\csc^{-1}(cx))\log\left(\frac{1}{x}\right) + \frac{(bd)\,\text{Subst}\left(\int \frac{\log(x)}{\sqrt{1-\frac{x^2}{c^2}}}\right.}{c}$$

$$= \frac{be\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} + \frac{1}{2}ex^2(a+b\csc^{-1}(cx)) + bd\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - d(a+b\csc$$

$$= \frac{be\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} + \frac{1}{2}ex^2(a+b\csc^{-1}(cx)) + bd\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - d(a+b\csc$$

$$= \frac{be\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} + \frac{1}{2}ibd\csc^{-1}(cx)^2 + \frac{1}{2}ex^2(a+b\csc^{-1}(cx)) + bd\csc^{-1}(cx)\log($$

$$= \frac{be\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} + \frac{1}{2}ibd\csc^{-1}(cx)^2 + \frac{1}{2}ex^2(a+b\csc^{-1}(cx)) - bd\csc^{-1}(cx)\log($$

$$= \frac{be\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} + \frac{1}{2}ibd\csc^{-1}(cx)^2 + \frac{1}{2}ex^2(a+b\csc^{-1}(cx)) - bd\csc^{-1}(cx)\log($$

$$= \frac{be\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} + \frac{1}{2}ibd\csc^{-1}(cx)^2 + \frac{1}{2}ex^2(a+b\csc^{-1}(cx)) - bd\csc^{-1}(cx)\log($$

Mathematica [A] time = 0.09, size = 108, normalized size = 0.87

$$ad\log(x)+\frac{1}{2}aex^2+\frac{bex\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{2c}+\frac{1}{2}ibd\left(\csc^{-1}(cx)^2+\text{Li}_2\left(e^{2i\csc^{-1}(cx)}\right)\right)-bd\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)+\frac{1}{2}bex^2\csc$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x,x]
```

```
[Out] (a*e*x^2)/2 + (b*e*x*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(2*c) + (b*e*x^2*ArcCs
c[c*x])/2 - b*d*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] + a*d*Log[x] + (
I/2)*b*d*(ArcCsc[c*x]^2 + PolyLog[2, E^((2*I)*ArcCsc[c*x])])
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{aex^2+ad+(bex^2+bd)\,\text{arccsc}(cx)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsc(c*x))/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(x)]Undef/Unsigne
d Inf encountered in limitEvaluation time: 0.55Limit: Max order reached or
unable to make series expansion Error: Bad Argument Value
```

maple [A] time = 3.30, size = 198, normalized size = 1.60

$$\frac{a\,x^{2}e}{2}+da\ln\left(cx\right)+\frac{ibd\mathrm{arccsc}\left(cx\right)^{2}}{2}+\frac{b\,\mathrm{arccsc}\left(cx\right)x^{2}e}{2}+\frac{b\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\;xe}{2c}-\frac{ibe}{2c^{2}}-bd\,\mathrm{arccsc}\left(cx\right)\ln\left(1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^{2}x}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccsc(c*x))/x,x)
```

```
[Out] 1/2*a*x^2*e+d*a*ln(c*x)+1/2*I*b*d*arccsc(c*x)^2+1/2*b*arccsc(c*x)*x^2*e+1/2
*b/c*((c^2*x^2-1)/c^2/x^2)^(1/2)*x*e-1/2*I*b/c^2*e-b*d*arccsc(c*x)*ln(1+I/c
/x+(1-1/c^2/x^2)^(1/2))-b*d*arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))+I*b
*d*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))+I*b*d*polylog(2,I/c/x+(1-1/c^2/x^2
)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,aex^{2}+ad\log(x)+\frac{2i\,bc^{2}d\log\left(-cx+1\right)\log(x)+2i\,bc^{2}d\log(x)^{2}+2i\,bc^{2}d\mathrm{Li}_{2}\left(cx\right)+2i\,bc^{2}d\mathrm{Li}_{2}\left(-cx\right)+\left(2\,bc^{2}\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/2*a*e*x^2 + a*d*log(x) + 1/4*(2*I*b*c^2*d*log(-c*x + 1)*log(x) + 2*I*b*c^
2*d*log(x)^2 + 2*I*b*c^2*d*dilog(c*x) + 2*I*b*c^2*d*dilog(-c*x) + (2*b*c^2*
arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 2*I*b*c^2*log(c))*e*x^2 - I*(b*e*
(log(c*x + 1)/c^2 + log(c*x - 1)/c^2) + 8*b*d*integrate(1/2*log(x)/(c^2*x^3
 - x), x))*c^2 + 4*c^2*integrate(1/2*(b*e*x^2 + 2*b*d*log(x))*sqrt(c*x + 1)
*sqrt(c*x - 1)/(c^2*x^3 - x), x) + I*b*e*log(c*x - 1) + (-I*b*c^2*e*x^2 - 2
*I*b*c^2*d*log(x))*log(c^2*x^2) + (2*I*b*c^2*d*log(x) + I*b*e)*log(c*x + 1)
 + (2*I*b*c^2*e*x^2 + (4*b*c^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*
I*b*c^2*log(c))*d)*log(x))/c^2
```

mupad [B] time = 0.92, size = 111, normalized size = 0.90

$$\frac{a\,e\,x^{2}}{2}-a\,d\,\ln\left(\frac{1}{x}\right)-b\,d\,\ln\left(1-\mathrm{e}^{\mathrm{asin}\left(\frac{1}{c\,x}\right)2\mathrm{i}}\right)\,\mathrm{asin}\left(\frac{1}{c\,x}\right)+\frac{b\,e\,x\,\left(\sqrt{1-\frac{1}{c^{2}\,x^{2}}}+c\,x\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{2\,c}+\frac{b\,d\,\mathrm{polylog}\left(2,\mathrm{e}^{\mathrm{asin}\left(\right.}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(a + b*asin(1/(c*x))))/x,x)
```

```
[Out] (b*d*polylog(2, exp(asin(1/(c*x))*2i))*1i)/2 - a*d*log(1/x) + (b*d*asin(1/(
c*x))^2*1i)/2 + (a*e*x^2)/2 - b*d*log(1 - exp(asin(1/(c*x))*2i))*asin(1/(c*
x)) + (b*e*x*((1 - 1/(c^2*x^2))^(1/2) + c*x*asin(1/(c*x))))/(2*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{acsc}\,(cx))\left(d + ex^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acsc(c*x))/x,x)
```

```
[Out] Integral((a + b*acsc(c*x))*(d + e*x**2)/x, x)
```

3.87 $\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^3}\,dx$

Optimal. Leaf size=137

$$-\frac{d\left(a+b\csc^{-1}(cx)\right)}{2x^2}-e\log\left(\frac{1}{x}\right)\left(a+b\csc^{-1}(cx)\right)-\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x}+\frac{1}{4}bc^2d\csc^{-1}(cx)+\frac{1}{2}ibe\text{Li}_2\left(e^{2i\csc^{-1}(cx)}\right)+\frac{1}{2}ibe$$

```
[Out] 1/4*b*c^2*d*arccsc(c*x)+1/2*I*b*e*arccsc(c*x)^2-1/2*d*(a+b*arccsc(c*x))/x^2
-b*e*arccsc(c*x)*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)+b*e*arccsc(c*x)*ln(1/x
)-e*(a+b*arccsc(c*x))*ln(1/x)+1/2*I*b*e*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2
))^2)-1/4*b*c*d*(1-1/c^2/x^2)^(1/2)/x
```

Rubi [A] time = 0.30, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {5241, 14, 4731, 12, 6742, 321, 216, 2326, 4625, 3717, 2190, 2279, 2391}

$$\frac{1}{2}ibe\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)-\frac{d\left(a+b\csc^{-1}(cx)\right)}{2x^2}-e\log\left(\frac{1}{x}\right)\left(a+b\csc^{-1}(cx)\right)-\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x}+\frac{1}{4}bc^2d\csc^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^3,x]
```

```
[Out] -(b*c*d*Sqrt[1 - 1/(c^2*x^2)])/(4*x) + (b*c^2*d*ArcCsc[c*x])/4 + (I/2)*b*e*
ArcCsc[c*x]^2 - (d*(a + b*ArcCsc[c*x]))/(2*x^2) - b*e*ArcCsc[c*x]*Log[1 - E
^((2*I)*ArcCsc[c*x])] + b*e*ArcCsc[c*x]*Log[x^(-1)] - e*(a + b*ArcCsc[c*x])
*Log[x^(-1)] + (I/2)*b*e*PolyLog[2, E^((2*I)*ArcCsc[c*x])]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
```

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2326

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[(ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/Rt[-e, 2], x
] - Dist[(b*n)/Rt[-e, 2], Int[ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4625

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n/Tan[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 4731

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSin[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 - c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e, 0] &
& IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Rule 5241

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSin[x/c])^n)/x^
(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)}{x^3}\,dx = -\text{Subst}\left(\int \frac{\left(e+dx^2\right)\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{x}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{d\left(a+b\csc^{-1}(cx)\right)}{2x^2} - e\left(a+b\csc^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{b\,\text{Subst}\left(\int \frac{dx^2+2e\log(x}{2\sqrt{1-\frac{x^2}{c^2}}}\right.}{c}$$

$$= -\frac{d\left(a+b\csc^{-1}(cx)\right)}{2x^2} - e\left(a+b\csc^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{b\,\text{Subst}\left(\int \frac{dx^2+2e\log(x}{\sqrt{1-\frac{x^2}{c^2}}}\right.}{2c}$$

$$= -\frac{d\left(a+b\csc^{-1}(cx)\right)}{2x^2} - e\left(a+b\csc^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{b\,\text{Subst}\left(\int\left(\frac{dx^2}{\sqrt{1-\frac{x^2}{c^2}}}+\right.\right.}{2c}$$

$$= -\frac{d\left(a+b\csc^{-1}(cx)\right)}{2x^2} - e\left(a+b\csc^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{(bd)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^2}{c^2}}}\right.}{2c}$$

$$= -\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{d\left(a+b\csc^{-1}(cx)\right)}{2x^2} + be\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - e\left(a+b\csc\right.$$

$$= -\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} + \frac{1}{4}bc^2d\csc^{-1}(cx) - \frac{d\left(a+b\csc^{-1}(cx)\right)}{2x^2} + be\csc^{-1}(cx)\log$$

$$= -\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} + \frac{1}{4}bc^2d\csc^{-1}(cx) + \frac{1}{2}ibe\csc^{-1}(cx)^2 - \frac{d\left(a+b\csc^{-1}(cx)\right)}{2x^2}$$

$$= -\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} + \frac{1}{4}bc^2d\csc^{-1}(cx) + \frac{1}{2}ibe\csc^{-1}(cx)^2 - \frac{d\left(a+b\csc^{-1}(cx)\right)}{2x^2}$$

$$= -\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} + \frac{1}{4}bc^2d\csc^{-1}(cx) + \frac{1}{2}ibe\csc^{-1}(cx)^2 - \frac{d\left(a+b\csc^{-1}(cx)\right)}{2x^2}$$

$$= -\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} + \frac{1}{4}bc^2d\csc^{-1}(cx) + \frac{1}{2}ibe\csc^{-1}(cx)^2 - \frac{d\left(a+b\csc^{-1}(cx)\right)}{2x^2}$$

Mathematica [A] time = 0.10, size = 125, normalized size = 0.91

$$-\frac{ad}{2x^2}+ae\log(x)-\frac{bcd\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{4x}+\frac{1}{4}bc^2d\sin^{-1}\left(\frac{1}{cx}\right)-\frac{bd\csc^{-1}(cx)}{2x^2}+\frac{1}{2}ibe\left(\csc^{-1}(cx)^2+\text{Li}_2\left(e^{2i\csc^{-1}(cx)}\right)\right)-be\csc^{-1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^3,x]
```

```
[Out] -1/2*(a*d)/x^2 - (b*c*d*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(4*x) - (b*d*ArcCsc
[c*x])/(2*x^2) + (b*c^2*d*ArcSin[1/(c*x)])/4 - b*e*ArcCsc[c*x]*Log[1 - E^((
2*I)*ArcCsc[c*x])] + a*e*Log[x] + (I/2)*b*e*(ArcCsc[c*x]^2 + PolyLog[2, E^(
(2*I)*ArcCsc[c*x])])
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{aex^2 + ad + \left(bex^2 + bd\right)\text{arccsc}\left(cx\right)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsc(c*x))/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2 + d\right)\left(b\,\text{arccsc}\left(cx\right) + a\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccsc(c*x) + a)/x^3, x)
```

maple [A] time = 0.78, size = 185, normalized size = 1.35

$$ae\ln\left(cx\right)-\frac{da}{2x^2}+\frac{ibe\,\text{arccsc}\left(cx\right)^2}{2}-be\,\text{arccsc}\left(cx\right)\ln\left(1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)-be\,\text{arccsc}\left(cx\right)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccsc(c*x))/x^3,x)
```

```
[Out] a*e*ln(c*x)-1/2*d*a/x^2+1/2*I*b*e*arccsc(c*x)^2-b*e*arccsc(c*x)*ln(1+I/c/x+
(1-1/c^2/x^2)^(1/2))-b*e*arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))+I*b*e*
polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))+I*b*e*polylog(2,I/c/x+(1-1/c^2/x^2)^(
1/2))+1/4*c^2*b*d*arccsc(c*x)*cos(2*arccsc(c*x))-1/8*c^2*b*sin(2*arccsc(c*x
))*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(c^2\int\frac{\sqrt{cx+1}\,\sqrt{cx-1}\,\log(x)}{c^4x^3-c^2x}\,dx+\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right)\log(x)\right)be+\frac{1}{4}\,bd\left(\frac{\frac{c^4x\sqrt{-\frac{1}{c^2x^2}+1}}{c^2x^2\left(\frac{1}{c^2x^2}-1\right)-1}-c^3\arctan\left(cx\sqrt{\right.}\right.}{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] (c^2*integrate(sqrt(c*x + 1)*sqrt(c*x - 1)*log(x)/(c^4*x^3 - c^2*x), x) + a
rctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(x))*b*e + 1/4*b*d*((c^4*x*sqrt(-
1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) - c^3*arctan(c*x*sqrt(-1/(
c^2*x^2) + 1)))/c - 2*arccsc(c*x)/x^2) + a*e*log(x) - 1/2*a*d/x^2
```

mupad [B] time = 0.99, size = 124, normalized size = 0.91

$$-a\,e\,\ln\left(\frac{1}{x}\right)-\frac{a\,d}{2\,x^2}-b\,e\,\ln\left(1-\mathrm{e}^{\mathrm{asin}\left(\frac{1}{c\,x}\right)2\mathrm{i}}\right)\,\mathrm{asin}\left(\frac{1}{c\,x}\right)-\frac{b\,c\,d\,\sqrt{1-\frac{1}{c^2\,x^2}}}{4\,x}-\frac{b\,c^2\,d\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\,\left(\frac{2}{c^2\,x^2}-1\right)}{4}+\frac{b\,e\,\mathrm{polylog}\left(2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(a + b*asin(1/(c*x))))/x^3,x)
```

```
[Out] (b*e*polylog(2, exp(asin(1/(c*x))*2i))*1i)/2 - a*e*log(1/x) + (b*e*asin(1/(
c*x))^2*1i)/2 - (a*d)/(2*x^2) - b*e*log(1 - exp(asin(1/(c*x))*2i))*asin(1/(
c*x)) - (b*c*d*(1 - 1/(c^2*x^2))^(1/2))/(4*x) - (b*c^2*d*asin(1/(c*x))*(2/(
c^2*x^2) - 1))/4
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{acsc}\,(cx))\left(d + ex^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acsc(c*x))/x**3,x)
```

```
[Out] Integral((a + b*acsc(c*x))*(d + e*x**2)/x**3, x)
```

3.88 $\int x^2 \left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right)\,dx$

Optimal. Leaf size=252

$$\frac{1}{3}d^2x^3\left(a+b\csc^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\csc^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\csc^{-1}(cx)\right)+\frac{be^2x^6\sqrt{c^2x^2-1}}{42c\sqrt{c^2x^2}}+\frac{bex^4\sqrt{c^2x^2-1}\,(84c}{840c^3\sqrt{c^2x^2}}$$

```
[Out] 1/3*d^2*x^3*(a+b*arccsc(c*x))+2/5*d*e*x^5*(a+b*arccsc(c*x))+1/7*e^2*x^7*(a+
b*arccsc(c*x))+1/1680*b*(280*c^4*d^2+252*c^2*d*e+75*e^2)*x*arctanh(c*x/(c^2
*x^2-1)^(1/2))/c^6/(c^2*x^2)^(1/2)+1/1680*b*(280*c^4*d^2+252*c^2*d*e+75*e^2
)*x^2*(c^2*x^2-1)^(1/2)/c^5/(c^2*x^2)^(1/2)+1/840*b*e*(84*c^2*d+25*e)*x^4*(
c^2*x^2-1)^(1/2)/c^3/(c^2*x^2)^(1/2)+1/42*b*e^2*x^6*(c^2*x^2-1)^(1/2)/c/(c^
2*x^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {270, 5239, 12, 1267, 459, 321, 217, 206}

$$\frac{1}{3}d^2x^3\left(a+b\csc^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\csc^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\csc^{-1}(cx)\right)+\frac{bx^2\sqrt{c^2x^2-1}\,\left(280c^4d^2+252c^2de+7\right.}{1680c^5\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*(280*c^4*d^2 + 252*c^2*d*e + 75*e^2)*x^2*Sqrt[-1 + c^2*x^2])/(1680*c^5*S
qrt[c^2*x^2]) + (b*e*(84*c^2*d + 25*e)*x^4*Sqrt[-1 + c^2*x^2])/(840*c^3*Sqr
t[c^2*x^2]) + (b*e^2*x^6*Sqrt[-1 + c^2*x^2])/(42*c*Sqrt[c^2*x^2]) + (d^2*x^
3*(a + b*ArcCsc[c*x]))/3 + (2*d*e*x^5*(a + b*ArcCsc[c*x]))/5 + (e^2*x^7*(a
+ b*ArcCsc[c*x]))/7 + (b*(280*c^4*d^2 + 252*c^2*d*e + 75*e^2)*x*ArcTanh[(c*
x)/Sqrt[-1 + c^2*x^2]])/(1680*c^6*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int x^2\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)dx=\frac{1}{3}d^2x^3\left(a+b\csc^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\csc^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\csc^{-1}\right.$$

$$=\frac{1}{3}d^2x^3\left(a+b\csc^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\csc^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\csc^{-1}\right.$$

$$=\frac{be^2x^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\csc^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\csc^{-1}(cx)\right)$$

$$=\frac{be\left(84c^2d+25e\right)x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}}+\frac{be^2x^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\,\mathrm{cs}\right.$$

$$=\frac{b\left(280c^4d^2+252c^2de+75e^2\right)x^2\sqrt{-1+c^2x^2}}{1680c^5\sqrt{c^2x^2}}+\frac{be\left(84c^2d+25e\right)x^4\sqrt{-}}{840c^3\sqrt{c^2x^2}}$$

$$=\frac{b\left(280c^4d^2+252c^2de+75e^2\right)x^2\sqrt{-1+c^2x^2}}{1680c^5\sqrt{c^2x^2}}+\frac{be\left(84c^2d+25e\right)x^4\sqrt{-}}{840c^3\sqrt{c^2x^2}}$$

$$=\frac{b\left(280c^4d^2+252c^2de+75e^2\right)x^2\sqrt{-1+c^2x^2}}{1680c^5\sqrt{c^2x^2}}+\frac{be\left(84c^2d+25e\right)x^4\sqrt{-}}{840c^3\sqrt{c^2x^2}}$$

Mathematica [A] time = 0.37, size = 184, normalized size = 0.73

$$\frac{c^2x^2\left(16ac^5x\left(35d^2+42dex^2+15e^2x^4\right)+b\sqrt{1-\frac{1}{c^2x^2}}\left(8c^4\left(35d^2+21dex^2+5e^2x^4\right)+2c^2e\left(126d+25ex^2\right)+75e^2\right.\right.}{1680c^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (c^2*x^2*(16*a*c^5*x*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4) + b*Sqrt[1 - 1/(c^2
*x^2)]*(75*e^2 + 2*c^2*e*(126*d + 25*e*x^2) + 8*c^4*(35*d^2 + 21*d*e*x^2 +
5*e^2*x^4))) + 16*b*c^7*x^3*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4)*ArcCsc[c*x]
+ b*(280*c^4*d^2 + 252*c^2*d*e + 75*e^2)*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x]
)/(1680*c^7)
```

fricas [A] time = 1.19, size = 273, normalized size = 1.08

$$240\,ac^7e^2x^7+672\,ac^7dex^5+560\,ac^7d^2x^3+16\left(15\,bc^7e^2x^7+42\,bc^7dex^5+35\,bc^7d^2x^3-35\,bc^7d^2-42\,bc^7de-15\,b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/1680*(240*a*c^7*e^2*x^7 + 672*a*c^7*d*e*x^5 + 560*a*c^7*d^2*x^3 + 16*(15*
b*c^7*e^2*x^7 + 42*b*c^7*d*e*x^5 + 35*b*c^7*d^2*x^3 - 35*b*c^7*d^2 - 42*b*c
^7*d*e - 15*b*c^7*e^2)*arccsc(c*x) - 32*(35*b*c^7*d^2 + 42*b*c^7*d*e + 15*b
*c^7*e^2)*arctan(-c*x + sqrt(c^2*x^2 - 1)) - (280*b*c^4*d^2 + 252*b*c^2*d*e
 + 75*b*e^2)*log(-c*x + sqrt(c^2*x^2 - 1)) + (40*b*c^5*e^2*x^5 + 2*(84*b*c^
5*d*e + 25*b*c^3*e^2)*x^3 + (280*b*c^5*d^2 + 252*b*c^3*d*e + 75*b*c*e^2)*x)
*sqrt(c^2*x^2 - 1))/c^7
```

giac [B] time = 4.95, size = 1573, normalized size = 6.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/13440*(15*b*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7*arcsin(1/(c*x))*e^2/c + 15
*a*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7*e^2/c + 168*b*d*x^5*(sqrt(-1/(c^2*x^2
) + 1) + 1)^5*arcsin(1/(c*x))*e/c + 5*b*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*
e^2/c^2 + 168*a*d*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*e/c + 105*b*x^5*(sqrt(
-1/(c^2*x^2) + 1) + 1)^5*arcsin(1/(c*x))*e^2/c^3 + 560*b*d^2*x^3*(sqrt(-1/(
c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x))/c + 105*a*x^5*(sqrt(-1/(c^2*x^2) + 1)
+ 1)^5*e^2/c^3 + 84*b*d*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*e/c^2 + 560*a*d^
2*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c + 840*b*d*x^3*(sqrt(-1/(c^2*x^2) + 1
) + 1)^3*arcsin(1/(c*x))*e/c^3 + 45*b*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*e^
2/c^4 + 840*a*d*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*e/c^3 + 560*b*d^2*x^2*(s
qrt(-1/(c^2*x^2) + 1) + 1)^2/c^2 + 315*b*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3
*arcsin(1/(c*x))*e^2/c^5 + 1680*b*d^2*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin
(1/(c*x))/c^3 + 315*a*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*e^2/c^5 + 672*b*d*
x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*e/c^4 + 1680*a*d^2*x*(sqrt(-1/(c^2*x^2)
+ 1) + 1)/c^3 + 1680*b*d*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))*e/c
^5 + 225*b*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*e^2/c^6 + 1680*a*d*x*(sqrt(-1
/(c^2*x^2) + 1) + 1)*e/c^5 + 2240*b*d^2*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4
 - 2240*b*d^2*log(1/(abs(c)*abs(x)))/c^4 + 525*b*x*(sqrt(-1/(c^2*x^2) + 1)
+ 1)*arcsin(1/(c*x))*e^2/c^7 + 2016*b*d*e*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c
^6 - 2016*b*d*e*log(1/(abs(c)*abs(x)))/c^6 + 1680*b*d^2*arcsin(1/(c*x))/(c^
5*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 525*a*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*e^
```

```
2/c^7 + 1680*a*d^2/(c^5*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1680*b*d*arcsin(1
/(c*x))*e/(c^7*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 600*b*e^2*log(sqrt(-1/(c^2
*x^2) + 1) + 1)/c^8 - 600*b*e^2*log(1/(abs(c)*abs(x)))/c^8 + 1680*a*d*e/(c^
7*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) - 560*b*d^2/(c^6*x^2*(sqrt(-1/(c^2*x^2) +
 1) + 1)^2) + 525*b*arcsin(1/(c*x))*e^2/(c^9*x*(sqrt(-1/(c^2*x^2) + 1) + 1)
) + 560*b*d^2*arcsin(1/(c*x))/(c^7*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 52
5*a*e^2/(c^9*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) - 672*b*d*e/(c^8*x^2*(sqrt(-1/
(c^2*x^2) + 1) + 1)^2) + 560*a*d^2/(c^7*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3)
 + 840*b*d*arcsin(1/(c*x))*e/(c^9*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) - 225
*b*e^2/(c^10*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 840*a*d*e/(c^9*x^3*(sqrt
(-1/(c^2*x^2) + 1) + 1)^3) + 315*b*arcsin(1/(c*x))*e^2/(c^11*x^3*(sqrt(-1/(
c^2*x^2) + 1) + 1)^3) + 315*a*e^2/(c^11*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3)
 - 84*b*d*e/(c^10*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) + 168*b*d*arcsin(1/(c
*x))*e/(c^11*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) - 45*b*e^2/(c^12*x^4*(sqrt
(-1/(c^2*x^2) + 1) + 1)^4) + 168*a*d*e/(c^11*x^5*(sqrt(-1/(c^2*x^2) + 1) +
1)^5) + 105*b*arcsin(1/(c*x))*e^2/(c^13*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5)
 + 105*a*e^2/(c^13*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) - 5*b*e^2/(c^14*x^6*
(sqrt(-1/(c^2*x^2) + 1) + 1)^6) + 15*b*arcsin(1/(c*x))*e^2/(c^15*x^7*(sqrt(
-1/(c^2*x^2) + 1) + 1)^7) + 15*a*e^2/(c^15*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)
^7))*c
```

maple [B] time = 0.06, size = 494, normalized size = 1.96

$$\frac{a e^2 x^7}{7}+\frac{2aed x^5}{5}+\frac{a x^3 d^2}{3}+\frac{b \operatorname{arccsc}(cx) e^2 x^7}{7}+\frac{2b \operatorname{arccsc}(cx) ed x^5}{5}+\frac{b \operatorname{arccsc}(cx) x^3 d^2}{3}+\frac{b x^6 e^2}{42c\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{b x^4 e^2}{168c^3\sqrt{\frac{c^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^2*(a+b*arccsc(c*x)),x)
```

```
[Out] 1/7*a*e^2*x^7+2/5*a*e*d*x^5+1/3*a*x^3*d^2+1/7*b*arccsc(c*x)*e^2*x^7+2/5*b*a
rccsc(c*x)*e*d*x^5+1/3*b*arccsc(c*x)*x^3*d^2+1/42/c*b/((c^2*x^2-1)/c^2/x^2)
^(1/2)*x^6*e^2+1/168/c^3*b/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^4*e^2+1/10/c*b/((c
^2*x^2-1)/c^2/x^2)^(1/2)*x^4*e*d+1/20/c^3*b/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^2
*e*d+1/6/c*b/((c^2*x^2-1)/c^2/x^2)^(1/2)*d^2*x^2-1/6/c^3*b/((c^2*x^2-1)/c^2
/x^2)^(1/2)*d^2+1/6/c^4*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d
^2*ln(c*x+(c^2*x^2-1)^(1/2))+5/336/c^5*b/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^2*e^
2-3/20/c^5*b/((c^2*x^2-1)/c^2/x^2)^(1/2)*e*d+3/20/c^6*b*(c^2*x^2-1)^(1/2)/(
(c^2*x^2-1)/c^2/x^2)^(1/2)/x*e*d*ln(c*x+(c^2*x^2-1)^(1/2))-5/112/c^7*b/((c^
2*x^2-1)/c^2/x^2)^(1/2)*e^2+5/112/c^8*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/
x^2)^(1/2)/x*e^2*ln(c*x+(c^2*x^2-1)^(1/2))
```

maxima [A] time = 0.34, size = 404, normalized size = 1.60

$$\frac{1}{7}ae^2x^7+\frac{2}{5}adex^5+\frac{1}{3}ad^2x^3+\frac{1}{12}\left(4x^3\operatorname{arccsc}(cx)+\frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}-\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bd^2+\frac{1}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*e^2*x^7 + 2/5*a*d*e*x^5 + 1/3*a*d^2*x^3 + 1/12*(4*x^3*arccsc(c*x) + (
2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x
^2) + 1) + 1)/c^2 - log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d^2 + 1/40*(1
```

```
6*x^5*arccsc(c*x) - (2*(3*(-1/(c^2*x^2) + 1)^(3/2) - 5*sqrt(-1/(c^2*x^2) +
1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(1/(c^2*x^2) - 1) + c^4) - 3*log(sqrt(
-1/(c^2*x^2) + 1) + 1)/c^4 + 3*log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*d*
e + 1/672*(96*x^7*arccsc(c*x) + (2*(15*(-1/(c^2*x^2) + 1)^(5/2) - 40*(-1/(c
^2*x^2) + 1)^(3/2) + 33*sqrt(-1/(c^2*x^2) + 1))/(c^6*(1/(c^2*x^2) - 1)^3 +
3*c^6*(1/(c^2*x^2) - 1)^2 + 3*c^6*(1/(c^2*x^2) - 1) + c^6) + 15*log(sqrt(-1
/(c^2*x^2) + 1) + 1)/c^6 - 15*log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^6)/c)*b*e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\left(e\,x^2+d\right)^2\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d + e*x^2)^2*(a + b*asin(1/(c*x))),x)
```

```
[Out] int(x^2*(d + e*x^2)^2*(a + b*asin(1/(c*x))), x)
```

sympy [A] time = 14.16, size = 542, normalized size = 2.15

$$\frac{ad^2x^3}{3}+\frac{2adex^5}{5}+\frac{ae^2x^7}{7}+\frac{bd^2x^3\operatorname{acsc}(cx)}{3}+\frac{2bdex^5\operatorname{acsc}(cx)}{5}+\frac{be^2x^7\operatorname{acsc}(cx)}{7}+\frac{bd^2\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\operatorname{acosh}(cx)}{2c^2} \\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\,\mathrm{as}}{}\end{cases}\right.}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**2*(a+b*acsc(c*x)),x)
```

```
[Out] a*d**2*x**3/3 + 2*a*d*e*x**5/5 + a*e**2*x**7/7 + b*d**2*x**3*acsc(c*x)/3 +
2*b*d*e*x**5*acsc(c*x)/5 + b*e**2*x**7*acsc(c*x)/7 + b*d**2*Piecewise((x*sq
rt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1), (-I*c*x
**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*asin(c*x)
/(2*c**2), True))/(3*c) + 2*b*d*e*Piecewise((c*x**5/(4*sqrt(c**2*x**2 - 1))
 + x**3/(8*c*sqrt(c**2*x**2 - 1)) - 3*x/(8*c**3*sqrt(c**2*x**2 - 1)) + 3*ac
osh(c*x)/(8*c**4), Abs(c**2*x**2) > 1), (-I*c*x**5/(4*sqrt(-c**2*x**2 + 1))
 - I*x**3/(8*c*sqrt(-c**2*x**2 + 1)) + 3*I*x/(8*c**3*sqrt(-c**2*x**2 + 1))
- 3*I*asin(c*x)/(8*c**4), True))/(5*c) + b*e**2*Piecewise((c*x**7/(6*sqrt(c
**2*x**2 - 1)) + x**5/(24*c*sqrt(c**2*x**2 - 1)) + 5*x**3/(48*c**3*sqrt(c**
2*x**2 - 1)) - 5*x/(16*c**5*sqrt(c**2*x**2 - 1)) + 5*acosh(c*x)/(16*c**6),
Abs(c**2*x**2) > 1), (-I*c*x**7/(6*sqrt(-c**2*x**2 + 1)) - I*x**5/(24*c*sqr
t(-c**2*x**2 + 1)) - 5*I*x**3/(48*c**3*sqrt(-c**2*x**2 + 1)) + 5*I*x/(16*c*
*5*sqrt(-c**2*x**2 + 1)) - 5*I*asin(c*x)/(16*c**6), True))/(7*c)
```

3.89 $\int \left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right) dx$

Optimal. Leaf size=191

$$d^2x\left(a+b\csc^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\csc^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\csc^{-1}(cx)\right)+\frac{be^2x^4\sqrt{c^2x^2-1}}{20c\sqrt{c^2x^2}}+\frac{bx\left(120c^4d^2+40c^2\right.}{12}$$

```
[Out] d^2*x*(a+b*arccsc(c*x))+2/3*d*e*x^3*(a+b*arccsc(c*x))+1/5*e^2*x^5*(a+b*arcc
sc(c*x))+1/120*b*(120*c^4*d^2+40*c^2*d*e+9*e^2)*x*arctanh(c*x/(c^2*x^2-1)^(
1/2))/c^4/(c^2*x^2)^(1/2)+1/120*b*e*(40*c^2*d+9*e)*x^2*(c^2*x^2-1)^(1/2)/c^
3/(c^2*x^2)^(1/2)+1/20*b*e^2*x^4*(c^2*x^2-1)^(1/2)/c/(c^2*x^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {194, 5229, 12, 1159, 388, 217, 206}

$$d^2x\left(a+b\csc^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\csc^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\csc^{-1}(cx)\right)+\frac{bx\left(120c^4d^2+40c^2de+9e^2\right)\tanh^{-1}\left(\right.}{120c^4\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*e*(40*c^2*d + 9*e)*x^2*Sqrt[-1 + c^2*x^2])/(120*c^3*Sqrt[c^2*x^2]) + (b*
e^2*x^4*Sqrt[-1 + c^2*x^2])/(20*c*Sqrt[c^2*x^2]) + d^2*x*(a + b*ArcCsc[c*x]
) + (2*d*e*x^3*(a + b*ArcCsc[c*x]))/3 + (e^2*x^5*(a + b*ArcCsc[c*x]))/5 + (
b*(120*c^4*d^2 + 40*c^2*d*e + 9*e^2)*x*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/(
120*c^4*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1159

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[(c^p*x^(4*p - 1)*(d + e*x^2)^(q + 1))/(e*(4*p + 2*q + 1))
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Rule 5229

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCsc[c*x], u, x]
+ Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1])
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1/2,
0])
```

Rubi steps

$$\int \left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right)dx = d^2x\left(a+b\csc^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\csc^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\csc^{-1}(cx)\right)+$$

$$= d^2x\left(a+b\csc^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\csc^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\csc^{-1}(cx)\right)+$$

$$= \frac{be^2x^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}}+d^2x\left(a+b\csc^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\csc^{-1}(cx)\right)+\frac{1}{5}e^2x^5$$

$$= \frac{be\left(40c^2d+9e\right)x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}}+\frac{be^2x^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}}+d^2x\left(a+b\csc^{-1}(cx)\right)+$$

$$= \frac{be\left(40c^2d+9e\right)x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}}+\frac{be^2x^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}}+d^2x\left(a+b\csc^{-1}(cx)\right)+$$

$$= \frac{be\left(40c^2d+9e\right)x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}}+\frac{be^2x^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}}+d^2x\left(a+b\csc^{-1}(cx)\right)+$$

Mathematica [A] time = 0.26, size = 151, normalized size = 0.79

$$\frac{c^2x\left(8ac^3\left(15d^2+10dex^2+3e^2x^4\right)+bex\sqrt{1-\frac{1}{c^2x^2}}\left(c^2\left(40d+6ex^2\right)+9e\right)\right)+8bc^5x\csc^{-1}(cx)\left(15d^2+10dex^2+3e\right.}{120c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (c^2*x*(8*a*c^3*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) + b*e*Sqrt[1 - 1/(c^2*x^2
)]*x*(9*e + c^2*(40*d + 6*e*x^2))) + 8*b*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^2
*x^4)*ArcCsc[c*x] + b*(120*c^4*d^2 + 40*c^2*d*e + 9*e^2)*Log[(1 + Sqrt[1 -
1/(c^2*x^2)])*x])/(120*c^5)
```

fricas [A] time = 2.60, size = 237, normalized size = 1.24

$$24\,ac^5e^2x^5+80\,ac^5dex^3+120\,ac^5d^2x+8\left(3\,bc^5e^2x^5+10\,bc^5dex^3+15\,bc^5d^2x-15\,bc^5d^2-10\,bc^5de-3\,bc^5e^2\right)\text{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="fricas")

[Out] 1/120*(24*a*c^5*e^2*x^5 + 80*a*c^5*d*e*x^3 + 120*a*c^5*d^2*x + 8*(3*b*c^5*e
^2*x^5 + 10*b*c^5*d*e*x^3 + 15*b*c^5*d^2*x - 15*b*c^5*d^2 - 10*b*c^5*d*e -
3*b*c^5*e^2)*arccsc(c*x) - 16*(15*b*c^5*d^2 + 10*b*c^5*d*e + 3*b*c^5*e^2)*a
rctan(-c*x + sqrt(c^2*x^2 - 1)) - (120*b*c^4*d^2 + 40*b*c^2*d*e + 9*b*e^2)*
log(-c*x + sqrt(c^2*x^2 - 1)) + (6*b*c^3*e^2*x^3 + (40*b*c^3*d*e + 9*b*c*e^
2)*x)*sqrt(c^2*x^2 - 1))/c^5

giac [B] time = 3.20, size = 1027, normalized size = 5.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="giac")

[Out] 1/960*(6*b*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*arcsin(1/(c*x))*e^2/c + 6*a*x
^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*e^2/c + 80*b*d*x^3*(sqrt(-1/(c^2*x^2) + 1
) + 1)^3*arcsin(1/(c*x))*e/c + 3*b*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*e^2/c
^2 + 80*a*d*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*e/c + 30*b*x^3*(sqrt(-1/(c^2
*x^2) + 1) + 1)^3*arcsin(1/(c*x))*e^2/c^3 + 480*b*d^2*x*(sqrt(-1/(c^2*x^2)
+ 1) + 1)*arcsin(1/(c*x))/c + 30*a*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*e^2/c
^3 + 80*b*d*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*e/c^2 + 480*a*d^2*x*(sqrt(-1
/(c^2*x^2) + 1) + 1)/c + 240*b*d*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c
*x))*e/c^3 + 24*b*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*e^2/c^4 + 240*a*d*x*(s
qrt(-1/(c^2*x^2) + 1) + 1)*e/c^3 + 960*b*d^2*log(sqrt(-1/(c^2*x^2) + 1) + 1
)/c^2 - 960*b*d^2*log(1/(abs(c)*abs(x)))/c^2 + 60*b*x*(sqrt(-1/(c^2*x^2) +
1) + 1)*arcsin(1/(c*x))*e^2/c^5 + 320*b*d*e*log(sqrt(-1/(c^2*x^2) + 1) + 1)
/c^4 - 320*b*d*e*log(1/(abs(c)*abs(x)))/c^4 + 480*b*d^2*arcsin(1/(c*x))/(c^
3*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 60*a*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*e^2
/c^5 + 480*a*d^2/(c^3*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 240*b*d*arcsin(1/(c
*x))*e/(c^5*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 72*b*e^2*log(sqrt(-1/(c^2*x^2
) + 1) + 1)/c^6 - 72*b*e^2*log(1/(abs(c)*abs(x)))/c^6 + 240*a*d*e/(c^5*x*(s
qrt(-1/(c^2*x^2) + 1) + 1)) + 60*b*arcsin(1/(c*x))*e^2/(c^7*x*(sqrt(-1/(c^2
*x^2) + 1) + 1)) + 60*a*e^2/(c^7*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) - 80*b*d*e
/(c^6*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 80*b*d*arcsin(1/(c*x))*e/(c^7*x
^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) - 24*b*e^2/(c^8*x^2*(sqrt(-1/(c^2*x^2) +
 1) + 1)^2) + 80*a*d*e/(c^7*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 30*b*arcs
in(1/(c*x))*e^2/(c^9*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 30*a*e^2/(c^9*x^
3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) - 3*b*e^2/(c^10*x^4*(sqrt(-1/(c^2*x^2) +
1) + 1)^4) + 6*b*arcsin(1/(c*x))*e^2/(c^11*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)
^5) + 6*a*e^2/(c^11*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))*c

maple [B] time = 0.06, size = 371, normalized size = 1.94

$$\frac{a\,e^2x^5}{5}+\frac{2a\,x^3de}{3}+ax\,d^2+\frac{b\,\operatorname{arccsc}(cx)\,e^2x^5}{5}+\frac{2b\,\operatorname{arccsc}(cx)\,x^3de}{3}+b\,\operatorname{arccsc}(cx)\,x\,d^2+\frac{b\sqrt{c^2x^2-1}\;d^2\ln\left(cx+\sqrt{c^2\cdots\right.}{c^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\;x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^2*(a+b*arccsc(c*x)),x)

[Out] 1/5*a*e^2*x^5+2/3*a*x^3*d*e+a*x*d^2+1/5*b*arccsc(c*x)*e^2*x^5+2/3*b*arccsc(
c*x)*x^3*d*e+b*arccsc(c*x)*x*d^2+1/c^2*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2
/x^2)^(1/2)/x*d^2*ln(c*x+(c^2*x^2-1)^(1/2))+1/20/c*b/((c^2*x^2-1)/c^2/x^2)^
(1/2)*x^4*e^2+1/40/c^3*b/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^2*e^2+1/3/c*b/((c^2*
x^2-1)/c^2/x^2)^(1/2)*e*d*x^2-1/3/c^3*b/((c^2*x^2-1)/c^2/x^2)^(1/2)*e*d+1/3
/c^4*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*e*d*ln(c*x+(c^2*x^2-
1)^(1/2))-3/40/c^5*b/((c^2*x^2-1)/c^2/x^2)^(1/2)*e^2+3/40/c^6*b*(c^2*x^2-1)
^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*e^2*ln(c*x+(c^2*x^2-1)^(1/2))

maxima [A] time = 0.34, size = 296, normalized size = 1.55

$$\frac{1}{5}ae^2x^5+\frac{2}{3}adex^3+\frac{1}{6}\left(4x^3\operatorname{arccsc}(cx)+\frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}-\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bde+\frac{1}{80}\left(16x^5\operatorname{arccsc}(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*e^2*x^5 + 2/3*a*d*e*x^3 + 1/6*(4*x^3*arccsc(c*x) + (2*sqrt(-1/(c^2*x^
2) + 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2
 - log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d*e + 1/80*(16*x^5*arccsc(c*x)
 - (2*(3*(-1/(c^2*x^2) + 1)^(3/2) - 5*sqrt(-1/(c^2*x^2) + 1))/(c^4*(1/(c^2*
x^2) - 1)^2 + 2*c^4*(1/(c^2*x^2) - 1) + c^4) - 3*log(sqrt(-1/(c^2*x^2) + 1)
 + 1)/c^4 + 3*log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*e^2 + a*d^2*x + 1/2
*(2*c*x*arccsc(c*x) + log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-sqrt(-1/(c^2*x
^2) + 1) + 1))*b*d^2/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(e\,x^2+d\right)^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2*(a + b*asin(1/(c*x))),x)
```

```
[Out] int((d + e*x^2)^2*(a + b*asin(1/(c*x))), x)
```

sympy [A] time = 10.12, size = 355, normalized size = 1.86

$$ad^2x+\frac{2adex^3}{3}+\frac{ae^2x^5}{5}+bd^2x\operatorname{acsc}(cx)+\frac{2bdex^3\operatorname{acsc}(cx)}{3}+\frac{be^2x^5\operatorname{acsc}(cx)}{5}+\frac{bd^2\left(\begin{cases}\operatorname{acosh}(cx) & \text{for } \left|c^2x^2\right|>1\\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}\right)}{c}+\frac{2\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acsc(c*x)),x)
```

```
[Out] a*d**2*x + 2*a*d*e*x**3/3 + a*e**2*x**5/5 + b*d**2*x*acsc(c*x) + 2*b*d*e*x*
*3*acsc(c*x)/3 + b*e**2*x**5*acsc(c*x)/5 + b*d**2*Piecewise((acosh(c*x), Ab
s(c**2*x**2) > 1), (-I*asin(c*x), True))/c + 2*b*d*e*Piecewise((x*sqrt(c**2
*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1), (-I*c*x**3/(2*
sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*asin(c*x)/(2*c**
2), True))/(3*c) + b*e**2*Piecewise((c*x**5/(4*sqrt(c**2*x**2 - 1)) + x**3/
(8*c*sqrt(c**2*x**2 - 1)) - 3*x/(8*c**3*sqrt(c**2*x**2 - 1)) + 3*acosh(c*x)
/(8*c**4), Abs(c**2*x**2) > 1), (-I*c*x**5/(4*sqrt(-c**2*x**2 + 1)) - I*x**
3/(8*c*sqrt(-c**2*x**2 + 1)) + 3*I*x/(8*c**3*sqrt(-c**2*x**2 + 1)) - 3*I*as
in(c*x)/(8*c**4), True))/(5*c)
```

3.90 $\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^2} dx$

Optimal. Leaf size=163

$$-\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{x}+2dex\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}e^2x^3\left(a+b\csc^{-1}(cx)\right)-\frac{bcd^2\sqrt{c^2x^2-1}}{\sqrt{c^2x^2}}+\frac{bex\left(12c^2d+e\right)\tanh^{-1}}{6c^2\sqrt{c^2x^2}}$$

[Out] -d^2*(a+b*arccsc(c*x))/x+2*d*e*x*(a+b*arccsc(c*x))+1/3*e^2*x^3*(a+b*arccsc(c*x))+1/6*b*e*(12*c^2*d+e)*x*arctanh(c*x/(c^2*x^2-1)^(1/2))/c^2/(c^2*x^2)^(1/2)-b*c*d^2*(c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)+1/6*b*e^2*x^2*(c^2*x^2-1)^(1/2)/c/(c^2*x^2)^(1/2)

Rubi [A] time = 0.13, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {270, 5239, 12, 1265, 388, 217, 206}

$$-\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{x}+2dex\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}e^2x^3\left(a+b\csc^{-1}(cx)\right)-\frac{bcd^2\sqrt{c^2x^2-1}}{\sqrt{c^2x^2}}+\frac{bex\left(12c^2d+e\right)\tanh^{-1}}{6c^2\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^2,x]

[Out] -((b*c*d^2*Sqrt[-1 + c^2*x^2])/Sqrt[c^2*x^2]) + (b*e^2*x^2*Sqrt[-1 + c^2*x^2])/(6*c*Sqrt[c^2*x^2]) - (d^2*(a + b*ArcCsc[c*x]))/x + 2*d*e*x*(a + b*ArcCsc[c*x]) + (e^2*x^3*(a + b*ArcCsc[c*x]))/3 + (b*e*(12*c^2*d + e)*x*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/(6*c^2*Sqrt[c^2*x^2])

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1265

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)}{x^2}\,dx = -\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{x} + 2dex\left(a+b\csc^{-1}(cx)\right) + \frac{1}{3}e^2x^3\left(a+b\csc^{-1}(cx)\right) + \cdots$$

$$= -\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{x} + 2dex\left(a+b\csc^{-1}(cx)\right) + \frac{1}{3}e^2x^3\left(a+b\csc^{-1}(cx)\right) + \cdots$$

$$= -\frac{bcd^2\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}} - \frac{d^2\left(a+b\csc^{-1}(cx)\right)}{x} + 2dex\left(a+b\csc^{-1}(cx)\right) + \frac{1}{3}e^2x^3\left(\cdots\right.$$

$$= -\frac{bcd^2\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}} + \frac{be^2x^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}} - \frac{d^2\left(a+b\csc^{-1}(cx)\right)}{x} + 2dex\left(a+b\cdots\right.$$

$$= -\frac{bcd^2\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}} + \frac{be^2x^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}} - \frac{d^2\left(a+b\csc^{-1}(cx)\right)}{x} + 2dex\left(a+b\cdots\right.$$

$$= -\frac{bcd^2\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}} + \frac{be^2x^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}} - \frac{d^2\left(a+b\csc^{-1}(cx)\right)}{x} + 2dex\left(a+b\cdots\right.$$

Mathematica [A] time = 0.21, size = 134, normalized size = 0.82

$$\frac{c^2\left(2ac\left(-3d^2+6dex^2+e^2x^4\right)+bx\sqrt{1-\frac{1}{c^2x^2}}\left(e^2x^2-6c^2d^2\right)\right)+2bc^3\csc^{-1}(cx)\left(-3d^2+6dex^2+e^2x^4\right)+bex\left(12c^2\cdots\right.}{6c^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^2,x]
```

```
[Out] (c^2*(b*Sqrt[1 - 1/(c^2*x^2)]*x*(-6*c^2*d^2 + e^2*x^2) + 2*a*c*(-3*d^2 + 6*
d*e*x^2 + e^2*x^4)) + 2*b*c^3*(-3*d^2 + 6*d*e*x^2 + e^2*x^4)*ArcCsc[c*x] +
b*e*(12*c^2*d + e)*x*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x])/(6*c^3*x)
```

fricas [A] time = 1.16, size = 232, normalized size = 1.42

$$\frac{2\,ac^3e^2x^4 - 6\,bc^4d^2x + 12\,ac^3dex^2 - 6\,ac^3d^2 + 4\left(3\,bc^3d^2 - 6\,bc^3de - bc^3e^2\right)x\arctan\left(-cx+\sqrt{c^2x^2-1}\right) - \left(12\,bc^2d\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*e^2*x^4 - 6*b*c^4*d^2*x + 12*a*c^3*d*e*x^2 - 6*a*c^3*d^2 + 4*(
3*b*c^3*d^2 - 6*b*c^3*d*e - b*c^3*e^2)*x*arctan(-c*x + sqrt(c^2*x^2 - 1)) -
 (12*b*c^2*d*e + b*e^2)*x*log(-c*x + sqrt(c^2*x^2 - 1)) + 2*(b*c^3*e^2*x^4
+ 6*b*c^3*d*e*x^2 - 3*b*c^3*d^2 + (3*b*c^3*d^2 - 6*b*c^3*d*e - b*c^3*e^2)*x
)*arccsc(c*x) - (6*b*c^3*d^2 - b*c*e^2*x^2)*sqrt(c^2*x^2 - 1))/(c^3*x)
```

giac [B] time = 2.15, size = 2494, normalized size = 15.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^2,x, algorithm="giac")
```

```
[Out] 1/24*(b*arcsin(1/(c*x))*e^2/(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*
x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5)) + a*e^2/(c/(x^3*(sqrt(-1/(c^2*x^2) + 1
) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5)) + 24*b*d*arcsin(1/(c*
x))*e/(x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) +
 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + b*e^2/(c*x*(sqrt(-1/(
c^2*x^2) + 1) + 1)*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt
(-1/(c^2*x^2) + 1) + 1)^5))) + 24*a*d*e/(x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2
*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1)
 + 1)^5))) - 24*b*c*d^2/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*(c/(x^3*(sqrt(-
1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 4*b
*arcsin(1/(c*x))*e^2/(c^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*(c/(x^3*(sqrt(
-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 48
*b*d*e*log(sqrt(-1/(c^2*x^2) + 1) + 1)/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^
3*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1
) + 1)^5))) - 48*b*d*e*log(1/(abs(c)*abs(x)))/(c*x^3*(sqrt(-1/(c^2*x^2) + 1
) + 1)^3*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x
^2) + 1) + 1)^5))) - 48*b*d^2*arcsin(1/(c*x))/(x^4*(sqrt(-1/(c^2*x^2) + 1)
+ 1)^4*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2
) + 1) + 1)^5))) + 4*a*e^2/(c^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*(c/(x^3*
(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))
) - 48*a*d^2/(x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*(c/(x^3*(sqrt(-1/(c^2*x^2)
 + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 48*b*d*arcsin(
1/(c*x))*e/(c^2*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*(c/(x^3*(sqrt(-1/(c^2*x^
2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 4*b*e^2*log(
sqrt(-1/(c^2*x^2) + 1) + 1)/(c^3*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*(c/(x^3
*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5)
)) - 4*b*e^2*log(1/(abs(c)*abs(x)))/(c^3*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3
*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1)
 + 1)^5))) + b*e^2/(c^3*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*(c/(x^3*(sqrt(-1
/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 48*a
*d*e/(c^2*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1
) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 24*b*d^2/(c*x^5*(s
qrt(-1/(c^2*x^2) + 1) + 1)^5*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c
*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 6*b*arcsin(1/(c*x))*e^2/(c^4*x^4*(
sqrt(-1/(c^2*x^2) + 1) + 1)^4*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(
c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 48*b*d*e*log(sqrt(-1/(c^2*x^2) +
1) + 1)/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*(c/(x^3*(sqrt(-1/(c^2*x^2)
+ 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) - 48*b*d*e*log(1/
(abs(c)*abs(x)))/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*(c/(x^3*(sqrt(-1/(
c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 6*a*e^
2/(c^4*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) +
 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 24*b*d*arcsin(1/(c*x)
)*e/(c^4*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1)
```

```
 + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 4*b*e^2*log(sqrt(-1
/(c^2*x^2) + 1) + 1)/(c^5*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*(c/(x^3*(sqrt(
-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) - 4*
b*e^2*log(1/(abs(c)*abs(x)))/(c^5*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*(c/(x^
3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5
))) - b*e^2/(c^5*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*(c/(x^3*(sqrt(-1/(c^2*x
^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 24*a*d*e/(c
^4*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^
3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 4*b*arcsin(1/(c*x))*e^2/(
c^6*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)
^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 4*a*e^2/(c^6*x^6*(sqrt(-
1/(c^2*x^2) + 1) + 1)^6*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*
(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) - b*e^2/(c^7*x^7*(sqrt(-1/(c^2*x^2) + 1)
+ 1)^7*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2
) + 1) + 1)^5))) + b*arcsin(1/(c*x))*e^2/(c^8*x^8*(sqrt(-1/(c^2*x^2) + 1) +
 1)^8*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2)
 + 1) + 1)^5))) + a*e^2/(c^8*x^8*(sqrt(-1/(c^2*x^2) + 1) + 1)^8*(c/(x^3*(sq
rt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))))*
c
```

maple [A] time = 0.06, size = 286, normalized size = 1.75

$$\frac{a\,e^2x^3}{3}+2aedx-\frac{a\,d^2}{x}+\frac{b\,\mathrm{arccsc}\,(cx)\,x^3e^2}{3}+2b\,\mathrm{arccsc}\,(cx)\,edx-\frac{b\,\mathrm{arccsc}\,(cx)\,d^2}{x}-\frac{cb\,d^2}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{b\,d^2}{c\,x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{2b\sqrt{c^2x^2-}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccsc(c*x))/x^2,x)
```

```
[Out] 1/3*a*e^2*x^3+2*a*e*d*x-a*d^2/x+1/3*b*arccsc(c*x)*x^3*e^2+2*b*arccsc(c*x)*e
*d*x-b*arccsc(c*x)*d^2/x-c*b/((c^2*x^2-1)/c^2/x^2)^(1/2)*d^2+b/c/x^2/((c^2*
x^2-1)/c^2/x^2)^(1/2)*d^2+2*b/c^2*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(
1/2)/x*e*d*ln(c*x+(c^2*x^2-1)^(1/2))+1/6/c*b*e^2/((c^2*x^2-1)/c^2/x^2)^(1/2
)*x^2-1/6/c^3*b*e^2/((c^2*x^2-1)/c^2/x^2)^(1/2)+1/6/c^4*b*e^2*(c^2*x^2-1)^(
1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*ln(c*x+(c^2*x^2-1)^(1/2))
```

maxima [A] time = 0.35, size = 197, normalized size = 1.21

$$\frac{1}{3}\,ae^2x^3-\left(c\sqrt{-\frac{1}{c^2x^2}+1}+\frac{\mathrm{arccsc}\,(cx)}{x}\right)bd^2+\frac{1}{12}\left(4\,x^3\,\mathrm{arccsc}\,(cx)+\frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}-\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}\right.}{c^2}}{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*a*e^2*x^3 - (c*sqrt(-1/(c^2*x^2) + 1) + arccsc(c*x)/x)*b*d^2 + 1/12*(4*
x^3*arccsc(c*x) + (2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) +
 log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^2)
/c)*b*e^2 + 2*a*d*e*x + (2*c*x*arccsc(c*x) + log(sqrt(-1/(c^2*x^2) + 1) + 1
) - log(-sqrt(-1/(c^2*x^2) + 1) + 1))*b*d*e/c - a*d^2/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(e\,x^2+d\right)}^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^2,x)
```

```
[Out] int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^2, x)
```

sympy [A] time = 8.82, size = 207, normalized size = 1.27

$$-\frac{ad^2}{x}+2adex+\frac{ae^2x^3}{3}-bcd^2\sqrt{1-\frac{1}{c^2x^2}}-\frac{bd^2\operatorname{acsc}(cx)}{x}+2bdex\operatorname{acsc}(cx)+\frac{be^2x^3\operatorname{acsc}(cx)}{3}+\frac{2bde\left(\begin{cases}\operatorname{acosh}(cx) \\ -i\operatorname{asin}(cx)\end{cases}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acsc(c*x))/x**2,x)
```

```
[Out] -a*d**2/x + 2*a*d*e*x + a*e**2*x**3/3 - b*c*d**2*sqrt(1 - 1/(c**2*x**2)) -
b*d**2*acsc(c*x)/x + 2*b*d*e*x*acsc(c*x) + b*e**2*x**3*acsc(c*x)/3 + 2*b*d*
e*Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin(c*x), True))/c + b*e
**2*Piecewise((x*sqrt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*
x**2) > 1), (-I*c*x**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2
+ 1)) - I*asin(c*x)/(2*c**2), True))/(3*c)
```

3.91 $\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x^4}\,dx$

Optimal. Leaf size=157

$$-\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\csc^{-1}(cx)\right)}{x}+e^2x\left(a+b\csc^{-1}(cx)\right)-\frac{bcd^2\sqrt{c^2x^2-1}}{9x^2\sqrt{c^2x^2}}-\frac{2bcd\sqrt{c^2x^2-1}\left(c^2d+9e\right)}{9\sqrt{c^2x^2}}+$$

```
[Out] -1/3*d^2*(a+b*arccsc(c*x))/x^3-2*d*e*(a+b*arccsc(c*x))/x+e^2*x*(a+b*arccsc(
c*x))+b*e^2*x*arctanh(c*x/(c^2*x^2-1)^(1/2))/(c^2*x^2)^(1/2)-2/9*b*c*d*(c^2
*d+9*e)*(c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)-1/9*b*c*d^2*(c^2*x^2-1)^(1/2)/x^2
/(c^2*x^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {270, 5239, 12, 1265, 451, 217, 206}

$$-\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\csc^{-1}(cx)\right)}{x}+e^2x\left(a+b\csc^{-1}(cx)\right)-\frac{bcd^2\sqrt{c^2x^2-1}}{9x^2\sqrt{c^2x^2}}-\frac{2bcd\sqrt{c^2x^2-1}\left(c^2d+9e\right)}{9\sqrt{c^2x^2}}+$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^4,x]
```

```
[Out] (-2*b*c*d*(c^2*d + 9*e)*Sqrt[-1 + c^2*x^2])/(9*Sqrt[c^2*x^2]) - (b*c*d^2*Sq
rt[-1 + c^2*x^2])/(9*x^2*Sqrt[c^2*x^2]) - (d^2*(a + b*ArcCsc[c*x]))/(3*x^3)
 - (2*d*e*(a + b*ArcCsc[c*x]))/x + e^2*x*(a + b*ArcCsc[c*x]) + (b*e^2*x*Arc
Tanh[(c*x)/Sqrt[-1 + c^2*x^2]])/Sqrt[c^2*x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
```

```
Q[m + n, -1]))
```

Rule 1265

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)}{x^4}\,dx = -\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\csc^{-1}(cx)\right)}{x}+e^2x\left(a+b\csc^{-1}(cx)\right)+ \cdots$$

$$= -\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\csc^{-1}(cx)\right)}{x}+e^2x\left(a+b\csc^{-1}(cx)\right)+ \cdots$$

$$= -\frac{bcd^2\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}}-\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\csc^{-1}(cx)\right)}{x}+e^2x\left(\cdots\right.$$

$$= -\frac{2bcd\left(c^2d+9e\right)\sqrt{-1+c^2x^2}}{9\sqrt{c^2x^2}}-\frac{bcd^2\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}}-\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{3x^3} \cdots$$

$$= -\frac{2bcd\left(c^2d+9e\right)\sqrt{-1+c^2x^2}}{9\sqrt{c^2x^2}}-\frac{bcd^2\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}}-\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{3x^3} \cdots$$

$$= -\frac{2bcd\left(c^2d+9e\right)\sqrt{-1+c^2x^2}}{9\sqrt{c^2x^2}}-\frac{bcd^2\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}}-\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{3x^3} \cdots$$

Mathematica [A] time = 0.29, size = 125, normalized size = 0.80

$$-\frac{3a\left(d^2+6dex^2-3e^2x^4\right)+bcdx\sqrt{1-\frac{1}{c^2x^2}}\left(2c^2dx^2+d+18ex^2\right)}{9x^3}+\frac{be^2\log\left(x\left(\sqrt{1-\frac{1}{c^2x^2}}+1\right)\right)}{c}-\frac{b\csc^{-1}(cx)\left(d \cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^4,x]
```

```
[Out] -1/9*(b*c*d*Sqrt[1 - 1/(c^2*x^2)]*x*(d + 2*c^2*d*x^2 + 18*e*x^2) + 3*a*(d^2
 + 6*d*e*x^2 - 3*e^2*x^4))/x^3 - (b*(d^2 + 6*d*e*x^2 - 3*e^2*x^4)*ArcCsc[c*
x])/(3*x^3) + (b*e^2*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x])/c
```

fricas [A] time = 0.51, size = 222, normalized size = 1.41

$$\frac{9\,ace^2x^4 - 9\,be^2x^3\log\left(-cx + \sqrt{c^2x^2 - 1}\right) - 18\,acdex^2 + 6\left(bcd^2 + 6\,bcde - 3\,bce^2\right)x^3\arctan\left(-cx + \sqrt{c^2x^2 - 1}\right) -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/9*(9*a*c*e^2*x^4 - 9*b*e^2*x^3*log(-c*x + sqrt(c^2*x^2 - 1)) - 18*a*c*d*e
*x^2 + 6*(b*c*d^2 + 6*b*c*d*e - 3*b*c*e^2)*x^3*arctan(-c*x + sqrt(c^2*x^2 -
 1)) - 3*a*c*d^2 - 2*(b*c^4*d^2 + 9*b*c^2*d*e)*x^3 + 3*(3*b*c*e^2*x^4 - 6*b
*c*d*e*x^2 - b*c*d^2 + (b*c*d^2 + 6*b*c*d*e - 3*b*c*e^2)*x^3)*arccsc(c*x) -
 (b*c*d^2 + 2*(b*c^3*d^2 + 9*b*c*d*e)*x^2)*sqrt(c^2*x^2 - 1))/(c*x^3)
```

giac [B] time = 14.42, size = 4280, normalized size = 27.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^4,x, algorithm="giac")
```

```
[Out] -1/18*(4*b*c^3*d^2/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)*(c/(x*(sqrt(-1/(c^2*x^2)
 + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-
1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) - 9
*b*arcsin(1/(c*x))*e^2/(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt
(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1
/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7)) - 9*a*e^2/(c/(x*(sqrt(-1/(c^2*x^
2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt
(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7)) +
36*b*c*d*e/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)*(c/(x*(sqrt(-1/(c^2*x^2) + 1) +
1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x
^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) + 72*b*d*ar
csin(1/(c*x))*e/(x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*(c/(x*(sqrt(-1/(c^2*x^2
) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(
-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) -
18*b*e^2*log(sqrt(-1/(c^2*x^2) + 1) + 1)/(c*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*
(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)
^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2
*x^2) + 1) + 1)^7))) + 18*b*e^2*log(1/(abs(c)*abs(x)))/(c*x*(sqrt(-1/(c^2*x
^2) + 1) + 1)*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*
x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7
*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) + 72*a*d*e/(x^2*(sqrt(-1/(c^2*x^2) + 1)
+ 1)^2*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) +
1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(
-1/(c^2*x^2) + 1) + 1)^7))) + 12*b*c*d^2/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^
3*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) +
1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c
^2*x^2) + 1) + 1)^7))) - 36*b*arcsin(1/(c*x))*e^2/(c^2*x^2*(sqrt(-1/(c^2*x^
2) + 1) + 1)^2*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2
*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^
7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) + 48*b*d^2*arcsin(1/(c*x))/(x^4*(sqrt(-
1/(c^2*x^2) + 1) + 1)^4*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqr
t(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) +
1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) - 36*a*e^2/(c^2*x^2*(sqrt(-1/(
c^2*x^2) + 1) + 1)^2*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-
1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(
c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) + 36*b*d*e/(c*x^3*(sqrt(-1/(c^2*x
^2) + 1) + 1)^3*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^
```

```
2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x
^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) + 48*a*d^2/(x^4*(sqrt(-1/(c^2*x^2) + 1
) + 1)^4*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2)
+ 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqr
t(-1/(c^2*x^2) + 1) + 1)^7))) + 144*b*d*arcsin(1/(c*x))*e/(c^2*x^4*(sqrt(-1
/(c^2*x^2) + 1) + 1)^4*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt
(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1
/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) - 54*b*e^2*log(sqrt(-1/(c^2*x^2
) + 1) + 1)/(c^3*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*(c/(x*(sqrt(-1/(c^2*x^2
) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(
-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) +
54*b*e^2*log(1/(abs(c)*abs(x)))/(c^3*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*(c/
(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3)
 + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^
2) + 1) + 1)^7))) + 144*a*d*e/(c^2*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*(c/(x
*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) +
 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2)
 + 1) + 1)^7))) - 12*b*d^2/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*(c/(x*(sqr
t(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c
^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1)
 + 1)^7))) - 54*b*arcsin(1/(c*x))*e^2/(c^4*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)
^4*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) +
 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(
c^2*x^2) + 1) + 1)^7))) - 54*a*e^2/(c^4*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*
(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)
^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2
*x^2) + 1) + 1)^7))) - 36*b*d*e/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*(c/
(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3)
 + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^
2) + 1) + 1)^7))) + 72*b*d*arcsin(1/(c*x))*e/(c^4*x^6*(sqrt(-1/(c^2*x^2) +
1) + 1)^6*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2)
 + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sq
rt(-1/(c^2*x^2) + 1) + 1)^7))) - 54*b*e^2*log(sqrt(-1/(c^2*x^2) + 1) + 1)/(
c^5*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1))
+ 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2)
+ 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) + 54*b*e^2*log(
1/(abs(c)*abs(x)))/(c^5*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*(c/(x*(sqrt(-1/(
c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5
*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^
7))) + 72*a*d*e/(c^4*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*(c/(x*(sqrt(-1/(c^2
*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(s
qrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))
) - 4*b*d^2/(c^3*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7*(c/(x*(sqrt(-1/(c^2*x^2
) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(
-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) -
36*b*arcsin(1/(c*x))*e^2/(c^6*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*(c/(x*(sqr
t(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c
^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1)
 + 1)^7))) - 36*a*e^2/(c^6*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*(c/(x*(sqrt(-
1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*
x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) +
1)^7))) - 36*b*d*e/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7*(c/(x*(sqrt(-1/(
c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5
*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^
7))) - 18*b*e^2*log(sqrt(-1/(c^2*x^2) + 1) + 1)/(c^7*x^7*(sqrt(-1/(c^2*x^2)
 + 1) + 1)^7*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x
^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*
(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) + 18*b*e^2*log(1/(abs(c)*abs(x)))/(c^7*x^
7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c
```

```
*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) +
 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) - 9*b*arcsin(1/(c*x))
*e^2/(c^8*x^8*(sqrt(-1/(c^2*x^2) + 1) + 1)^8*(c/(x*(sqrt(-1/(c^2*x^2) + 1)
+ 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2
*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) - 9*a*e^2
/(c^8*x^8*(sqrt(-1/(c^2*x^2) + 1) + 1)^8*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)
) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2
) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))))*c
```

maple [A] time = 0.07, size = 254, normalized size = 1.62

$$ax\,e^2-\frac{a\,d^2}{3x^3}-\frac{2aed}{x}+b\,\mathrm{arccsc}\,(cx)\,x\,e^2-\frac{b\,\mathrm{arccsc}\,(cx)\,d^2}{3x^3}-\frac{2b\,\mathrm{arccsc}\,(cx)\,ed}{x}-\frac{2c^3b\,d^2}{9\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{cb\,d^2}{9x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{2cbed}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{\cdots}{c\sqrt{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccsc(c*x))/x^4,x)
```

```
[Out] a*x*e^2-1/3*a*d^2/x^3-2*a*e*d/x+b*arccsc(c*x)*x*e^2-1/3*b*arccsc(c*x)*d^2/x
^3-2*b*arccsc(c*x)*e*d/x-2/9*c^3*b/((c^2*x^2-1)/c^2/x^2)^(1/2)*d^2+1/9*c*b/
x^2/((c^2*x^2-1)/c^2/x^2)^(1/2)*d^2-2*c*b/((c^2*x^2-1)/c^2/x^2)^(1/2)*e*d+2
/c*b/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^2*e*d+1/9/c*b/((c^2*x^2-1)/c^2/x^2)^(1/2
)/x^4*d^2+1/c^2*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*e^2*ln(c*
x+(c^2*x^2-1)^(1/2))
```

maxima [A] time = 0.33, size = 158, normalized size = 1.01

$$-2\left(c\sqrt{-\frac{1}{c^2x^2}+1}+\frac{\mathrm{arccsc}\,(cx)}{x}\right)bde+ae^2x+\frac{1}{9}\,bd^2\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c}-\frac{3\,\mathrm{arccsc}\,(cx)}{x^3}\right)+\frac{\left(2\,cx\,\mathrm{arc}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] -2*(c*sqrt(-1/(c^2*x^2) + 1) + arccsc(c*x)/x)*b*d*e + a*e^2*x + 1/9*b*d^2*(
(c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c - 3*arccsc(
c*x)/x^3) + 1/2*(2*c*x*arccsc(c*x) + log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(
-sqrt(-1/(c^2*x^2) + 1) + 1))*b*e^2/c - 2*a*d*e/x - 1/3*a*d^2/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(e\,x^2+d\right)}^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^4,x)
```

```
[Out] int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^4, x)
```

sympy [A] time = 8.43, size = 211, normalized size = 1.34

$$-\frac{ad^2}{3x^3}-\frac{2ade}{x}+ae^2x-2bcde\sqrt{1-\frac{1}{c^2x^2}}-\frac{bd^2\,\mathrm{acsc}\,(cx)}{3x^3}-\frac{2bde\,\mathrm{acsc}\,(cx)}{x}+be^2x\,\mathrm{acsc}\,(cx)-\frac{bd^2\left(\begin{cases}\frac{2c^3\sqrt{c^2x^2-1}}{3x}+\frac{c\sqrt{c^2x^2-1}}{3x^3} & \cdots\\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x}+\frac{ic\sqrt{-c^2x^2\cdots}}{3x^3} & \cdots\end{cases}\right)}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acsc(c*x))/x**4,x)
```

```
[Out] -a*d**2/(3*x**3) - 2*a*d*e/x + a*e**2*x - 2*b*c*d*e*sqrt(1 - 1/(c**2*x**2))
 - b*d**2*acsc(c*x)/(3*x**3) - 2*b*d*e*acsc(c*x)/x + b*e**2*x*acsc(c*x) - b
*d**2*Piecewise((2*c**3*sqrt(c**2*x**2 - 1)/(3*x) + c*sqrt(c**2*x**2 - 1)/(
3*x**3), Abs(c**2*x**2) > 1), (2*I*c**3*sqrt(-c**2*x**2 + 1)/(3*x) + I*c*sq
rt(-c**2*x**2 + 1)/(3*x**3), True))/(3*c) + b*e**2*Piecewise((acosh(c*x), A
bs(c**2*x**2) > 1), (-I*asin(c*x), True))/c
```

3.92 $\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^6} dx$

Optimal. Leaf size=183

$$-\frac{d^2(a+b\csc^{-1}(cx))}{5x^5}-\frac{2de(a+b\csc^{-1}(cx))}{3x^3}-\frac{e^2(a+b\csc^{-1}(cx))}{x}-\frac{bcd^2\sqrt{c^2x^2-1}}{25x^4\sqrt{c^2x^2}}-\frac{2bcd\sqrt{c^2x^2-1}(6c^2d+25e)}{225x^2\sqrt{c^2x^2}}$$

```
[Out] -1/5*d^2*(a+b*arccsc(c*x))/x^5-2/3*d*e*(a+b*arccsc(c*x))/x^3-e^2*(a+b*arccs
c(c*x))/x-1/225*b*c*(24*c^4*d^2+100*c^2*d*e+225*e^2)*(c^2*x^2-1)^(1/2)/(c^2
*x^2)^(1/2)-1/25*b*c*d^2*(c^2*x^2-1)^(1/2)/x^4/(c^2*x^2)^(1/2)-2/225*b*c*d*
(6*c^2*d+25*e)*(c^2*x^2-1)^(1/2)/x^2/(c^2*x^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {270, 5239, 12, 1265, 453, 264}

$$-\frac{d^2(a+b\csc^{-1}(cx))}{5x^5}-\frac{2de(a+b\csc^{-1}(cx))}{3x^3}-\frac{e^2(a+b\csc^{-1}(cx))}{x}-\frac{bc\sqrt{c^2x^2-1}(24c^4d^2+100c^2de+225e^2)}{225\sqrt{c^2x^2}}-\frac{bcc}{2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^6,x]
```

```
[Out] -(b*c*(24*c^4*d^2 + 100*c^2*d*e + 225*e^2)*Sqrt[-1 + c^2*x^2])/(225*Sqrt[c^
2*x^2]) - (b*c*d^2*Sqrt[-1 + c^2*x^2])/(25*x^4*Sqrt[c^2*x^2]) - (2*b*c*d*(6
*c^2*d + 25*e)*Sqrt[-1 + c^2*x^2])/(225*x^2*Sqrt[c^2*x^2]) - (d^2*(a + b*Ar
cCsc[c*x]))/(5*x^5) - (2*d*e*(a + b*ArcCsc[c*x]))/(3*x^3) - (e^2*(a + b*Arc
Csc[c*x]))/x
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1265

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
```

```
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)}{x^6}\,dx = -\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{5x^5} - \frac{2de\left(a+b\csc^{-1}(cx)\right)}{3x^3} - \frac{e^2\left(a+b\csc^{-1}(cx)\right)}{x} + \frac{(b\cdots}{\cdots}$$

$$= -\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{5x^5} - \frac{2de\left(a+b\csc^{-1}(cx)\right)}{3x^3} - \frac{e^2\left(a+b\csc^{-1}(cx)\right)}{x} + \frac{(b\cdots}{\cdots}$$

$$= -\frac{bcd^2\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}} - \frac{d^2\left(a+b\csc^{-1}(cx)\right)}{5x^5} - \frac{2de\left(a+b\csc^{-1}(cx)\right)}{3x^3} - \frac{e^2\left(a\cdots\right.}{\cdots}$$

$$= -\frac{bcd^2\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}} - \frac{2bcd\left(6c^2d+25e\right)\sqrt{-1+c^2x^2}}{225x^2\sqrt{c^2x^2}} - \frac{d^2\left(a+b\csc^{-1}(cx\cdots\right.}{5x^5}$$

$$= -\frac{bc\left(225e^2+4c^2d\left(6c^2d+25e\right)\right)\sqrt{-1+c^2x^2}}{225\sqrt{c^2x^2}} - \frac{bcd^2\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}} - \frac{2bcd\cdots}{\cdots}$$

Mathematica [A] time = 0.28, size = 127, normalized size = 0.69

$$-\frac{15a\left(3d^2+10dex^2+15e^2x^4\right)+bcx\sqrt{1-\frac{1}{c^2x^2}}\left(50dex^2\left(2c^2x^2+1\right)+3d^2\left(8c^4x^4+4c^2x^2+3\right)+225e^2x^4\right)+15\cdots}{225x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^6,x]
```

```
[Out] -1/225*(15*a*(3*d^2 + 10*d*e*x^2 + 15*e^2*x^4) + b*c*Sqrt[1 - 1/(c^2*x^2)]*
x*(225*e^2*x^4 + 50*d*e*x^2*(1 + 2*c^2*x^2) + 3*d^2*(3 + 4*c^2*x^2 + 8*c^4*
x^4)) + 15*b*(3*d^2 + 10*d*e*x^2 + 15*e^2*x^4)*ArcCsc[c*x])/x^5
```

fricas [A] time = 0.69, size = 126, normalized size = 0.69

$$-\frac{225\,ae^2x^4+150\,adex^2+45\,ad^2+15\left(15\,be^2x^4+10\,bdex^2+3\,bd^2\right)\operatorname{arccsc}(cx)+\left(\left(24\,bc^4d^2+100\,bc^2de+22\cdots\right.\right.}{225\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] -1/225*(225*a*e^2*x^4 + 150*a*d*e*x^2 + 45*a*d^2 + 15*(15*b*e^2*x^4 + 10*b*
d*e*x^2 + 3*b*d^2)*arccsc(c*x) + ((24*b*c^4*d^2 + 100*b*c^2*d*e + 225*b*e^2
)*x^4 + 9*b*d^2 + 2*(6*b*c^2*d^2 + 25*b*d*e)*x^2)*sqrt(c^2*x^2 - 1))/x^5
```

giac [A] time = 0.16, size = 316, normalized size = 1.73

$$-\frac{1}{225}\left(9\,bc^4d^2\left(\frac{1}{c^2x^2}-1\right)^2\sqrt{-\frac{1}{c^2x^2}+1}-30\,bc^4d^2\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+\frac{45\,bc^3d^2\left(\frac{1}{c^2x^2}-1\right)^2\arcsin\left(\frac{1}{cx}\right)}{x}+45\,bc^4d^2\sqrt{-\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^6,x, algorithm="giac")
```

```
[Out] -1/225*(9*b*c^4*d^2*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1) - 30*b*c^4*d
^2*(-1/(c^2*x^2) + 1)^(3/2) + 45*b*c^3*d^2*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*
x))/x + 45*b*c^4*d^2*sqrt(-1/(c^2*x^2) + 1) + 90*b*c^3*d^2*(1/(c^2*x^2) - 1
)*arcsin(1/(c*x))/x - 50*b*c^2*d*(-1/(c^2*x^2) + 1)^(3/2)*e + 45*b*c^3*d^2*
arcsin(1/(c*x))/x + 150*b*c^2*d*sqrt(-1/(c^2*x^2) + 1)*e + 150*b*c*d*(1/(c^
2*x^2) - 1)*arcsin(1/(c*x))*e/x + 150*b*c*d*arcsin(1/(c*x))*e/x + 225*b*sqr
t(-1/(c^2*x^2) + 1)*e^2 + 225*b*arcsin(1/(c*x))*e^2/(c*x) + 225*a*e^2/(c*x)
 + 150*a*d*e/(c*x^3) + 45*a*d^2/(c*x^5))*c
```

maple [A] time = 0.06, size = 191, normalized size = 1.04

$$c^5\left(\frac{a\left(-\frac{2ed}{3c\,x^3}-\frac{e^2}{cx}-\frac{d^2}{5c\,x^5}\right)}{c^4}+\frac{b\left(-\frac{2\,\mathrm{arccsc}(cx)ed}{3c\,x^3}-\frac{\mathrm{arccsc}(cx)e^2}{cx}-\frac{\mathrm{arccsc}(cx)d^2}{5c\,x^5}-\frac{\left(c^2x^2-1\right)\left(24x^4c^8d^2+100c^6ed\,x^4+12x^2c^6d^2+225c^4e^2x^4+\cdots\right.}{225\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^6x^6}\right.}{c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccsc(c*x))/x^6,x)
```

```
[Out] c^5*(a/c^4*(-2/3/c*e*d/x^3-e^2/c/x-1/5*d^2/c/x^5)+b/c^4*(-2/3*arccsc(c*x)/c
*e*d/x^3-arccsc(c*x)*e^2/c/x-1/5*arccsc(c*x)*d^2/c/x^5-1/225*(c^2*x^2-1)*(2
4*c^8*d^2*x^4+100*c^6*d*e*x^4+12*c^6*d^2*x^2+225*c^4*e^2*x^4+50*c^4*d*e*x^2
+9*c^4*d^2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^6/x^6))
```

maxima [A] time = 0.35, size = 181, normalized size = 0.99

$$-\left(c\sqrt{-\frac{1}{c^2x^2}+1}+\frac{\mathrm{arccsc}\,(cx)}{x}\right)be^2-\frac{1}{75}\,bd^2\left(\frac{3\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,c^6\sqrt{-\frac{1}{c^2x^2}+1}}{c}+\frac{15\,\mathrm{arccsc}\cdots}{x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] -(c*sqrt(-1/(c^2*x^2) + 1) + arccsc(c*x)/x)*b*e^2 - 1/75*b*d^2*((3*c^6*(-1/
(c^2*x^2) + 1)^(5/2) - 10*c^6*(-1/(c^2*x^2) + 1)^(3/2) + 15*c^6*sqrt(-1/(c^
2*x^2) + 1))/c + 15*arccsc(c*x)/x^5) + 2/9*b*d*e*((c^4*(-1/(c^2*x^2) + 1)^(
3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c - 3*arccsc(c*x)/x^3) - a*e^2/x - 2/3
*a*d*e/x^3 - 1/5*a*d^2/x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(e\,x^2+d\right)^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^6,x)
```

```
[Out] int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^6, x)
```

sympy [A] time = 10.89, size = 335, normalized size = 1.83

$$-\frac{ad^2}{5x^5}-\frac{2ade}{3x^3}-\frac{ae^2}{x}-bce^2\sqrt{1-\frac{1}{c^2x^2}}-\frac{bd^2\operatorname{acsc}(cx)}{5x^5}-\frac{2bde\operatorname{acsc}(cx)}{3x^3}-\frac{be^2\operatorname{acsc}(cx)}{x}-\frac{bd^2\left(\begin{cases}\frac{8c^5\sqrt{c^2x^2-1}}{15x}+\frac{4c^3\sqrt{c^2x^2-1}}{15x^3} \\ \frac{8ic^5\sqrt{-c^2x^2+1}}{15x}+\frac{4ic^3\sqrt{-c^2}}{15x^3}\end{cases}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acsc(c*x))/x**6,x)
```

```
[Out] -a*d**2/(5*x**5) - 2*a*d*e/(3*x**3) - a*e**2/x - b*c*e**2*sqrt(1 - 1/(c**2*
x**2)) - b*d**2*acsc(c*x)/(5*x**5) - 2*b*d*e*acsc(c*x)/(3*x**3) - b*e**2*ac
sc(c*x)/x - b*d**2*Piecewise((8*c**5*sqrt(c**2*x**2 - 1)/(15*x) + 4*c**3*sq
rt(c**2*x**2 - 1)/(15*x**3) + c*sqrt(c**2*x**2 - 1)/(5*x**5), Abs(c**2*x**2
) > 1), (8*I*c**5*sqrt(-c**2*x**2 + 1)/(15*x) + 4*I*c**3*sqrt(-c**2*x**2 +
1)/(15*x**3) + I*c*sqrt(-c**2*x**2 + 1)/(5*x**5), True))/(5*c) - 2*b*d*e*Pi
ecewise((2*c**3*sqrt(c**2*x**2 - 1)/(3*x) + c*sqrt(c**2*x**2 - 1)/(3*x**3),
 Abs(c**2*x**2) > 1), (2*I*c**3*sqrt(-c**2*x**2 + 1)/(3*x) + I*c*sqrt(-c**2
*x**2 + 1)/(3*x**3), True))/(3*c)
```

3.93 $\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x^8}\,dx$

Optimal. Leaf size=241

$$-\frac{d^2(a+b\csc^{-1}(cx))}{7x^7}-\frac{2de(a+b\csc^{-1}(cx))}{5x^5}-\frac{e^2(a+b\csc^{-1}(cx))}{3x^3}-\frac{bcd^2\sqrt{c^2x^2-1}}{49x^6\sqrt{c^2x^2}}-\frac{2bcd\sqrt{c^2x^2-1}\,(15c^2d+49e)}{1225x^4\sqrt{c^2x^2}}$$

```
[Out] -1/7*d^2*(a+b*arccsc(c*x))/x^7-2/5*d*e*(a+b*arccsc(c*x))/x^5-1/3*e^2*(a+b*a
rccsc(c*x))/x^3-2/11025*b*c^3*(360*c^4*d^2+1176*c^2*d*e+1225*e^2)*(c^2*x^2-
1)^(1/2)/(c^2*x^2)^(1/2)-1/49*b*c*d^2*(c^2*x^2-1)^(1/2)/x^6/(c^2*x^2)^(1/2)
-2/1225*b*c*d*(15*c^2*d+49*e)*(c^2*x^2-1)^(1/2)/x^4/(c^2*x^2)^(1/2)-1/11025
*b*c*(360*c^4*d^2+1176*c^2*d*e+1225*e^2)*(c^2*x^2-1)^(1/2)/x^2/(c^2*x^2)^(1
/2)
```

Rubi [A] time = 0.20, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {270, 5239, 12, 1265, 453, 271, 264}

$$-\frac{d^2(a+b\csc^{-1}(cx))}{7x^7}-\frac{2de(a+b\csc^{-1}(cx))}{5x^5}-\frac{e^2(a+b\csc^{-1}(cx))}{3x^3}-\frac{2bc^3\sqrt{c^2x^2-1}\,(360c^4d^2+1176c^2de+1225e^2}{11025\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^8,x]
```

```
[Out] (-2*b*c^3*(360*c^4*d^2 + 1176*c^2*d*e + 1225*e^2)*Sqrt[-1 + c^2*x^2])/(1102
5*Sqrt[c^2*x^2]) - (b*c*d^2*Sqrt[-1 + c^2*x^2])/(49*x^6*Sqrt[c^2*x^2]) - (2
*b*c*d*(15*c^2*d + 49*e)*Sqrt[-1 + c^2*x^2])/(1225*x^4*Sqrt[c^2*x^2]) - (b*
c*(360*c^4*d^2 + 1176*c^2*d*e + 1225*e^2)*Sqrt[-1 + c^2*x^2])/(11025*x^2*Sq
rt[c^2*x^2]) - (d^2*(a + b*ArcCsc[c*x]))/(7*x^7) - (2*d*e*(a + b*ArcCsc[c*x
]))/(5*x^5) - (e^2*(a + b*ArcCsc[c*x]))/(3*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 1265

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)}{x^8}\,dx = -\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\csc^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\csc^{-1}(cx)\right)}{3x^3}+\frac{(b\cdots}{\cdots}$$

$$= -\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\csc^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\csc^{-1}(cx)\right)}{3x^3}+\frac{(b\cdots}{\cdots}$$

$$= -\frac{bcd^2\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}}-\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\csc^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a\cdots\right.}{\cdots}$$

$$= -\frac{bcd^2\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}}-\frac{2bcd\left(15c^2d+49e\right)\sqrt{-1+c^2x^2}}{1225x^4\sqrt{c^2x^2}}-\frac{d^2\left(a+b\csc^{-1}(c\cdots\right.}{7x^7}$$

$$= -\frac{bcd^2\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}}-\frac{2bcd\left(15c^2d+49e\right)\sqrt{-1+c^2x^2}}{1225x^4\sqrt{c^2x^2}}-\frac{bc\left(1225e^2+24c^2\cdots\right.}{1\cdots}$$

$$= -\frac{2bc^3\left(1225e^2+24c^2d\left(15c^2d+49e\right)\right)\sqrt{-1+c^2x^2}}{11025\sqrt{c^2x^2}}-\frac{bcd^2\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}}-\cdots$$

Mathematica [A] time = 0.28, size = 153, normalized size = 0.63

$$-\frac{105a\left(15d^2+42dex^2+35e^2x^4\right)+bcx\sqrt{1-\frac{1}{c^2x^2}}\left(1225e^2x^4\left(2c^2x^2+1\right)+294dex^2\left(8c^4x^4+4c^2x^2+3\right)+45d^{2}\cdots\right.}{11025x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^8,x]
```

```
[Out] -1/11025*(105*a*(15*d^2 + 42*d*e*x^2 + 35*e^2*x^4) + b*c*Sqrt[1 - 1/(c^2*x^
2)]*x*(1225*e^2*x^4*(1 + 2*c^2*x^2) + 294*d*e*x^2*(3 + 4*c^2*x^2 + 8*c^4*x^
4) + 45*d^2*(5 + 6*c^2*x^2 + 8*c^4*x^4 + 16*c^6*x^6)) + 105*b*(15*d^2 + 42*
d*e*x^2 + 35*e^2*x^4)*ArcCsc[c*x])/x^7
```

fricas [A] time = 0.69, size = 158, normalized size = 0.66

$$-\frac{3675\,ae^2x^4 + 4410\,adex^2 + 1575\,ad^2 + 105\left(35\,be^2x^4 + 42\,bdex^2 + 15\,bd^2\right)\operatorname{arccsc}(cx) + \left(2\left(360\,bc^6d^2 + 1176\,bc\right.\right.}{1102}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^8,x, algorithm="fricas")
```

```
[Out] -1/11025*(3675*a*e^2*x^4 + 4410*a*d*e*x^2 + 1575*a*d^2 + 105*(35*b*e^2*x^4
+ 42*b*d*e*x^2 + 15*b*d^2)*arccsc(c*x) + (2*(360*b*c^6*d^2 + 1176*b*c^4*d*e
 + 1225*b*c^2*e^2)*x^6 + (360*b*c^4*d^2 + 1176*b*c^2*d*e + 1225*b*e^2)*x^4
+ 225*b*d^2 + 18*(15*b*c^2*d^2 + 49*b*d*e)*x^2)*sqrt(c^2*x^2 - 1))/x^7
```

giac [B] time = 0.17, size = 493, normalized size = 2.05

$$-\frac{1}{11025}\left(225\,bc^6d^2\left(\frac{1}{c^2x^2}-1\right)^3\sqrt{-\frac{1}{c^2x^2}+1} + 945\,bc^6d^2\left(\frac{1}{c^2x^2}-1\right)^2\sqrt{-\frac{1}{c^2x^2}+1} + \frac{1575\,bc^5d^2\left(\frac{1}{c^2x^2}-1\right)^3\arcsin}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^8,x, algorithm="giac")
```

```
[Out] -1/11025*(225*b*c^6*d^2*(1/(c^2*x^2) - 1)^3*sqrt(-1/(c^2*x^2) + 1) + 945*b*
c^6*d^2*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1) + 1575*b*c^5*d^2*(1/(c^2
*x^2) - 1)^3*arcsin(1/(c*x))/x - 1575*b*c^6*d^2*(-1/(c^2*x^2) + 1)^(3/2) +
4725*b*c^5*d^2*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*x))/x + 1575*b*c^6*d^2*sqrt(
-1/(c^2*x^2) + 1) + 882*b*c^4*d*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1)*
e + 4725*b*c^5*d^2*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))/x - 2940*b*c^4*d*(-1/(
c^2*x^2) + 1)^(3/2)*e + 1575*b*c^5*d^2*arcsin(1/(c*x))/x + 4410*b*c^3*d*(1/
(c^2*x^2) - 1)^2*arcsin(1/(c*x))*e/x + 4410*b*c^4*d*sqrt(-1/(c^2*x^2) + 1)*
e + 8820*b*c^3*d*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))*e/x + 4410*b*c^3*d*arcsi
n(1/(c*x))*e/x - 1225*b*c^2*(-1/(c^2*x^2) + 1)^(3/2)*e^2 + 3675*b*c^2*sqrt(
-1/(c^2*x^2) + 1)*e^2 + 3675*b*c*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))*e^2/x +
3675*b*c*arcsin(1/(c*x))*e^2/x + 3675*a*e^2/(c*x^3) + 4410*a*d*e/(c*x^5) +
1575*a*d^2/(c*x^7))*c
```

maple [A] time = 0.07, size = 223, normalized size = 0.93

$$c^7\left(\frac{a\left(-\frac{d^2}{7c^3x^7}-\frac{e^2}{3c^3x^3}-\frac{2ed}{5c^3x^5}\right)}{c^4} + \frac{b\left(-\frac{\operatorname{arccsc}(cx)d^2}{7c^3x^7}-\frac{\operatorname{arccsc}(cx)e^2}{3c^3x^3}-\frac{2\operatorname{arccsc}(cx)ed}{5c^3x^5}-\frac{\left(c^2x^2-1\right)\left(720c^{10}d^2x^6+2352c^8ed\,x^6+360x^4c^8d^2+24\right.}{1}\right.}{c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccsc(c*x))/x^8,x)
```

```
[Out] c^7*(a/c^4*(-1/7*d^2/c^3/x^7-1/3*e^2/c^3/x^3-2/5/c^3*e*d/x^5)+b/c^4*(-1/7*a
rccsc(c*x)*d^2/c^3/x^7-1/3*arccsc(c*x)*e^2/c^3/x^3-2/5*arccsc(c*x)/c^3*e*d/
x^5-1/11025*(c^2*x^2-1)*(720*c^10*d^2*x^6+2352*c^8*d*e*x^6+360*c^8*d^2*x^4+
2450*c^6*e^2*x^6+1176*c^6*d*e*x^4+270*c^6*d^2*x^2+1225*c^4*e^2*x^4+882*c^4*
d*e*x^2+225*c^4*d^2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^8/x^8))
```

maxima [A] time = 0.34, size = 241, normalized size = 1.00

$$\frac{1}{245}bd^2\left(\frac{5c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{7}{2}}-21c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+35c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-35c^8\sqrt{-\frac{1}{c^2x^2}+1}}{c}-\frac{35\operatorname{arccsc}(cx)}{x^7}\right)-\frac{2}{75}b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^8,x, algorithm="maxima")
```

```
[Out] 1/245*b*d^2*((5*c^8*(-1/(c^2*x^2) + 1)^(7/2) - 21*c^8*(-1/(c^2*x^2) + 1)^(5
/2) + 35*c^8*(-1/(c^2*x^2) + 1)^(3/2) - 35*c^8*sqrt(-1/(c^2*x^2) + 1))/c -
35*arccsc(c*x)/x^7) - 2/75*b*d*e*((3*c^6*(-1/(c^2*x^2) + 1)^(5/2) - 10*c^6*
(-1/(c^2*x^2) + 1)^(3/2) + 15*c^6*sqrt(-1/(c^2*x^2) + 1))/c + 15*arccsc(c*x
)/x^5) + 1/9*b*e^2*((c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2)
 + 1))/c - 3*arccsc(c*x)/x^3) - 1/3*a*e^2/x^3 - 2/5*a*d*e/x^5 - 1/7*a*d^2/x
^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^8,x)
```

```
[Out] int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^8, x)
```

sympy [A] time = 59.50, size = 510, normalized size = 2.12

$$-\frac{ad^2}{7x^7}-\frac{2ade}{5x^5}-\frac{ae^2}{3x^3}-\frac{bd^2\operatorname{acsc}(cx)}{7x^7}-\frac{2bde\operatorname{acsc}(cx)}{5x^5}-\frac{be^2\operatorname{acsc}(cx)}{3x^3}-\frac{bd^2\left(\begin{cases}\frac{16c^7\sqrt{c^2x^2-1}}{35x}+\frac{8c^5\sqrt{c^2x^2-1}}{35x^3}+\frac{6c^3\sqrt{c^2x^2-1}}{35x^5}+ \\ \frac{16ic^7\sqrt{-c^2x^2+1}}{35x}+\frac{8ic^5\sqrt{-c^2x^2+1}}{35x^3}+\frac{6ic^3\sqrt{-c^2x^2}}{35x^5}\end{cases}\right)}{7c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acsc(c*x))/x**8,x)
```

```
[Out] -a*d**2/(7*x**7) - 2*a*d*e/(5*x**5) - a*e**2/(3*x**3) - b*d**2*acsc(c*x)/(7
*x**7) - 2*b*d*e*acsc(c*x)/(5*x**5) - b*e**2*acsc(c*x)/(3*x**3) - b*d**2*Pi
ecewise((16*c**7*sqrt(c**2*x**2 - 1)/(35*x) + 8*c**5*sqrt(c**2*x**2 - 1)/(3
5*x**3) + 6*c**3*sqrt(c**2*x**2 - 1)/(35*x**5) + c*sqrt(c**2*x**2 - 1)/(7*x
**7), Abs(c**2*x**2) > 1), (16*I*c**7*sqrt(-c**2*x**2 + 1)/(35*x) + 8*I*c**
5*sqrt(-c**2*x**2 + 1)/(35*x**3) + 6*I*c**3*sqrt(-c**2*x**2 + 1)/(35*x**5)
+ I*c*sqrt(-c**2*x**2 + 1)/(7*x**7), True))/(7*c) - 2*b*d*e*Piecewise((8*c*
*5*sqrt(c**2*x**2 - 1)/(15*x) + 4*c**3*sqrt(c**2*x**2 - 1)/(15*x**3) + c*sq
rt(c**2*x**2 - 1)/(5*x**5), Abs(c**2*x**2) > 1), (8*I*c**5*sqrt(-c**2*x**2
+ 1)/(15*x) + 4*I*c**3*sqrt(-c**2*x**2 + 1)/(15*x**3) + I*c*sqrt(-c**2*x**2
 + 1)/(5*x**5), True))/(5*c) - b*e**2*Piecewise((2*c**3*sqrt(c**2*x**2 - 1)
/(3*x) + c*sqrt(c**2*x**2 - 1)/(3*x**3), Abs(c**2*x**2) > 1), (2*I*c**3*sqr
t(-c**2*x**2 + 1)/(3*x) + I*c*sqrt(-c**2*x**2 + 1)/(3*x**3), True))/(3*c)
```

3.94 $\int x^3 \left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right)\,dx$

Optimal. Leaf size=242

$$\frac{1}{4}d^2x^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\csc^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\csc^{-1}(cx)\right)+\frac{bex\left(c^2x^2-1\right)^{5/2}\left(8c^2d+9e\right)}{120c^7\sqrt{c^2x^2}}+\frac{be^2x\left(c\right.}{56c}$$

```
[Out] 1/4*d^2*x^4*(a+b*arccsc(c*x))+1/3*d*e*x^6*(a+b*arccsc(c*x))+1/8*e^2*x^8*(a+
b*arccsc(c*x))+1/72*b*(6*c^4*d^2+16*c^2*d*e+9*e^2)*x*(c^2*x^2-1)^(3/2)/c^7/
(c^2*x^2)^(1/2)+1/120*b*e*(8*c^2*d+9*e)*x*(c^2*x^2-1)^(5/2)/c^7/(c^2*x^2)^(
1/2)+1/56*b*e^2*x*(c^2*x^2-1)^(7/2)/c^7/(c^2*x^2)^(1/2)+1/24*b*(6*c^4*d^2+8
*c^2*d*e+3*e^2)*x*(c^2*x^2-1)^(1/2)/c^7/(c^2*x^2)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {266, 43, 5239, 12, 1251, 771}

$$\frac{1}{4}d^2x^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\csc^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\csc^{-1}(cx)\right)+\frac{bx\left(c^2x^2-1\right)^{3/2}\left(6c^4d^2+16c^2de+9e^2\right.}{72c^7\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*(6*c^4*d^2 + 8*c^2*d*e + 3*e^2)*x*Sqrt[-1 + c^2*x^2])/(24*c^7*Sqrt[c^2*x
^2]) + (b*(6*c^4*d^2 + 16*c^2*d*e + 9*e^2)*x*(-1 + c^2*x^2)^(3/2))/(72*c^7*
Sqrt[c^2*x^2]) + (b*e*(8*c^2*d + 9*e)*x*(-1 + c^2*x^2)^(5/2))/(120*c^7*Sqrt
[c^2*x^2]) + (b*e^2*x*(-1 + c^2*x^2)^(7/2))/(56*c^7*Sqrt[c^2*x^2]) + (d^2*x
^4*(a + b*ArcCsc[c*x]))/4 + (d*e*x^6*(a + b*ArcCsc[c*x]))/3 + (e^2*x^8*(a +
 b*ArcCsc[c*x]))/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 771

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int x^3\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)dx=\frac{1}{4}d^2x^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\csc^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\csc^{-1}\right.$$

$$=\frac{1}{4}d^2x^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\csc^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\csc^{-1}\right.$$

$$=\frac{1}{4}d^2x^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\csc^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\csc^{-1}\right.$$

$$=\frac{1}{4}d^2x^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\csc^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\csc^{-1}\right.$$

$$=\frac{b\left(6c^4d^2+8c^2de+3e^2\right)x\sqrt{-1+c^2x^2}}{24c^7\sqrt{c^2x^2}}+\frac{b\left(6c^4d^2+16c^2de+9e^2\right)x\left(-1\right.}{72c^7\sqrt{c^2x^2}}$$

Mathematica [A] time = 0.34, size = 159, normalized size = 0.66

$$\frac{x\left(105ax^3\left(6d^2+8dex^2+3e^2x^4\right)+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\left(3c^6\left(70d^2x^2+56dex^4+15e^2x^6\right)+c^4\left(420d^2+224dex^2+54e^2x^4\right)+8c^2e\left(56d+9ex^2\right)+144e^2\right)}{c^7}\right.}{2520}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (x*(105*a*x^3*(6*d^2 + 8*d*e*x^2 + 3*e^2*x^4) + (b*Sqrt[1 - 1/(c^2*x^2)]*(1
44*e^2 + 8*c^2*e*(56*d + 9*e*x^2) + c^4*(420*d^2 + 224*d*e*x^2 + 54*e^2*x^4
) + 3*c^6*(70*d^2*x^2 + 56*d*e*x^4 + 15*e^2*x^6)))/c^7 + 105*b*x^3*(6*d^2 +
 8*d*e*x^2 + 3*e^2*x^4)*ArcCsc[c*x]))/2520
```

fricas [A] time = 0.75, size = 186, normalized size = 0.77

$$\frac{315\,ac^8e^2x^8+840\,ac^8dex^6+630\,ac^8d^2x^4+105\left(3\,bc^8e^2x^8+8\,bc^8dex^6+6\,bc^8d^2x^4\right)\operatorname{arccsc}(cx)+\left(45\,bc^6e^2x^6\right.}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/2520*(315*a*c^8*e^2*x^8 + 840*a*c^8*d*e*x^6 + 630*a*c^8*d^2*x^4 + 105*(3*
b*c^8*e^2*x^8 + 8*b*c^8*d*e*x^6 + 6*b*c^8*d^2*x^4)*arccsc(c*x) + (45*b*c^6*
e^2*x^6 + 420*b*c^4*d^2 + 448*b*c^2*d*e + 6*(28*b*c^6*d*e + 9*b*c^4*e^2)*x^
4 + 144*b*e^2 + 2*(105*b*c^6*d^2 + 112*b*c^4*d*e + 36*b*c^2*e^2)*x^2)*sqrt(
c^2*x^2 - 1))/c^8
```

giac [B] time = 0.45, size = 1700, normalized size = 7.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/645120*(315*b*x^8*(sqrt(-1/(c^2*x^2) + 1) + 1)^8*arcsin(1/(c*x))*e^2/c +
315*a*x^8*(sqrt(-1/(c^2*x^2) + 1) + 1)^8*e^2/c + 3360*b*d*x^6*(sqrt(-1/(c^2
*x^2) + 1) + 1)^6*arcsin(1/(c*x))*e/c + 90*b*x^7*(sqrt(-1/(c^2*x^2) + 1) +
1)^7*e^2/c^2 + 3360*a*d*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*e/c + 2520*b*x^6
*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*arcsin(1/(c*x))*e^2/c^3 + 10080*b*d^2*x^4*(
sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))/c + 2520*a*x^6*(sqrt(-1/(c^2*
x^2) + 1) + 1)^6*e^2/c^3 + 1344*b*d*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*e/c^
2 + 10080*a*d^2*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c + 20160*b*d*x^4*(sqrt(
-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))*e/c^3 + 882*b*x^5*(sqrt(-1/(c^2*x^
2) + 1) + 1)^5*e^2/c^4 + 20160*a*d*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*e/c^3
 + 6720*b*d^2*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^2 + 8820*b*x^4*(sqrt(-1/
(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))*e^2/c^5 + 40320*b*d^2*x^2*(sqrt(-1/(c
^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c^3 + 8820*a*x^4*(sqrt(-1/(c^2*x^2) + 1
) + 1)^4*e^2/c^5 + 11200*b*d*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*e/c^4 + 403
20*a*d^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^3 + 50400*b*d*x^2*(sqrt(-1/(c
^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))*e/c^5 + 4410*b*x^3*(sqrt(-1/(c^2*x^2) +
 1) + 1)^3*e^2/c^6 + 50400*a*d*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*e/c^5 + 6
0480*b*d^2*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 + 17640*b*x^2*(sqrt(-1/(c^2*x
^2) + 1) + 1)^2*arcsin(1/(c*x))*e^2/c^7 + 60480*b*d^2*arcsin(1/(c*x))/c^5 +
 17640*a*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*e^2/c^7 + 67200*b*d*x*(sqrt(-1/
(c^2*x^2) + 1) + 1)*e/c^6 + 60480*a*d^2/c^5 + 67200*b*d*arcsin(1/(c*x))*e/c
^7 + 22050*b*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*e^2/c^8 + 67200*a*d*e/c^7 - 604
80*b*d^2/(c^6*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 22050*b*arcsin(1/(c*x))*e^2
/c^9 + 40320*b*d^2*arcsin(1/(c*x))/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2)
 + 22050*a*e^2/c^9 - 67200*b*d*e/(c^8*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 403
20*a*d^2/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 50400*b*d*arcsin(1/(c*x
))*e/(c^9*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) - 22050*b*e^2/(c^10*x*(sqrt(-
1/(c^2*x^2) + 1) + 1)) + 50400*a*d*e/(c^9*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^
2) - 6720*b*d^2/(c^8*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 17640*b*arcsin(1
/(c*x))*e^2/(c^11*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 10080*b*d^2*arcsin(
1/(c*x))/(c^9*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) + 17640*a*e^2/(c^11*x^2*(
sqrt(-1/(c^2*x^2) + 1) + 1)^2) - 11200*b*d*e/(c^10*x^3*(sqrt(-1/(c^2*x^2) +
 1) + 1)^3) + 10080*a*d^2/(c^9*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) + 20160*
b*d*arcsin(1/(c*x))*e/(c^11*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) - 4410*b*e^
2/(c^12*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 20160*a*d*e/(c^11*x^4*(sqrt(-
1/(c^2*x^2) + 1) + 1)^4) + 8820*b*arcsin(1/(c*x))*e^2/(c^13*x^4*(sqrt(-1/(c
^2*x^2) + 1) + 1)^4) + 8820*a*e^2/(c^13*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4)
 - 1344*b*d*e/(c^12*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 3360*b*d*arcsin(1
/(c*x))*e/(c^13*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6) - 882*b*e^2/(c^14*x^5*(
sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 3360*a*d*e/(c^13*x^6*(sqrt(-1/(c^2*x^2) +
1) + 1)^6) + 2520*b*arcsin(1/(c*x))*e^2/(c^15*x^6*(sqrt(-1/(c^2*x^2) + 1) +
 1)^6) + 2520*a*e^2/(c^15*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6) - 90*b*e^2/(c
^16*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7) + 315*b*arcsin(1/(c*x))*e^2/(c^17*x
^8*(sqrt(-1/(c^2*x^2) + 1) + 1)^8) + 315*a*e^2/(c^17*x^8*(sqrt(-1/(c^2*x^2)
 + 1) + 1)^8))*c
```

maple [A] time = 0.06, size = 214, normalized size = 0.88

$$\frac{\frac{a\left(\frac{1}{8}e^2c^8x^8+\frac{1}{3}c^8ed\,x^6+\frac{1}{4}x^4c^8d^2\right)}{c^4}+\frac{b\left(\frac{\mathrm{arccsc}(cx)e^2c^8x^8}{8}+\frac{\mathrm{arccsc}(cx)c^8ed\,x^6}{3}+\frac{\mathrm{arccsc}(cx)c^8x^4d^2}{4}+\frac{(c^2x^2-1)(45e^2c^6x^6+168c^6ed\,x^4+210x^2c^6d^2+54c^4e^2x^4+224c^4de\,x^2\cdots}{2520\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right.}{c^4}}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^2*(a+b*arccsc(c*x)),x)
```

```
[Out] 1/c^4*(a/c^4*(1/8*e^2*c^8*x^8+1/3*c^8*e*d*x^6+1/4*x^4*c^8*d^2)+b/c^4*(1/8*a
rccsc(c*x)*e^2*c^8*x^8+1/3*arccsc(c*x)*c^8*e*d*x^6+1/4*arccsc(c*x)*c^8*x^4*
d^2+1/2520*(c^2*x^2-1)*(45*c^6*e^2*x^6+168*c^6*d*e*x^4+210*c^6*d^2*x^2+54*c
^4*e^2*x^4+224*c^4*d*e*x^2+420*c^4*d^2+72*c^2*e^2*x^2+448*c^2*d*e+144*e^2)/
((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x))
```

maxima [A] time = 0.35, size = 253, normalized size = 1.05

$$\frac{1}{8}\,ae^2x^8+\frac{1}{3}\,adex^6+\frac{1}{4}\,ad^2x^4+\frac{1}{12}\left(3\,x^4\,\mathrm{arccsc}\left(cx\right)+\frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+3\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)bd^2+\frac{1}{45}\left(15\,x^6\,\mathrm{arccsc}\,(\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*e^2*x^8 + 1/3*a*d*e*x^6 + 1/4*a*d^2*x^4 + 1/12*(3*x^4*arccsc(c*x) + (
c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*b*d^2 +
 1/45*(15*x^6*arccsc(c*x) + (3*c^4*x^5*(-1/(c^2*x^2) + 1)^(5/2) + 10*c^2*x^
3*(-1/(c^2*x^2) + 1)^(3/2) + 15*x*sqrt(-1/(c^2*x^2) + 1))/c^5)*b*d*e + 1/28
0*(35*x^8*arccsc(c*x) + (5*c^6*x^7*(-1/(c^2*x^2) + 1)^(7/2) + 21*c^4*x^5*(-
1/(c^2*x^2) + 1)^(5/2) + 35*c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 35*x*sqrt(-1
/(c^2*x^2) + 1))/c^7)*b*e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\left(e\,x^2+d\right)^2\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d + e*x^2)^2*(a + b*asin(1/(c*x))),x)
```

```
[Out] int(x^3*(d + e*x^2)^2*(a + b*asin(1/(c*x))), x)
```

sympy [A] time = 9.22, size = 493, normalized size = 2.04

$$\frac{ad^2x^4}{4}+\frac{adex^6}{3}+\frac{ae^2x^8}{8}+\frac{bd^2x^4\,\mathrm{acsc}\,(cx)}{4}+\frac{bdex^6\,\mathrm{acsc}\,(cx)}{3}+\frac{be^2x^8\,\mathrm{acsc}\,(cx)}{8}+\frac{bd^2\left(\begin{cases}\frac{x^2\sqrt{c^2x^2-1}}{3c}+\frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{fo}\cdots\\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c}+\frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{ot}\cdots\end{cases}\right.}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**2*(a+b*acsc(c*x)),x)
```

```
[Out] a*d**2*x**4/4 + a*d*e*x**6/3 + a*e**2*x**8/8 + b*d**2*x**4*acsc(c*x)/4 + b*
d*e*x**6*acsc(c*x)/3 + b*e**2*x**8*acsc(c*x)/8 + b*d**2*Piecewise((x**2*sqr
```

```
t(c**2*x**2 - 1)/(3*c) + 2*sqrt(c**2*x**2 - 1)/(3*c**3), Abs(c**2*x**2) > 1
), (I*x**2*sqrt(-c**2*x**2 + 1)/(3*c) + 2*I*sqrt(-c**2*x**2 + 1)/(3*c**3),
True))/(4*c) + b*d*e*Piecewise((x**4*sqrt(c**2*x**2 - 1)/(5*c) + 4*x**2*sqr
t(c**2*x**2 - 1)/(15*c**3) + 8*sqrt(c**2*x**2 - 1)/(15*c**5), Abs(c**2*x**2
) > 1), (I*x**4*sqrt(-c**2*x**2 + 1)/(5*c) + 4*I*x**2*sqrt(-c**2*x**2 + 1)/
(15*c**3) + 8*I*sqrt(-c**2*x**2 + 1)/(15*c**5), True))/(3*c) + b*e**2*Piece
wise((x**6*sqrt(c**2*x**2 - 1)/(7*c) + 6*x**4*sqrt(c**2*x**2 - 1)/(35*c**3)
 + 8*x**2*sqrt(c**2*x**2 - 1)/(35*c**5) + 16*sqrt(c**2*x**2 - 1)/(35*c**7),
 Abs(c**2*x**2) > 1), (I*x**6*sqrt(-c**2*x**2 + 1)/(7*c) + 6*I*x**4*sqrt(-c
**2*x**2 + 1)/(35*c**3) + 8*I*x**2*sqrt(-c**2*x**2 + 1)/(35*c**5) + 16*I*sq
rt(-c**2*x**2 + 1)/(35*c**7), True))/(8*c)
```

3.95 $\int x\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)\,dx$

Optimal. Leaf size=195

$$\frac{\left(d+ex^2\right)^3\left(a+b\csc^{-1}(cx)\right)}{6e}+\frac{bcd^3x\tan^{-1}\left(\sqrt{c^2x^2-1}\right)}{6e\sqrt{c^2x^2}}+\frac{bex\left(c^2x^2-1\right)^{3/2}\left(3c^2d+2e\right)}{18c^5\sqrt{c^2x^2}}+\frac{be^2x\left(c^2x^2-1\right)^{5/2}}{30c^5\sqrt{c^2x^2}}+\frac{bx}{}$$

```
[Out] 1/6*(e*x^2+d)^3*(a+b*arccsc(c*x))/e+1/18*b*e*(3*c^2*d+2*e)*x*(c^2*x^2-1)^(3
/2)/c^5/(c^2*x^2)^(1/2)+1/30*b*e^2*x*(c^2*x^2-1)^(5/2)/c^5/(c^2*x^2)^(1/2)+
1/6*b*c*d^3*x*arctan((c^2*x^2-1)^(1/2))/e/(c^2*x^2)^(1/2)+1/6*b*(3*c^4*d^2+
3*c^2*d*e+e^2)*x*(c^2*x^2-1)^(1/2)/c^5/(c^2*x^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5237, 446, 88, 63, 205}

$$\frac{\left(d+ex^2\right)^3\left(a+b\csc^{-1}(cx)\right)}{6e}+\frac{bx\sqrt{c^2x^2-1}\left(3c^4d^2+3c^2de+e^2\right)}{6c^5\sqrt{c^2x^2}}+\frac{bcd^3x\tan^{-1}\left(\sqrt{c^2x^2-1}\right)}{6e\sqrt{c^2x^2}}+\frac{bex\left(c^2x^2-1\right)^{3/2}(}{18c^5\sqrt{c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*(3*c^4*d^2 + 3*c^2*d*e + e^2)*x*Sqrt[-1 + c^2*x^2])/(6*c^5*Sqrt[c^2*x^2]
) + (b*e*(3*c^2*d + 2*e)*x*(-1 + c^2*x^2)^(3/2))/(18*c^5*Sqrt[c^2*x^2]) + (
b*e^2*x*(-1 + c^2*x^2)^(5/2))/(30*c^5*Sqrt[c^2*x^2]) + ((d + e*x^2)^3*(a +
b*ArcCsc[c*x]))/(6*e) + (b*c*d^3*x*ArcTan[Sqrt[-1 + c^2*x^2]])/(6*e*Sqrt[c^
2*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5237

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCsc[c*x]))/(2*e*(p + 1)), x
] + Dist[(b*c*x)/(2*e*(p + 1)*Sqrt[c^2*x^2]), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\int x\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)dx=\frac{\left(d+ex^2\right)^3\left(a+b\csc^{-1}(cx)\right)}{6e}+\frac{(bcx)\int\frac{\left(d+ex^2\right)^3}{x\sqrt{-1+c^2x^2}}dx}{6e\sqrt{c^2x^2}}$$

$$=\frac{\left(d+ex^2\right)^3\left(a+b\csc^{-1}(cx)\right)}{6e}+\frac{(bcx)\,\text{Subst}\left(\int\frac{(d+ex)^3}{x\sqrt{-1+c^2x}}dx,x,x^2\right)}{12e\sqrt{c^2x^2}}$$

$$=\frac{\left(d+ex^2\right)^3\left(a+b\csc^{-1}(cx)\right)}{6e}+\frac{(bcx)\,\text{Subst}\left(\int\left(\frac{e\left(3c^4d^2+3c^2de+e^2\right)}{c^4\sqrt{-1+c^2x}}+\frac{d^3}{x\sqrt{-1+c^2x}}\right.\right.}{12e\sqrt{}}$$

$$=\frac{b\left(3c^4d^2+3c^2de+e^2\right)x\sqrt{-1+c^2x^2}}{6c^5\sqrt{c^2x^2}}+\frac{be\left(3c^2d+2e\right)x\left(-1+c^2x^2\right)^{3/2}}{18c^5\sqrt{c^2x^2}}+$$

$$=\frac{b\left(3c^4d^2+3c^2de+e^2\right)x\sqrt{-1+c^2x^2}}{6c^5\sqrt{c^2x^2}}+\frac{be\left(3c^2d+2e\right)x\left(-1+c^2x^2\right)^{3/2}}{18c^5\sqrt{c^2x^2}}+$$

$$=\frac{b\left(3c^4d^2+3c^2de+e^2\right)x\sqrt{-1+c^2x^2}}{6c^5\sqrt{c^2x^2}}+\frac{be\left(3c^2d+2e\right)x\left(-1+c^2x^2\right)^{3/2}}{18c^5\sqrt{c^2x^2}}+$$

Mathematica [A] time = 0.29, size = 124, normalized size = 0.64

$$\frac{1}{90}x\left(15ax\left(3d^2+3dex^2+e^2x^4\right)+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\left(3c^4\left(15d^2+5dex^2+e^2x^4\right)+2c^2e\left(15d+2ex^2\right)+8e^2\right)}{c^5}+15bx\csc\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (x*(15*a*x*(3*d^2 + 3*d*e*x^2 + e^2*x^4) + (b*Sqrt[1 - 1/(c^2*x^2)]*(8*e^2
+ 2*c^2*e*(15*d + 2*e*x^2) + 3*c^4*(15*d^2 + 5*d*e*x^2 + e^2*x^4)))/c^5 + 1
5*b*x*(3*d^2 + 3*d*e*x^2 + e^2*x^4)*ArcCsc[c*x]))/90
```

fricas [A] time = 0.48, size = 152, normalized size = 0.78

$$\frac{15\,ac^6e^2x^6+45\,ac^6dex^4+45\,ac^6d^2x^2+15\left(bc^6e^2x^6+3\,bc^6dex^4+3\,bc^6d^2x^2\right)\text{arccsc}\left(cx\right)+\left(3\,bc^4e^2x^4+45\,bc^4d^2\right.}{90\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/90*(15*a*c^6*e^2*x^6 + 45*a*c^6*d*e*x^4 + 45*a*c^6*d^2*x^2 + 15*(b*c^6*e^
2*x^6 + 3*b*c^6*d*e*x^4 + 3*b*c^6*d^2*x^2)*arccsc(c*x) + (3*b*c^4*e^2*x^4 +
 45*b*c^4*d^2 + 30*b*c^2*d*e + 8*b*e^2 + (15*b*c^4*d*e + 4*b*c^2*e^2)*x^2)*
sqrt(c^2*x^2 - 1))/c^6
```

giac [B] time = 0.32, size = 1154, normalized size = 5.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/5760*(15*b*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*arcsin(1/(c*x))*e^2/c + 15*
a*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*e^2/c + 180*b*d*x^4*(sqrt(-1/(c^2*x^2)
 + 1) + 1)^4*arcsin(1/(c*x))*e/c + 6*b*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*e
^2/c^2 + 180*a*d*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*e/c + 90*b*x^4*(sqrt(-1
/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))*e^2/c^3 + 720*b*d^2*x^2*(sqrt(-1/(c^
2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c + 90*a*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1
)^4*e^2/c^3 + 120*b*d*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*e/c^2 + 720*a*d^2*
x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c + 720*b*d*x^2*(sqrt(-1/(c^2*x^2) + 1)
+ 1)^2*arcsin(1/(c*x))*e/c^3 + 50*b*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*e^2/
c^4 + 720*a*d*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*e/c^3 + 1440*b*d^2*x*(sqrt
(-1/(c^2*x^2) + 1) + 1)/c^2 + 225*b*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcs
in(1/(c*x))*e^2/c^5 + 1440*b*d^2*arcsin(1/(c*x))/c^3 + 225*a*x^2*(sqrt(-1/(
c^2*x^2) + 1) + 1)^2*e^2/c^5 + 1080*b*d*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*e/c^
4 + 1440*a*d^2/c^3 + 1080*b*d*arcsin(1/(c*x))*e/c^5 + 300*b*x*(sqrt(-1/(c^2
*x^2) + 1) + 1)*e^2/c^6 + 1080*a*d*e/c^5 - 1440*b*d^2/(c^4*x*(sqrt(-1/(c^2*
x^2) + 1) + 1)) + 300*b*arcsin(1/(c*x))*e^2/c^7 + 720*b*d^2*arcsin(1/(c*x))
/(c^5*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 300*a*e^2/c^7 - 1080*b*d*e/(c^6
*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 720*a*d^2/(c^5*x^2*(sqrt(-1/(c^2*x^2) +
1) + 1)^2) + 720*b*d*arcsin(1/(c*x))*e/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1
)^2) - 300*b*e^2/(c^8*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 720*a*d*e/(c^7*x^2*
(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 225*b*arcsin(1/(c*x))*e^2/(c^9*x^2*(sqrt(
-1/(c^2*x^2) + 1) + 1)^2) + 225*a*e^2/(c^9*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)
^2) - 120*b*d*e/(c^8*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 180*b*d*arcsin(1
/(c*x))*e/(c^9*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) - 50*b*e^2/(c^10*x^3*(sq
rt(-1/(c^2*x^2) + 1) + 1)^3) + 180*a*d*e/(c^9*x^4*(sqrt(-1/(c^2*x^2) + 1) +
 1)^4) + 90*b*arcsin(1/(c*x))*e^2/(c^11*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4)
 + 90*a*e^2/(c^11*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) - 6*b*e^2/(c^12*x^5*(
sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 15*b*arcsin(1/(c*x))*e^2/(c^13*x^6*(sqrt(-
1/(c^2*x^2) + 1) + 1)^6) + 15*a*e^2/(c^13*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^
6))*c
```

maple [A] time = 0.06, size = 182, normalized size = 0.93

$$\frac{\frac{a\left(\frac{1}{6}e^{2}c^{6}x^{6}+\frac{1}{2}c^{6}ed\,x^{4}+\frac{1}{2}x^{2}c^{6}d^{2}\right)}{c^{4}}+\frac{b\left(\frac{\operatorname{arccsc}(cx)e^{2}c^{6}x^{6}}{6}+\frac{\operatorname{arccsc}(cx)c^{6}x^{4}de}{2}+\frac{\operatorname{arccsc}(cx)c^{6}x^{2}d^{2}}{2}+\frac{\left(c^{2}x^{2}-1\right)\left(3c^{4}e^{2}x^{4}+15c^{4}de\,x^{2}+45d^{2}c^{4}+4c^{2}e^{2}x^{2}+30c^{2}ed+8e^{2}\right)}{90\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,cx}\right)}{c^{4}}}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^2*(a+b*arccsc(c*x)),x)
```

```
[Out] 1/c^2*(a/c^4*(1/6*e^2*c^6*x^6+1/2*c^6*e*d*x^4+1/2*x^2*c^6*d^2)+b/c^4*(1/6*a
rccsc(c*x)*e^2*c^6*x^6+1/2*arccsc(c*x)*c^6*x^4*d*e+1/2*arccsc(c*x)*c^6*x^2*
d^2+1/90*(c^2*x^2-1)*(3*c^4*e^2*x^4+15*c^4*d*e*x^2+45*c^4*d^2+4*c^2*e^2*x^2
+30*c^2*d*e+8*e^2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x))
```

maxima [A] time = 0.34, size = 189, normalized size = 0.97

$$\frac{1}{6}\,ae^{2}x^{6}+\frac{1}{2}\,adex^{4}+\frac{1}{2}\,ad^{2}x^{2}+\frac{1}{2}\left(x^{2}\operatorname{arccsc}(cx)+\frac{x\sqrt{-\frac{1}{c^{2}x^{2}}+1}}{c}\right)bd^{2}+\frac{1}{6}\left(3\,x^{4}\operatorname{arccsc}(cx)+\frac{c^{2}x^{3}\left(-\frac{1}{c^{2}x^{2}}+1\right)^{\frac{3}{2}}+3}{c^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*e^2*x^6 + 1/2*a*d*e*x^4 + 1/2*a*d^2*x^2 + 1/2*(x^2*arccsc(c*x) + x*sq
rt(-1/(c^2*x^2) + 1)/c)*b*d^2 + 1/6*(3*x^4*arccsc(c*x) + (c^2*x^3*(-1/(c^2*
x^2) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*b*d*e + 1/90*(15*x^6*arc
csc(c*x) + (3*c^4*x^5*(-1/(c^2*x^2) + 1)^(5/2) + 10*c^2*x^3*(-1/(c^2*x^2) +
 1)^(3/2) + 15*x*sqrt(-1/(c^2*x^2) + 1))/c^5)*b*e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(e\,x^2+d\right)^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d + e*x^2)^2*(a + b*asin(1/(c*x))),x)
```

```
[Out] int(x*(d + e*x^2)^2*(a + b*asin(1/(c*x))), x)
```

sympy [A] time = 6.38, size = 352, normalized size = 1.81

$$\frac{ad^2x^2}{2}+\frac{adex^4}{2}+\frac{ae^2x^6}{6}+\frac{bd^2x^2\operatorname{acsc}(cx)}{2}+\frac{bdex^4\operatorname{acsc}(cx)}{2}+\frac{be^2x^6\operatorname{acsc}(cx)}{6}+\frac{bd^2\left(\begin{cases}\frac{\sqrt{c^2x^2-1}}{c} & \text{for } \left|c^2x^2\right|>1\\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)}{2c}+\frac{bde\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**2*(a+b*acsc(c*x)),x)
```

```
[Out] a*d**2*x**2/2 + a*d*e*x**4/2 + a*e**2*x**6/6 + b*d**2*x**2*acsc(c*x)/2 + b*
d*e*x**4*acsc(c*x)/2 + b*e**2*x**6*acsc(c*x)/6 + b*d**2*Piecewise((sqrt(c**
2*x**2 - 1)/c, Abs(c**2*x**2) > 1), (I*sqrt(-c**2*x**2 + 1)/c, True))/(2*c)
 + b*d*e*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3*c) + 2*sqrt(c**2*x**2 - 1)/
(3*c**3), Abs(c**2*x**2) > 1), (I*x**2*sqrt(-c**2*x**2 + 1)/(3*c) + 2*I*sqr
t(-c**2*x**2 + 1)/(3*c**3), True))/(2*c) + b*e**2*Piecewise((x**4*sqrt(c**2
*x**2 - 1)/(5*c) + 4*x**2*sqrt(c**2*x**2 - 1)/(15*c**3) + 8*sqrt(c**2*x**2
- 1)/(15*c**5), Abs(c**2*x**2) > 1), (I*x**4*sqrt(-c**2*x**2 + 1)/(5*c) + 4
*I*x**2*sqrt(-c**2*x**2 + 1)/(15*c**3) + 8*I*sqrt(-c**2*x**2 + 1)/(15*c**5)
, True))/(6*c)
```

3.96 $\int \frac{\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=186

$$-d^2\log\left(\frac{1}{x}\right)\left(a+b\csc^{-1}(cx)\right)+dex^2\left(a+b\csc^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\csc^{-1}(cx)\right)+\frac{be^2x^3\sqrt{1-\frac{1}{c^2x^2}}}{12c}+\frac{bex\sqrt{1-\frac{1}{c^2x^2}}}{6c}$$

```
[Out] 1/2*I*b*d^2*arccsc(c*x)^2+d*e*x^2*(a+b*arccsc(c*x))+1/4*e^2*x^4*(a+b*arccsc
(c*x))-b*d^2*arccsc(c*x)*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)+b*d^2*arccsc(c
*x)*ln(1/x)-d^2*(a+b*arccsc(c*x))*ln(1/x)+1/2*I*b*d^2*polylog(2,(I/c/x+(1-1
/c^2/x^2)^(1/2))^2)+1/6*b*e*(6*c^2*d+e)*x*(1-1/c^2/x^2)^(1/2)/c^3+1/12*b*e^
2*x^3*(1-1/c^2/x^2)^(1/2)/c
```

Rubi [A] time = 0.42, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 13, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {5241, 266, 43, 4731, 6742, 453, 264, 2326, 4625, 3717, 2190, 2279, 2391}

$$\frac{1}{2}ibd^2\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)-d^2\log\left(\frac{1}{x}\right)\left(a+b\csc^{-1}(cx)\right)+dex^2\left(a+b\csc^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\csc^{-1}(cx)\right)+\frac{b}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x,x]
```

```
[Out] (b*e*(6*c^2*d + e)*Sqrt[1 - 1/(c^2*x^2)]*x)/(6*c^3) + (b*e^2*Sqrt[1 - 1/(c^
2*x^2)]*x^3)/(12*c) + (I/2)*b*d^2*ArcCsc[c*x]^2 + d*e*x^2*(a + b*ArcCsc[c*x
]) + (e^2*x^4*(a + b*ArcCsc[c*x]))/4 - b*d^2*ArcCsc[c*x]*Log[1 - E^((2*I)*A
rcCsc[c*x])] + b*d^2*ArcCsc[c*x]*Log[x^(-1)] - d^2*(a + b*ArcCsc[c*x])*Log[
x^(-1)] + (I/2)*b*d^2*PolyLog[2, E^((2*I)*ArcCsc[c*x])]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2326

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[(ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/Rt[-e, 2], x
] - Dist[(b*n)/Rt[-e, 2], Int[ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4625

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n/Tan[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 4731

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSin[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 - c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e, 0] &
& IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Rule 5241

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSin[x/c])^n)/x^
(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right)}{x}\,dx = -\text{Subst}\left(\int \frac{\left(e+dx^2\right)^2 \left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{x^5}\,dx, x, \frac{1}{x}\right)$$

$$= dex^2\left(a+b\csc^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\csc^{-1}(cx)\right)-d^2\left(a+b\csc^{-1}(cx)\right)\log$$

$$= dex^2\left(a+b\csc^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\csc^{-1}(cx)\right)-d^2\left(a+b\csc^{-1}(cx)\right)\log$$

$$= dex^2\left(a+b\csc^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\csc^{-1}(cx)\right)-d^2\left(a+b\csc^{-1}(cx)\right)\log$$

$$= \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c}+dex^2\left(a+b\csc^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\csc^{-1}(cx)\right)+bd^2$$

$$= \frac{be\left(6c^2d+e\right)\sqrt{1-\frac{1}{c^2x^2}}\,x}{6c^3}+\frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c}+dex^2\left(a+b\csc^{-1}(cx)\right)+\frac{1}{4}$$

$$= \frac{be\left(6c^2d+e\right)\sqrt{1-\frac{1}{c^2x^2}}\,x}{6c^3}+\frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c}+\frac{1}{2}ibd^2\csc^{-1}(cx)^2+dex^2\left(a\right.$$

$$= \frac{be\left(6c^2d+e\right)\sqrt{1-\frac{1}{c^2x^2}}\,x}{6c^3}+\frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c}+\frac{1}{2}ibd^2\csc^{-1}(cx)^2+dex^2\left(a\right.$$

$$= \frac{be\left(6c^2d+e\right)\sqrt{1-\frac{1}{c^2x^2}}\,x}{6c^3}+\frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c}+\frac{1}{2}ibd^2\csc^{-1}(cx)^2+dex^2\left(a\right.$$

$$= \frac{be\left(6c^2d+e\right)\sqrt{1-\frac{1}{c^2x^2}}\,x}{6c^3}+\frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c}+\frac{1}{2}ibd^2\csc^{-1}(cx)^2+dex^2\left(a\right.$$

Mathematica [A] time = 0.39, size = 157, normalized size = 0.84

$$ad^2\log(x)+adex^2+\frac{1}{4}ae^2x^4+\frac{bdex\left(\sqrt{1-\frac{1}{c^2x^2}}+cx\csc^{-1}(cx)\right)}{c}+\frac{be^2x\left(3c^3x^3\csc^{-1}(cx)+\sqrt{1-\frac{1}{c^2x^2}}\left(c^2x^2+2\right)\right)}{12c^3}+$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x,x]
```

```
[Out] a*d*e*x^2 + (a*e^2*x^4)/4 + (b*d*e*x*(Sqrt[1 - 1/(c^2*x^2)] + c*x*ArcCsc[c*
x]))/c + (b*e^2*x*(Sqrt[1 - 1/(c^2*x^2)]*(2 + c^2*x^2) + 3*c^3*x^3*ArcCsc[c
*x]))/(12*c^3) + a*d^2*Log[x] + (I/2)*b*d^2*(ArcCsc[c*x]*(ArcCsc[c*x] + (2*
I)*Log[1 - E^((2*I)*ArcCsc[c*x])]) + PolyLog[2, E^((2*I)*ArcCsc[c*x])])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ae^2x^4 + 2\,adex^2 + ad^2 + \left(be^2x^4 + 2\,bdex^2 + bd^2\right)\operatorname{arccsc}(cx)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^
2)*arccsc(c*x))/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(x)]Undef/Unsigne
d Inf encountered in limitEvaluation time: 5.5Limit: Max order reached or u
nable to make series expansion Error: Bad Argument Value
```

maple [A] time = 6.18, size = 301, normalized size = 1.62

$$\frac{a\,e^2x^4}{4}+aed\,x^2+a\,d^2\ln(cx)+\frac{ib\,d^2\operatorname{arccsc}(cx)^2}{2}+\frac{b\operatorname{arccsc}(cx)\,e^2x^4}{4}+b\operatorname{arccsc}(cx)\,x^2de+\frac{b\sqrt{\frac{c^2x^2-1}{c^2x^2}}\;x^3e^2}{12c}+\frac{b\sqrt{\frac{c^2x^2-1}{c^2x^2}}\;x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccsc(c*x))/x,x)
```

```
[Out] 1/4*a*e^2*x^4+a*e*d*x^2+a*d^2*ln(c*x)+1/2*I*b*d^2*arccsc(c*x)^2+1/4*b*arccs
c(c*x)*e^2*x^4+b*arccsc(c*x)*x^2*d*e+1/12*b/c*((c^2*x^2-1)/c^2/x^2)^(1/2)*x
^3*e^2+b/c*((c^2*x^2-1)/c^2/x^2)^(1/2)*x*d*e-I*b/c^2*d*e+1/6*b/c^3*((c^2*x^
2-1)/c^2/x^2)^(1/2)*x*e^2-1/6*I*b/c^4*e^2-b*d^2*arccsc(c*x)*ln(1+I/c/x+(1-1
/c^2/x^2)^(1/2))-b*d^2*arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))+I*b*d^2*
polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))+I*b*d^2*polylog(2,I/c/x+(1-1/c^2/x^2)
^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\,ae^2x^4+adex^2+ad^2\log(x)+\frac{4i\,bc^4d^2\log(-cx+1)\log(x)+4i\,bc^4d^2\log(x)^2+4i\,bc^4d^2\mathrm{Li}_2(cx)+4i\,bc^4d^2\mathrm{Li}_2(-cx)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/4*a*e^2*x^4 + a*d*e*x^2 + a*d^2*log(x) + 1/8*(4*I*b*c^4*d^2*log(-c*x + 1)
*log(x) + 4*I*b*c^4*d^2*log(x)^2 + 4*I*b*c^4*d^2*dilog(c*x) + 4*I*b*c^4*d^2
*dilog(-c*x) + (2*b*c^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 2*I*b*c^4
*log(c))*e^2*x^4 - I*(b*e^2*(x^2/c^2 + log(c*x + 1)/c^4 + log(c*x - 1)/c^4)
 + 4*b*d*e*(log(c*x + 1)/c^2 + log(c*x - 1)/c^2) + 32*b*d^2*integrate(1/4*l
og(x)/(c^2*x^3 - x), x))*c^4 + 8*c^4*integrate(1/4*(b*e^2*x^4 + 4*b*d*e*x^2
 + 4*b*d^2*log(x))*sqrt(c*x + 1)*sqrt(c*x - 1)/(c^2*x^3 - x), x) + (I*b*c^2
*e^2 + (8*b*c^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 8*I*b*c^4*log(c))
```

```
*d*e)*x^2 + (-I*b*c^4*e^2*x^4 - 4*I*b*c^4*d*e*x^2 - 4*I*b*c^4*d^2*log(x))*l
og(c^2*x^2) + (4*I*b*c^4*d^2*log(x) + 4*I*b*c^2*d*e + I*b*e^2)*log(c*x + 1)
 + (4*I*b*c^2*d*e + I*b*e^2)*log(c*x - 1) + (2*I*b*c^4*e^2*x^4 + 8*I*b*c^4*
d*e*x^2 + (8*b*c^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 8*I*b*c^4*log(
c))*d^2)*log(x))/c^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(e\,x^2+d\right)}^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x,x)
```

```
[Out] int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{acsc}{\left(c x \right)}\right) \left(d + e x^{2}\right)^{2}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acsc(c*x))/x,x)
```

```
[Out] Integral((a + b*acsc(c*x))*(d + e*x**2)**2/x, x)
```

3.97 $\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x^3}\,dx$

Optimal. Leaf size=189

$$-\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{2x^2}-2de\log\left(\frac{1}{x}\right)\left(a+b\csc^{-1}(cx)\right)+\frac{1}{2}e^2x^2\left(a+b\csc^{-1}(cx)\right)-\frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}}{4x}+\frac{1}{4}bc^2d^2\csc^{-1}(cx)-$$

```
[Out] 1/4*b*c^2*d^2*arccsc(c*x)+I*b*d*e*arccsc(c*x)^2-1/2*d^2*(a+b*arccsc(c*x))/x
^2+1/2*e^2*x^2*(a+b*arccsc(c*x))-2*b*d*e*arccsc(c*x)*ln(1-(I/c/x+(1-1/c^2/x
^2)^(1/2))^2)+2*b*d*e*arccsc(c*x)*ln(1/x)-2*d*e*(a+b*arccsc(c*x))*ln(1/x)+I
*b*d*e*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)-1/4*b*c*d^2*(1-1/c^2/x^2)^(
1/2)/x+1/2*b*e^2*x*(1-1/c^2/x^2)^(1/2)/c
```

Rubi [A] time = 0.43, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5241, 266, 43, 4731, 12, 6742, 264, 321, 216, 2326, 4625, 3717, 2190, 2279, 2391}

$$ibde\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)-\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{2x^2}-2de\log\left(\frac{1}{x}\right)\left(a+b\csc^{-1}(cx)\right)+\frac{1}{2}e^2x^2\left(a+b\csc^{-1}(cx)\right)-\frac{bcd^2\sqrt{}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^3,x]
```

```
[Out] -(b*c*d^2*Sqrt[1 - 1/(c^2*x^2)])/(4*x) + (b*e^2*Sqrt[1 - 1/(c^2*x^2)]*x)/(2
*c) + (b*c^2*d^2*ArcCsc[c*x])/4 + I*b*d*e*ArcCsc[c*x]^2 - (d^2*(a + b*ArcCs
c[c*x]))/(2*x^2) + (e^2*x^2*(a + b*ArcCsc[c*x]))/2 - 2*b*d*e*ArcCsc[c*x]*Lo
g[1 - E^((2*I)*ArcCsc[c*x])] + 2*b*d*e*ArcCsc[c*x]*Log[x^(-1)] - 2*d*e*(a +
 b*ArcCsc[c*x])*Log[x^(-1)] + I*b*d*e*PolyLog[2, E^((2*I)*ArcCsc[c*x])]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2326

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[(ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/Rt[-e, 2], x
] - Dist[(b*n)/Rt[-e, 2], Int[ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4625

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n/Tan[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 4731

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSin[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 - c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e, 0] &
& IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Rule 5241

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSin[x/c])^n)/x^
(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)}{x^3}\,dx=-\text{Subst}\left(\int \frac{\left(e+dx^2\right)^2\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{x^3}\,dx,x,\frac{1}{x}\right)$$

$$=-\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{2x^2}+\frac{1}{2}e^2x^2\left(a+b\csc^{-1}(cx)\right)-2de\left(a+b\csc^{-1}(cx)\right)\log\Big($$

$$=-\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{2x^2}+\frac{1}{2}e^2x^2\left(a+b\csc^{-1}(cx)\right)-2de\left(a+b\csc^{-1}(cx)\right)\log\Big($$

$$=-\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{2x^2}+\frac{1}{2}e^2x^2\left(a+b\csc^{-1}(cx)\right)-2de\left(a+b\csc^{-1}(cx)\right)\log\Big($$

$$=-\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{2x^2}+\frac{1}{2}e^2x^2\left(a+b\csc^{-1}(cx)\right)-2de\left(a+b\csc^{-1}(cx)\right)\log\Big($$

$$=-\frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}}{4x}+\frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c}-\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{2x^2}+\frac{1}{2}e^2x^2\Big(a+b\csc$$

$$=-\frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}}{4x}+\frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c}+\frac{1}{4}bc^2d^2\csc^{-1}(cx)-\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{2x^2}$$

$$=-\frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}}{4x}+\frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c}+\frac{1}{4}bc^2d^2\csc^{-1}(cx)+ibde\csc^{-1}(cx)^2-\frac{d^2}{}$$

$$=-\frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}}{4x}+\frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c}+\frac{1}{4}bc^2d^2\csc^{-1}(cx)+ibde\csc^{-1}(cx)^2-\frac{d^2}{}$$

$$=-\frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}}{4x}+\frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c}+\frac{1}{4}bc^2d^2\csc^{-1}(cx)+ibde\csc^{-1}(cx)^2-\frac{d^2}{}$$

$$=-\frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}}{4x}+\frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c}+\frac{1}{4}bc^2d^2\csc^{-1}(cx)+ibde\csc^{-1}(cx)^2-\frac{d^2}{}$$

Mathematica [A] time = 0.68, size = 187, normalized size = 0.99

$$\frac{1}{4}\left(-\frac{2ad^2}{x^2}+8ade\log(x)+2ae^2x^2-\frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{c^2x^2\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{\sqrt{1-c^2x^2}}+1\right)}{x}+\frac{2be^2x\left(\sqrt{1-\frac{1}{c^2x^2}}+cx\csc^{-1}(cx\right.}{c}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^3,x]
```

```
[Out] ((-2*a*d^2)/x^2 + 2*a*e^2*x^2 - (2*b*d^2*ArcCsc[c*x])/x^2 + (2*b*e^2*x*(Sqr
t[1 - 1/(c^2*x^2)] + c*x*ArcCsc[c*x]))/c - (b*c*d^2*Sqrt[1 - 1/(c^2*x^2)]*(
1 + (c^2*x^2*ArcTanh[Sqrt[1 - c^2*x^2]])/Sqrt[1 - c^2*x^2]))/x - 8*b*d*e*Ar
cCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] + 8*a*d*e*Log[x] + (4*I)*b*d*e*(Ar
cCsc[c*x]^2 + PolyLog[2, E^((2*I)*ArcCsc[c*x])]))/4
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ae^2x^4+2\,adex^2+ad^2+\left(be^2x^4+2\,bdex^2+bd^2\right)\text{arccsc}\,(cx)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^
2)*arccsc(c*x))/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(ex^2+d\right)^2(b\,\text{arccsc}\,(cx)+a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccsc(c*x) + a)/x^3, x)
```

maple [A] time = 4.30, size = 276, normalized size = 1.46

$$\frac{a\,x^2e^2}{2}+2aed\ln(cx)-\frac{a\,d^2}{2x^2}+ibde\,\text{arccsc}\,(cx)^2-\frac{cb\,d^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{4x}+\frac{b\,c^2d^2\text{arccsc}\,(cx)}{4}-\frac{b\,\text{arccsc}\,(cx)\,d^2}{2x^2}+\frac{b\,\text{arccsc}\,(cx)\,x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccsc(c*x))/x^3,x)
```

```
[Out] 1/2*a*x^2*e^2+2*a*e*d*ln(c*x)-1/2*a*d^2/x^2+I*b*d*e*arccsc(c*x)^2-1/4*c*b*d
^2/x*((c^2*x^2-1)/c^2/x^2)^(1/2)+1/4*b*c^2*d^2*arccsc(c*x)-1/2*b*arccsc(c*x
)*d^2/x^2+1/2*b*arccsc(c*x)*x^2*e^2+1/2/c*b*((c^2*x^2-1)/c^2/x^2)^(1/2)*x*e
^2-1/2*I/c^2*b*e^2-2*b*e*d*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))-2*b*
e*d*arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))+2*I*b*e*d*polylog(2,-I/c/x-
(1-1/c^2/x^2)^(1/2))+2*I*b*e*d*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,ae^2x^2+\frac{1}{4}\,bd^2\left(\frac{\frac{c^4x\sqrt{-\frac{1}{c^2x^2}+1}}{c^2x^2\left(\frac{1}{c^2x^2}-1\right)-1}-c^3\arctan\left(cx\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c}-\frac{2\,\mathrm{arccsc}\,(cx)}{x^2}\right)+2\,ade\log(x)-\frac{ad^2}{2\,x^2}+\frac{4i\,bc^2de\log\left(-cx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*e^2*x^2 + 1/4*b*d^2*((c^4*x*sqrt(-1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x
^2) - 1) - 1) - c^3*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)))/c - 2*arccsc(c*x)/x
^2) + 2*a*d*e*log(x) - 1/2*a*d^2/x^2 + 1/4*(4*I*b*c^2*d*e*log(-c*x + 1)*log
(x) + 4*I*b*c^2*d*e*log(x)^2 + 4*I*b*c^2*d*e*dilog(c*x) + 4*I*b*c^2*d*e*dil
og(-c*x) + (2*b*c^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 2*I*b*c^2*log
(c))*e^2*x^2 + I*b*e^2*log(c*x - 1) - I*(b*e^2*(log(c*x + 1)/c^2 + log(c*x
- 1)/c^2) + 16*b*d*e*integrate(1/2*log(x)/(c^2*x^3 - x), x))*c^2 + 4*c^2*in
tegrate(1/2*(b*e^2*x^2 + 4*b*d*e*log(x))*sqrt(c*x + 1)*sqrt(c*x - 1)/(c^2*x
^3 - x), x) + (-I*b*c^2*e^2*x^2 - 4*I*b*c^2*d*e*log(x))*log(c^2*x^2) + (4*I
*b*c^2*d*e*log(x) + I*b*e^2)*log(c*x + 1) + (2*I*b*c^2*e^2*x^2 + (8*b*c^2*a
rctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 8*I*b*c^2*log(c))*d*e)*log(x))/c^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(e\,x^2+d\right)}^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^3,x)
```

```
[Out] int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{acsc}{\left(c x \right)}\right) \left(d + e x^{2}\right)^{2}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acsc(c*x))/x**3,x)
```

```
[Out] Integral((a + b*acsc(c*x))*(d + e*x**2)**2/x**3, x)
```

3.98 $\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{d+ex^2}\,dx$

Optimal. Leaf size=565

$$-\frac{\sqrt{-d}\,\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}}+\frac{\sqrt{-d}\,\left(a+b\csc^{-1}(cx)\right)\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}}-\frac{\sqrt{-d}\,\left(a+b\right.}{}$$

```
[Out] x*(a+b*arccsc(c*x))/e+b*arctanh((1-1/c^2/x^2)^(1/2))/c/e-1/2*(a+b*arccsc(c*
x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2
)))*(-d)^(1/2)/e^(3/2)+1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^
(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2*(a+b*ar
ccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+
e)^(1/2)))*(-d)^(1/2)/e^(3/2)+1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^
2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2*
I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e
)^(1/2)))*(-d)^(1/2)/e^(3/2)+1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/
2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2*I*b*polylo
g(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*
(-d)^(1/2)/e^(3/2)+1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(
1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)
```

Rubi [A] time = 1.26, antiderivative size = 565, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5241, 4733, 4627, 266, 63, 208, 4667, 4741, 4519, 2190, 2279, 2391}

$$-\frac{ib\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}}+\frac{ib\sqrt{-d}\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}}-\frac{ib\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2),x]
```

```
[Out] (x*(a + b*ArcCsc[c*x]))/e + (b*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/(c*e) - (Sqr
t[-d]*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e]
 - Sqrt[c^2*d + e])])/(2*e^(3/2)) + (Sqrt[-d]*(a + b*ArcCsc[c*x])*Log[1 + (
I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^(3/2)) -
 (Sqrt[-d]*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sq
rt[e] + Sqrt[c^2*d + e])])/(2*e^(3/2)) + (Sqrt[-d]*(a + b*ArcCsc[c*x])*Log[
1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^(3/
2)) - ((I/2)*b*Sqrt[-d]*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqr
t[e] - Sqrt[c^2*d + e])])/e^(3/2) + ((I/2)*b*Sqrt[-d]*PolyLog[2, (I*c*Sqrt[
-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/e^(3/2) - ((I/2)*b*Sqr
t[-d]*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d
+ e])])/e^(3/2) + ((I/2)*b*Sqrt[-d]*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*
x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^(3/2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rule 4627

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4667

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4733

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4741

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[((a + b*x)^n*Cos[x])/(c*d + e*Sin[x]), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5241

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSin[x/c])^n)/x^
(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{d+ex^2}\,dx = -\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{x^2\left(e+dx^2\right)}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{ex^2} - \frac{d\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{e\left(e+dx^2\right)}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{x^2}\,dx, x, \frac{1}{x}\right)}{e} + \frac{d\,\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e}$$

$$= \frac{x\left(a+b\csc^{-1}(cx)\right)}{e} - \frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{ce} + \frac{d\,\text{Subst}\left(\int \left(\frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}\,x\right)} + \right.\right.}{e}$$

$$= \frac{x\left(a+b\csc^{-1}(cx)\right)}{e} + \frac{d\,\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}\,x}\,dx, x, \frac{1}{x}\right)}{2e^{3/2}} + \frac{d\,\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}\,x}\,dx\right.}{2e^{3/2}}$$

$$= \frac{x\left(a+b\csc^{-1}(cx)\right)}{e} + \frac{d\,\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\,\sin(x)}\,dx, x, \csc^{-1}(cx)\right)}{2e^{3/2}} + \frac{d\,\text{Subst}\left(\int \frac{(a}{\frac{\sqrt{e}}{c}}\right.}{}$$

$$= \frac{x\left(a+b\csc^{-1}(cx)\right)}{e} + \frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce} + \frac{d\,\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}\,e^{ix}}\,dx, x\right.}{2e^{3/2}}$$

$$= \frac{x\left(a+b\csc^{-1}(cx)\right)}{e} + \frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce} - \frac{\sqrt{-d}\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{}}{\sqrt{}}\right.}{2e^{3/2}}$$

$$= \frac{x\left(a+b\csc^{-1}(cx)\right)}{e} + \frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce} - \frac{\sqrt{-d}\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{}}{\sqrt{}}\right.}{2e^{3/2}}$$

$$= \frac{x\left(a+b\csc^{-1}(cx)\right)}{e} + \frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce} - \frac{\sqrt{-d}\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{}}{\sqrt{}}\right.}{2e^{3/2}}$$

Mathematica [B] time = 1.82, size = 1260, normalized size = 2.23

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2),x]
```

```
[Out] ((I/4)*((-4*I)*a*c*Sqrt[e]*x - (4*I)*b*c*Sqrt[e]*x*ArcCsc[c*x] + (4*I)*a*c*
Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + (8*I)*b*c*Sqrt[d]*ArcSin[Sqrt[1 - (I*
Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi +
 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - (8*I)*b*c*Sqrt[d]*ArcSin[Sqrt[1 + (I
*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2
*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] + b*c*Sqrt[d]*Pi*Log[1 + (Sqrt[e] - Sqrt
[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 2*b*c*Sqrt[d]*ArcCsc[c*x]*Log
[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*b*c*Sqr
t[d]*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - S
qrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - b*c*Sqrt[d]*Pi*Log[1 + (-S
qrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 2*b*c*Sqrt[d]*Ar
cCsc[c*x]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x])
)] - 4*b*c*Sqrt[d]*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1
+ (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - b*c*Sqrt[d]
*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 2*
b*c*Sqrt[d]*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I
*ArcCsc[c*x]))] + 4*b*c*Sqrt[d]*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sq
rt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] +
 b*c*Sqrt[d]*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[
c*x]))] - 2*b*c*Sqrt[d]*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*
Sqrt[d]*E^(I*ArcCsc[c*x]))] - 4*b*c*Sqrt[d]*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*
Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcC
sc[c*x]))] + b*c*Sqrt[d]*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] - b*c*Sqrt[d]*Pi*L
og[Sqrt[e] + (I*Sqrt[d])/x] - (4*I)*b*Sqrt[e]*Log[Cos[ArcCsc[c*x]/2]] + (4*
I)*b*Sqrt[e]*Log[Sin[ArcCsc[c*x]/2]] + (2*I)*b*c*Sqrt[d]*PolyLog[2, (Sqrt[e
] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (2*I)*b*c*Sqrt[d]*Pol
yLog[2, (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (2*I)
*b*c*Sqrt[d]*PolyLog[2, -((Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCs
c[c*x])))] + (2*I)*b*c*Sqrt[d]*PolyLog[2, (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sq
rt[d]*E^(I*ArcCsc[c*x]))]))/(c*e^(3/2))
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{bx^2\,\mathrm{arccsc}\,(cx)+ax^2}{ex^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccsc(c*x) + a*x^2)/(e*x^2 + d), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 4.27, size = 407, normalized size = 0.72

$$\frac{ax}{e}-\frac{ad\arctan\left(\frac{ex}{\sqrt{de}}\right)}{e\sqrt{de}}+\frac{b\,\mathrm{arccsc}\,(cx)\,x}{e}+\frac{b\ln\left(1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce}-\frac{b\ln\left(\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}-1\right)}{ce}+\frac{cbd\left(\sum_{\_R1=\mathrm{RootOf}(c^2d\_}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsc(c*x))/(e*x^2+d),x)
```

```
[Out] a*x/e-a*d/e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+b*arccsc(c*x)/e*x+1/c*b/e*l
n(1+I/c/x+(1-1/c^2/x^2)^(1/2))-1/c*b/e*ln(I/c/x+(1-1/c^2/x^2)^(1/2)-1)+1/8*
c*b/e^2*d*sum((_R1^2*c^2*d+4*_R1^2*e-c^2*d)/_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*
arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c
^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-1/8*
c*b/e^2*d*sum((_R1^2*c^2*d-c^2*d-4*e)/_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc
(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2
)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-a\left(\frac{d\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,e}-\frac{x}{e}\right)+b\int\frac{x^2\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right)}{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] -a*(d*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e) - x/e) + b*integrate(x^2*arctan2(
1, sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x^2 + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^2\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2),x)
```

```
[Out] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2\left(a+b\,\mathrm{acsc}\,(cx)\right)}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsc(c*x))/(e*x**2+d),x)
```

```
[Out] Integral(x**2*(a + b*acsc(c*x))/(d + e*x**2), x)
```

3.99 $\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{d+ex^2}\,dx$

Optimal. Leaf size=507

$$\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e}+\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e}+\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic}{}\right.}{2e}$$

```
[Out] -(a+b*arccsc(c*x))*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)/e+1/2*(a+b*arccsc(c*
x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2
)))/e+1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)
/(e^(1/2)-(c^2*d+e)^(1/2)))/e+1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^
2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e+1/2*(a+b*arccsc(c*x))
*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))
/e+1/2*I*b*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)/e-1/2*I*b*polylog(2,-I*
c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e-1/2*I
*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^
(1/2)))/e-1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^
(1/2)+(c^2*d+e)^(1/2)))/e-1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))
*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e
```

Rubi [A] time = 1.14, antiderivative size = 507, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {5241, 4733, 4625, 3717, 2190, 2279, 2391, 4741, 4519}

$$-\frac{ib\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e}-\frac{ib\text{PolyLog}\left(2,\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e}-\frac{ib\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e}-\frac{ib\text{PolyLog}\left(2,\right.}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2),x]
```

```
[Out] ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sq
rt[c^2*d + e])])/(2*e) + ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*Ar
cCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e) + ((a + b*ArcCsc[c*x])*Log[
1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e) +
((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sq
rt[c^2*d + e])])/(2*e) - ((a + b*ArcCsc[c*x])*Log[1 - E^((2*I)*ArcCsc[c*x])
])/e - ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - S
qrt[c^2*d + e])])/e - ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/
(Sqrt[e] - Sqrt[c^2*d + e])])/e - ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I
*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e - ((I/2)*b*PolyLog[2, (I*c*S
qrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e + ((I/2)*b*PolyL
og[2, E^((2*I)*ArcCsc[c*x])])/e
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rule 4625

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n/Tan[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 4733

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4741

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[((a + b*x)^n*Cos[x])/(c*d + e*Sin[x]), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5241

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSin[x/c])^n)/x^
(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{d+ex^2}\,dx = -\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{x\left(e+dx^2\right)}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{ex} - \frac{dx\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{e\left(e+dx^2\right)}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)}{e} + \frac{d\,\text{Subst}\left(\int \frac{x\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e}$$

$$= -\frac{\text{Subst}\left(\int (a+bx)\cot(x)\,dx, x, \csc^{-1}(cx)\right)}{e} + \frac{d\,\text{Subst}\left(\int \left(-\frac{\sqrt{-d}\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}\,x\right)} + \frac{\sqrt{-d}\left(a+\right.}{2d\left(\sqrt{e}\right.}\right.}{e}$$

$$= \frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2be} + \frac{(2i)\,\text{Subst}\left(\int \frac{e^{2ix}(a+bx)}{1-e^{2ix}}\,dx, x, \csc^{-1}(cx)\right)}{e} - \frac{\sqrt{-d}\,\text{Subst}\left(\int \frac{a+b\sin}{\sqrt{e}-\sqrt{}}\right.}{2e}$$

$$= \frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2be} - \frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{e} + \frac{b\,\text{Subst}\left(\int \log\left(1-e^{2ix}\right.\right.}{e}$$

$$= -\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{e} - \frac{(ib)\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2i\csc^{-1}(cx)}\right)}{2e} + \frac{\sqrt{}}{}$$

$$= \frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} + \frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e}$$

$$= \frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} + \frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e}$$

$$= \frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} + \frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e}$$

Mathematica [B] time = 0.43, size = 1123, normalized size = 2.21

$$\frac{8ib\csc^{-1}(cx)^2 + 4b\log\left(\frac{e^{-i\csc^{-1}(cx)}\left(\sqrt{e}-\sqrt{dc^2+e}\right)}{c\sqrt{d}}+1\right)\csc^{-1}(cx) + 4b\log\left(\frac{e^{-i\csc^{-1}(cx)}\left(\sqrt{dc^2+e}-\sqrt{e}\right)}{c\sqrt{d}}+1\right)\csc^{-1}(cx) + 4b\log}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2),x]
```

```
[Out] (I*b*Pi^2 - (4*I)*b*Pi*ArcCsc[c*x] + (8*I)*b*ArcCsc[c*x]^2 - (16*I)*b*ArcSi
n[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt
[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - (16*I)*b*ArcSin[Sqrt[1
 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(P
i + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - 2*b*Pi*Log[1 + (Sqrt[e] - Sqrt[c^
2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*b*ArcCsc[c*x]*Log[1 + (Sqrt[e]
 - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*b*ArcSin[Sqrt[1 - (I
*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt
```

```
[d]*E^(I*ArcCsc[c*x]))] - 2*b*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sq
rt[d]*E^(I*ArcCsc[c*x]))] + 4*b*ArcCsc[c*x]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d
+ e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*S
qrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcC
sc[c*x]))] - 2*b*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*Arc
Csc[c*x]))] + 4*b*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d
]*E^(I*ArcCsc[c*x]))] + 8*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2
]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 2*b
*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*
b*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*
x]))] + 8*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt
[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*b*ArcCsc[c*x]*Log
[1 - E^((2*I)*ArcCsc[c*x])] + 2*b*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] + 2*b*Pi*
Log[Sqrt[e] + (I*Sqrt[d])/x] + 4*a*Log[d + e*x^2] + (4*I)*b*PolyLog[2, (Sqr
t[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*b*PolyLog[2,
 (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*b*Poly
Log[2, -((Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x])))] + (4*I
)*b*PolyLog[2, (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] +
 (4*I)*b*PolyLog[2, E^((2*I)*ArcCsc[c*x])])/(8*e)
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx\,\text{arccsc}\,(cx) + ax}{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x*arccsc(c*x) + a*x)/(e*x^2 + d), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sy
m(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Va
lue
```

maple [C] time = 1.12, size = 401, normalized size = 0.79

$$\frac{a\ln\left(c^2e\,x^2+c^2d\right)}{2e}-\frac{b\,\text{arccsc}\,(cx)\ln\left(1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{e}+\frac{ib\,\text{dilog}\left(1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{e}-\frac{ib\left(\sum_{\_R1=\text{RootOf}\left(c^2d\_Z^4\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsc(c*x))/(e*x^2+d),x)
```

```
[Out] 1/2*a/e*ln(c^2*e*x^2+c^2*d)-b/e*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))
+I*b/e*dilog(1+I/c/x+(1-1/c^2/x^2)^(1/2))-1/4*I*b*sum((_R1^2*c^2*d-c^2*d-4*
e)/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2)
)/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-
```

```
2*c^2*d-4*e)*_Z^2+c^2*d))/e-I*b*dilog(I/c/x+(1-1/c^2/x^2)^(1/2))/e-1/4*I*c^
2*b*sum((_R1^2-1)/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1
/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootO
f(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))*d/e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\int \frac{x \arctan\left(1, \sqrt{cx+1}\,\sqrt{cx-1}\right)}{ex^2+d}\,dx + \frac{a\log\left(ex^2+d\right)}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] b*integrate(x*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x^2 + d), x) + 1/2
*a*log(e*x^2 + d)/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asin(1/(c*x))))/(d + e*x^2),x)
```

```
[Out] int((x*(a + b*asin(1/(c*x))))/(d + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\,(a+b\,\mathrm{acsc}\,(cx))}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsc(c*x))/(e*x**2+d),x)
```

```
[Out] Integral(x*(a + b*acsc(c*x))/(d + e*x**2), x)
```

3.100 $\int \frac{a+b\csc^{-1}(cx)}{d+ex^2}\,dx$

Optimal. Leaf size=529

$$-\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1\right.}{2\sqrt{-d}\sqrt{}}$$

```
[Out] -1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(
1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I
/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/
e^(1/2)-1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/
2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*(a+b*arccsc(c*x))*ln(1
+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)
^(1/2)/e^(1/2)-1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2
)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*I*b*polylog(2,I*c*(I/c/
x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(
1/2)-1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)
+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2
/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)
```

Rubi [A] time = 0.87, antiderivative size = 529, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5231, 4667, 4741, 4519, 2190, 2279, 2391}

$$-\frac{ib\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{ib\text{PolyLog}\left(2,\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{ib\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{ib\text{PolyLog}\left(\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(d + e*x^2),x]
```

```
[Out] -((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - S
qrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) + ((a + b*ArcCsc[c*x])*Log[1 + (I*c*
Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[
e]) - ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e
] + Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) + ((a + b*ArcCsc[c*x])*Log[1 +
(I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*Sqrt[-d]*
Sqrt[e]) - ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e]
 - Sqrt[c^2*d + e])])/(Sqrt[-d]*Sqrt[e]) + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d
]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(Sqrt[-d]*Sqrt[e]) - ((I
/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d
+ e])])/(Sqrt[-d]*Sqrt[e]) + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[
c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(Sqrt[-d]*Sqrt[e])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rule 4667

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4741

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[((a + b*x)^n*Cos[x])/(c*d + e*Sin[x]), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5231

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSin[x/c])^n)/x^(2*(p + 1))
, x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{d+ex^2}\,dx = -\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int\left(\frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}\,x\right)}+\frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}\,x\right)}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}\,x}\,dx, x, \frac{1}{x}\right)}{2\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}\,x}\,dx, x, \frac{1}{x}\right)}{2\sqrt{e}}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\,\sin(x)}\,dx, x, \csc^{-1}(cx)\right)}{2\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\,\sin(x)}\,dx, x, \csc^{-1}(cx)\right)}{2\sqrt{e}}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}\,e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{2\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}\,e^{ix}}\,dx, x, \csc\right.}{2\sqrt{e}}$$

$$= -\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}} + \frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}}$$

$$= -\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}} + \frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}}$$

$$= -\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}} + \frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}}$$

Mathematica [B] time = 0.44, size = 1068, normalized size = 2.02

$$-\frac{i\left(4ia\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)+8ib\sin^{-1}\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\tan^{-1}\left(\frac{\left(\sqrt{e}-ic\sqrt{d}\right)\cot\left(\frac{1}{4}\left(2\csc^{-1}(cx)+\pi\right)\right)}{\sqrt{dc^2+e}}\right)-8ib\sin^{-1}\left(\frac{\sqrt{\frac{i\sqrt{e}}{c\sqrt{d}}+1}}{\sqrt{2}}\right)\tan^{-1}\left(\frac{(i\sqrt{}}{}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(d + e*x^2),x]
```

```
[Out] ((-1/4*I)*((4*I)*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + (8*I)*b*ArcSin[Sqrt[1 - (I
*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi
+ 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - (8*I)*b*ArcSin[Sqrt[1 + (I*Sqrt[e])
/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c
*x])/4])/Sqrt[c^2*d + e]] + b*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqr
t[d]*E^(I*ArcCsc[c*x]))] - 2*b*ArcCsc[c*x]*Log[1 + (Sqrt[e] - Sqrt[c^2*d +
e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqr
t[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[
c*x]))] - b*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[
c*x]))] + 2*b*ArcCsc[c*x]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E
^(I*ArcCsc[c*x]))] - 4*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*
Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - b*Pi*
```

```
Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 2*b*Ar
cCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))
] + 4*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (Sqrt[e]
+ Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + b*Pi*Log[1 + (Sqrt[e] +
 Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 2*b*ArcCsc[c*x]*Log[1 +
(Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 4*b*ArcSin[Sqr
t[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])
/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + b*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] - b*Pi*
Log[Sqrt[e] + (I*Sqrt[d])/x] + (2*I)*b*PolyLog[2, (Sqrt[e] - Sqrt[c^2*d + e
])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (2*I)*b*PolyLog[2, (-Sqrt[e] + Sqrt[c^2
*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (2*I)*b*PolyLog[2, -((Sqrt[e] + S
qrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x])))] + (2*I)*b*PolyLog[2, (Sqrt[
e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))]))/(Sqrt[d]*Sqrt[e])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\mathrm{arccsc}\,(cx)+a}{ex^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)/(e*x^2 + d), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sy
m(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Va
lue
```

maple [C] time = 2.04, size = 272, normalized size = 0.51

$$\frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}-\frac{cb\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(-2c^2d-4e\right)\_Z^2+c^2d\right)}\frac{i\,\mathrm{arccsc}(cx)\ln\left(\frac{\_R1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}}{\_R1}\right)}{\_R1\left(\_R1^2c^2d-c^2d-2e\right)}\right)}{2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/(e*x^2+d),x)
```

```
[Out] a/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-1/2*c*b*sum(1/_R1/(_R1^2*c^2*d-c^2*d-
2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/
x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*
d))-1/2*c*b*sum(_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1
-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=Roo
tOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\int\frac{\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right)}{ex^2+d}\,dx+\frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] b*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x^2 + d), x) + a*arc
tan(e*x/sqrt(d*e))/sqrt(d*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{e\,x^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x^2),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(d + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{acsc}\,(cx)}{d + ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/(e*x**2+d),x)
```

```
[Out] Integral((a + b*acsc(c*x))/(d + e*x**2), x)
```

3.101 $\int \frac{a+b\csc^{-1}(cx)}{x(d+ex^2)}\,dx$

Optimal. Leaf size=479

$$-\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}-\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}-\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{i}{}\right.}{2d}$$

```
[Out] 1/2*I*(a+b*arccsc(c*x))^2/b/d-1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^
2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d-1/2*(a+b*arccsc(c*x))
*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))
/d-1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e
^(1/2)+(c^2*d+e)^(1/2)))/d-1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x
^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d+1/2*I*b*polylog(2,-I*c*(
I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d+1/2*I*b*
polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/
2)))/d+1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/
2)+(c^2*d+e)^(1/2)))/d+1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-
d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d
```

Rubi [A] time = 0.90, antiderivative size = 479, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5241, 4733, 4741, 4519, 2190, 2279, 2391}

$$\frac{ib\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}+\frac{ib\text{PolyLog}\left(2,\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}+\frac{ib\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d}+\frac{ib\text{PolyLog}\left(2,\frac{ic}{}\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)),x]
```

```
[Out] ((I/2)*(a + b*ArcCsc[c*x])^2)/(b*d) - ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqr
t[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d) - ((a + b*ArcC
sc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e
])])/(2*d) - ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/
(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d) - ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqr
t[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d) + ((I/2)*b*Pol
yLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/d
 + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2
*d + e])])/d + ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqr
t[e] + Sqrt[c^2*d + e])])/d + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc
[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/d
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rule 4733

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4741

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[((a + b*x)^n*Cos[x])/(c*d + e*Sin[x]), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5241

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSin[x/c])^n)/x^
(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)}\,dx = -\text{Subst}\left(\int \frac{x\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(-\frac{\sqrt{-d}\,\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}\,x\right)}+\frac{\sqrt{-d}\,\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}+\sqrt{-d}\,x\right)}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}\,x}\,dx, x, \frac{1}{x}\right)}{2\sqrt{-d}}+\frac{\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}\,x}\,dx, x, \frac{1}{x}\right)}{2\sqrt{-d}}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\,\sin(x)}\,dx, x, \csc^{-1}(cx)\right)}{2\sqrt{-d}}+\frac{\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\,\sin(x)}\,dx, x, \csc^{-1}(cx)\right)}{2\sqrt{-d}}$$

$$= \frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2bd}+\frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}\,e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{2\sqrt{-d}}+\frac{\text{Subst}\left(\int \frac{e^{ix}(a+\cdots}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}\cdots}\right.}{\cdots}$$

$$= \frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2bd}-\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}-\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1\cdots\right.}{2d}$$

$$= \frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2bd}-\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}-\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1\cdots\right.}{2d}$$

$$= \frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2bd}-\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}-\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1\cdots\right.}{2d}$$

Mathematica [A] time = 0.94, size = 407, normalized size = 0.85

$$\frac{-2a\log\left(d+ex^2\right)+4a\log(x)+ib\left(\text{Li}_2\left(\frac{\left(dc^2+2e-2\sqrt{e\left(dc^2+e\right)}\right)e^{2i\csc^{-1}(cx)}}{c^2d}\right)+\text{Li}_2\left(\frac{\left(dc^2+2\left(e+\sqrt{e\left(dc^2+e\right)}\right)\right)e^{2i\csc^{-1}(cx)}}{c^2d}\right)+4\sin^{-1}\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)),x]
```

```
[Out] (4*a*Log[x] - 2*a*Log[d + e*x^2] + I*b*(2*ArcCsc[c*x]^2 + 4*ArcSin[Sqrt[-(e
/(c^2*d))]]*ArcTan[Sqrt[e*(c^2*d + e)]/(c*e*Sqrt[1 - 1/(c^2*x^2)]*x)] + (2*
I)*ArcCsc[c*x]*Log[1 - ((c^2*d + 2*e - 2*Sqrt[e*(c^2*d + e)])*E^((2*I)*ArcC
sc[c*x]))/(c^2*d)] - (2*I)*ArcSin[Sqrt[-(e/(c^2*d))]]*Log[1 - ((c^2*d + 2*e
 - 2*Sqrt[e*(c^2*d + e)])*E^((2*I)*ArcCsc[c*x]))/(c^2*d)] + (2*I)*ArcCsc[c*
x]*Log[1 - ((c^2*d + 2*(e + Sqrt[e*(c^2*d + e)]))*E^((2*I)*ArcCsc[c*x]))/(c
^2*d)] + (2*I)*ArcSin[Sqrt[-(e/(c^2*d))]]*Log[1 - ((c^2*d + 2*(e + Sqrt[e*(
c^2*d + e)]))*E^((2*I)*ArcCsc[c*x]))/(c^2*d)] + PolyLog[2, ((c^2*d + 2*e -
2*Sqrt[e*(c^2*d + e)])*E^((2*I)*ArcCsc[c*x]))/(c^2*d)] + PolyLog[2, ((c^2*d
 + 2*(e + Sqrt[e*(c^2*d + e)]))*E^((2*I)*ArcCsc[c*x]))/(c^2*d)]))/(4*d)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arccsc}\,(cx)+a}{ex^3+dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)/(e*x^3 + d*x), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sy
m(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Va
lue
```

maple [C] time = 1.07, size = 2875, normalized size = 6.00

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x/(e*x^2+d),x)
```

```
[Out] 1/2*b*c^2*ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(
1/2)+2*e))*arccsc(c*x)/(c^2*d+e)-1/4*I*b*c^2*polylog(2,c^2*d*(I/c/x+(1-1/c^
2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))/(c^2*d+e)-1/2*I*b*c^2*ar
ccsc(c*x)^2/(c^2*d+e)+5/2*b*ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d
-2*(e*(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)*e/d/(c^2*d+e)+I*b/c^4*polylog(2,c^
2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*e^2/d^
3+I*b/c^2*polylog(2,c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+
e))^(1/2)+2*e))/d^2*e-2*b/c^2*e*ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c
^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)/d^2-2*b/c^4*e^2*ln(1-c^2*d*(I/
c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)/d
^3-b/c^2*ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1
/2)+2*e))*arccsc(c*x)/d^2*(e*(c^2*d+e))^(1/2)+I*b/c^2*arccsc(c*x)^2/d^2*(e*
(c^2*d+e))^(1/2)+3/2*b*ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e
*(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)/d/(c^2*d+e)*(e*(c^2*d+e))^(1/2)-I*b*(e*
(c^2*d+e))^(1/2)/d/(c^2*d+e)*arccsc(c*x)^2-5/4*I*b*polylog(2,c^2*d*(I/c/x+(
1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*e/d/(c^2*d+e)-5/2*
I*b*arccsc(c*x)^2*e/d/(c^2*d+e)-3/4*I*b*polylog(2,c^2*d*(I/c/x+(1-1/c^2/x^2
)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))/d/(c^2*d+e)*(e*(c^2*d+e))^(1/
2)+2*I*b/c^4*arccsc(c*x)^2*e^2/d^3+1/2*I*b/d*sum((_R1^2*c^2*d-2*c^2*d-4*e)/
(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_
R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c
^2*d-4*e)*_Z^2+c^2*d))+1/4*I*b*polylog(2,c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^
2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))/d-1/2*b*ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2
)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)/d+I*b*arccsc(c*x)
^2/d-2*b/c^4*e*ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+
e))^(1/2)+2*e))*arccsc(c*x)/d^3*(e*(c^2*d+e))^(1/2)+1/4*b*c^2*ln(1-c^2*d*(I
/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)/
e/(c^2*d+e)*(e*(c^2*d+e))^(1/2)-1/4*b*c^2*(e*(c^2*d+e))^(1/2)/e/(c^2*d+e)*a
rccsc(c*x)*ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d+2*(e*(c^2*d+e))^
(1/2)+2*e))-4*I*b/c^2*e^2*arccsc(c*x)^2/d^2/(c^2*d+e)+I*b/c^4*polylog(2,c^2
*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*e/d^3*(
e*(c^2*d+e))^(1/2)+2*b/c^4*e^3*ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^
2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)/d^3/(c^2*d+e)+4*b/c^2*e^2*ln(1-
c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*arcc
```

```
sc(c*x)/d^2/(c^2*d+e)-I*b/c^4*e^3*polylog(2,c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2
))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))/d^3/(c^2*d+e)+2*I*b/c^4*arccsc(c*x)
^2*e/d^3*(e*(c^2*d+e))^(1/2)-2*I*b/c^4*e^3*arccsc(c*x)^2/d^3/(c^2*d+e)-1/8*
I*b*c^2*polylog(2,c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e)
)^(1/2)+2*e))/e/(c^2*d+e)*(e*(c^2*d+e))^(1/2)+1/8*I*b*c^2*(e*(c^2*d+e))^(1/
2)/e/(c^2*d+e)*polylog(2,c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d+2*(e*(c
^2*d+e))^(1/2)+2*e))-2*I*b/c^2*e^2*polylog(2,c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/
2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))/d^2/(c^2*d+e)+1/2*I*b/c^2*polylog(
2,c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))/d^
2*(e*(c^2*d+e))^(1/2)+2*I*b/c^2*arccsc(c*x)^2/d^2*e+1/4*I*b*(e*(c^2*d+e))^(
1/2)/d/(c^2*d+e)*polylog(2,c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d+2*(e*
(c^2*d+e))^(1/2)+2*e))-1/2*b*(e*(c^2*d+e))^(1/2)/d/(c^2*d+e)*arccsc(c*x)*ln
(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e))+3
*b/c^2*e*ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1
/2)+2*e))*arccsc(c*x)/d^2/(c^2*d+e)*(e*(c^2*d+e))^(1/2)-I*b/c^4*e^2*polylog
(2,c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))/d
^3/(c^2*d+e)*(e*(c^2*d+e))^(1/2)+2*b/c^4*e^2*ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2
)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)/d^3/(c^2*d+e)*(e*
(c^2*d+e))^(1/2)-2*I*b/c^4*e^2*arccsc(c*x)^2/d^3/(c^2*d+e)*(e*(c^2*d+e))^(1
/2)-3/2*I*b/c^2*e*polylog(2,c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e
*(c^2*d+e))^(1/2)+2*e))/d^2/(c^2*d+e)*(e*(c^2*d+e))^(1/2)-3*I*b/c^2*e*arccs
c(c*x)^2/d^2/(c^2*d+e)*(e*(c^2*d+e))^(1/2)-1/2*a/d*ln(c^2*e*x^2+c^2*d)+a/d*
ln(c*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\,a\left(\frac{\log\left(ex^2+d\right)}{d}-\frac{2\,\log(x)}{d}\right)+b\int\frac{\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right)}{ex^3+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/2*a*(log(e*x^2 + d)/d - 2*log(x)/d) + b*integrate(arctan2(1, sqrt(c*x +
1)*sqrt(c*x - 1))/(e*x^3 + d*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x\,\left(e\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\,\mathrm{acsc}\,(cx)}{x\left(d+ex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x/(e*x**2+d),x)
```

```
[Out] Integral((a + b*acsc(c*x))/(x*(d + e*x**2)), x)
```

3.102 $\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex^2)}\,dx$

Optimal. Leaf size=572

$$-\frac{\sqrt{e}\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}+\frac{\sqrt{e}\left(a+b\csc^{-1}(cx)\right)\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}-\frac{\sqrt{e}\left(a+b\csc^{-1}(\right.}{}$$

```
[Out] -a/d/x-b*arccsc(c*x)/d/x-1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2
)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/2*(a+b*
arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*
d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)-1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/
c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/
2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2
)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)-1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c
^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/2
*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e
)^(1/2)))*e^(1/2)/(-d)^(3/2)-1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1
/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/2*I*b*polyl
og(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*
e^(1/2)/(-d)^(3/2)-b*c*(1-1/c^2/x^2)^(1/2)/d
```

Rubi [A] time = 1.10, antiderivative size = 572, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {5241, 4733, 4619, 261, 4667, 4741, 4519, 2190, 2279, 2391}

$$-\frac{ib\sqrt{e}\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}+\frac{ib\sqrt{e}\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}-\frac{ib\sqrt{e}\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2(-d)^{3/2}}+\frac{ib}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(x^2*(d + e*x^2)),x]
```

```
[Out] -((b*c*Sqrt[1 - 1/(c^2*x^2)])/d) - a/(d*x) - (b*ArcCsc[c*x])/(d*x) - (Sqrt[
e]*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] -
Sqrt[c^2*d + e])])/(2*(-d)^(3/2)) + (Sqrt[e]*(a + b*ArcCsc[c*x])*Log[1 + (I
*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*(-d)^(3/2))
 - (Sqrt[e]*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(S
qrt[e] + Sqrt[c^2*d + e])])/(2*(-d)^(3/2)) + (Sqrt[e]*(a + b*ArcCsc[c*x])*L
og[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*(-
d)^(3/2)) - ((I/2)*b*Sqrt[e]*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))
/(Sqrt[e] - Sqrt[c^2*d + e])])/(-d)^(3/2) + ((I/2)*b*Sqrt[e]*PolyLog[2, (I*
c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(-d)^(3/2) - ((
I/2)*b*Sqrt[e]*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sq
rt[c^2*d + e])])/(-d)^(3/2) + ((I/2)*b*Sqrt[e]*PolyLog[2, (I*c*Sqrt[-d]*E^(
I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(-d)^(3/2)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rule 4619

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4667

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4733

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4741

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[((a + b*x)^n*Cos[x])/(c*d + e*Sin[x]), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5241

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSin[x/c])^n)/x^
(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex^2)}\,dx = -\text{Subst}\left(\int \frac{x^2\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{d} - \frac{e\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)dx, x, \frac{1}{x}\right)}{d} + \frac{e\,\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d}$$

$$= -\frac{a}{dx} - \frac{b\,\text{Subst}\left(\int \sin^{-1}\left(\frac{x}{c}\right)dx, x, \frac{1}{x}\right)}{d} + \frac{e\,\text{Subst}\left(\int \left(\frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}\,x\right)} + \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}\,x\right)}\right) d}{d}$$

$$= -\frac{a}{dx} - \frac{b\csc^{-1}(cx)}{dx} + \frac{b\,\text{Subst}\left(\int \frac{x}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{cd} + \frac{\sqrt{e}\,\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}\,x}\,dx, x, \frac{1}{x}\right)}{2d}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b\csc^{-1}(cx)}{dx} + \frac{\sqrt{e}\,\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\,\sin(x)}\,dx, x, \csc^{-1}(cx)\right)}{2d} + \sqrt{e}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b\csc^{-1}(cx)}{dx} + \frac{\sqrt{e}\,\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}\,e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{2d}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b\csc^{-1}(cx)}{dx} - \frac{\sqrt{e}\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} + \sqrt{}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b\csc^{-1}(cx)}{dx} - \frac{\sqrt{e}\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} + \sqrt{}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b\csc^{-1}(cx)}{dx} - \frac{\sqrt{e}\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} + \sqrt{}$$

Mathematica [B] time = 1.75, size = 1241, normalized size = 2.17

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x^2*(d + e*x^2)),x]
```

```
[Out] -(a/(d*x)) - (a*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(3/2) + b*(-((c*Sqrt
[1 - 1/(c^2*x^2)]*x + ArcCsc[c*x])/(d*x)) + (Sqrt[e]*(Pi^2 - 4*Pi*ArcCsc[c*
x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]
*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d
 + e]] + (4*I)*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCs
c[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d
]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqr
t[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] +
(4*I)*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))]
 - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*Ar
```

```
cCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log
[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (8*I)*Arc
Csc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] - (4*I)*Pi*Log[Sqrt[e] + (I*Sqrt[d]
)/x] + 8*PolyLog[2, (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x
]))] + 8*PolyLog[2, -((Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*
x])))] + 4*PolyLog[2, E^((2*I)*ArcCsc[c*x])]))/(16*d^(3/2)) - (Sqrt[e]*(Pi^
2 - 4*Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*
Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])
/4])/Sqrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sq
rt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (-Sqrt[e] + Sqrt[c^2*
d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])
/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I
*ArcCsc[c*x]))] + (4*I)*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E
^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/
(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt
[d])]/Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c
*x]))] + (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] - (4*I)*Pi*Log[Sq
rt[e] - (I*Sqrt[d])/x] + 8*PolyLog[2, (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d
]*E^(I*ArcCsc[c*x]))] + 8*PolyLog[2, (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]
*E^(I*ArcCsc[c*x]))] + 4*PolyLog[2, E^((2*I)*ArcCsc[c*x])]))/(16*d^(3/2)))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sy
m(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Va
lue
```

maple [C] time = 3.98, size = 332, normalized size = 0.58

$$-\frac{ae\arctan\left(\frac{ex}{\sqrt{de}}\right)}{d\sqrt{de}}-\frac{a}{dx}-\frac{cb\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{d}-\frac{b\,\mathrm{arccsc}\left(cx\right)}{dx}+\frac{cbe\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\,\_Z^4+\left(-2c^2d-4e\right)\_Z^2+c^2d\right)}\frac{i\,\mathrm{arccsc}(cx)\ln\left(\frac{\_R1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2 \cdots}}}{\_R1}\right)\cdots}{\_R1\left(\_R1^2c \cdots\right.}\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x^2/(e*x^2+d),x)
```

```
[Out] -a*e/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-a/d/x-c*b/d*((c^2*x^2-1)/c^2/x^2
)^(1/2)-b*arccsc(c*x)/d/x+1/2*c*b*e/d*sum(1/_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*
arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c
^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))+1/2*
```

```
c*b*e/d*sum(_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c
^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(
c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-a\left(\frac{e\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,d}+\frac{1}{dx}\right)+b\int\frac{\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right)}{ex^4+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] -a*(e*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d) + 1/(d*x)) + b*integrate(arctan2(
1, sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x^4 + d*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^2\,\left(e\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x^2)),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\,\mathrm{acsc}\,(cx)}{x^2\left(d+ex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x**2/(e*x**2+d),x)
```

```
[Out] Integral((a + b*acsc(c*x))/(x**2*(d + e*x**2)), x)
```

3.103 $\int \frac{x^5\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=628

$$-\frac{d\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}-\frac{d\left(a+b\csc^{-1}(cx)\right)\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}-\frac{d\left(a+b\csc^{-1}(cx)\right)\log\left(\right.}{e^3}$$

```
[Out] 1/2*d*(a+b*arccsc(c*x))/e^2/(e+d/x^2)+1/2*x^2*(a+b*arccsc(c*x))/e^2+2*d*(a+
b*arccsc(c*x))*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)/e^3-d*(a+b*arccsc(c*x))*
ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/
e^3-d*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^
(1/2)-(c^2*d+e)^(1/2)))/e^3-d*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^
2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3-d*(a+b*arccsc(c*x))*ln(
1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3
-I*b*d*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)/e^3+I*b*d*polylog(2,-I*c*(I
/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3+I*b*d*p
olylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2
)))/e^3+I*b*d*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2
)+(c^2*d+e)^(1/2)))/e^3+I*b*d*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d
)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3-1/2*b*d*arctan((c^2*d+e)^(1/2)/c/x/e
^(1/2)/(1-1/c^2/x^2)^(1/2))/e^(5/2)/(c^2*d+e)^(1/2)+1/2*b*x*(1-1/c^2/x^2)^(
1/2)/c/e^2
```

Rubi [A] time = 1.32, antiderivative size = 628, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 14, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5241, 4733, 4627, 264, 4625, 3717, 2190, 2279, 2391, 4729, 377, 205, 4741, 4519}

$$\frac{ibd\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}+\frac{ibd\text{PolyLog}\left(2,\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}+\frac{ibd\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{e^3}+\frac{ibd\text{PolyLog}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (b*Sqrt[1 - 1/(c^2*x^2)]*x)/(2*c*e^2) + (d*(a + b*ArcCsc[c*x]))/(2*e^2*(e +
 d/x^2)) + (x^2*(a + b*ArcCsc[c*x]))/(2*e^2) - (b*d*ArcTan[Sqrt[c^2*d + e]/
(c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)])/(2*e^(5/2)*Sqrt[c^2*d + e]) - (d*(a +
 b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^
2*d + e])])/e^3 - (d*(a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[
c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/e^3 - (d*(a + b*ArcCsc[c*x])*Log[1 - (
I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3 - (d*(a +
 b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^
2*d + e])])/e^3 + (2*d*(a + b*ArcCsc[c*x])*Log[1 - E^((2*I)*ArcCsc[c*x])])/
e^3 + (I*b*d*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt
[c^2*d + e])])/e^3 + (I*b*d*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sq
rt[e] - Sqrt[c^2*d + e])])/e^3 + (I*b*d*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*Ar
cCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3 + (I*b*d*PolyLog[2, (I*c*Sqrt
[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3 - (I*b*d*PolyLog[
2, E^((2*I)*ArcCsc[c*x])])/e^3
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rule 4625

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n/Tan[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 4627

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c*n
```

```
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4729

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x]))/(2*e*(p + 1)), x]
 - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/Sqrt[1 - c^2*x^2], x],
 x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 4733

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4741

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[((a + b*x)^n*Cos[x])/(c*d + e*Sin[x]), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5241

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSin[x/c])^n)/x^
(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^5 \left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^2} dx = -\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{x^3\left(e+dx^2\right)^2} dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{e^2x^3} - \frac{2d\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{e^3x} + \frac{d^2x\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{e^2\left(e+dx^2\right)^2} + \frac{2d\cdots}{\cdots}\right.\right.$$

$$= \frac{(2d)\,\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{x} dx, x, \frac{1}{x}\right)}{e^3} - \frac{\left(2d^2\right)\text{Subst}\left(\int \frac{x\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2} dx, x, \frac{1}{x}\right)}{e^3} - \frac{\text{Sub}\cdots}{\cdots}$$

$$= \frac{d\left(a+b\csc^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{2e^2} + \frac{(2d)\,\text{Subst}\left(\int (a+bx)\cot(x)\,dx, x, \text{cs}\cdots\right.}{e^3}$$

$$= \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x}{2ce^2} + \frac{d\left(a+b\csc^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{2e^2} - \frac{id\left(a+b\csc^{-1}(cx)\right)^2}{be^3}\cdots$$

$$= \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x}{2ce^2} + \frac{d\left(a+b\csc^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{2e^2} - \frac{id\left(a+b\csc^{-1}(cx)\right)^2}{be^3}\cdots$$

$$= \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x}{2ce^2} + \frac{d\left(a+b\csc^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{2e^2} - \frac{bd\tan^{-1}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\cdots}\right.}{2e^{5/2}\sqrt{c^2d+e}}$$

$$= \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x}{2ce^2} + \frac{d\left(a+b\csc^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{2e^2} - \frac{bd\tan^{-1}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\cdots}\right.}{2e^{5/2}\sqrt{c^2d+e}}$$

$$= \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x}{2ce^2} + \frac{d\left(a+b\csc^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{2e^2} - \frac{bd\tan^{-1}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\cdots}\right.}{2e^{5/2}\sqrt{c^2d+e}}$$

$$= \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x}{2ce^2} + \frac{d\left(a+b\csc^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{2e^2} - \frac{bd\tan^{-1}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\cdots}\right.}{2e^{5/2}\sqrt{c^2d+e}}$$

Mathematica [B] time = 6.24, size = 1480, normalized size = 2.36

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^5*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/4*(-2*a*e*x^2 + (2*a*d^2)/(d + e*x^2) + 4*a*d*Log[d + e*x^2] + b*(I*d*Pi
^2 - (2*e*Sqrt[1 - 1/(c^2*x^2)]*x)/c - (4*I)*d*Pi*ArcCsc[c*x] - 2*e*x^2*Arc
Csc[c*x] + (d^(3/2)*ArcCsc[c*x])/(Sqrt[d] - I*Sqrt[e]*x) + (d^(3/2)*ArcCsc[
c*x])/(Sqrt[d] + I*Sqrt[e]*x) + (8*I)*d*ArcCsc[c*x]^2 - 2*d*ArcSin[1/(c*x)]
 - (16*I)*d*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)
*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - (16*I
)*d*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d]
+ Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - 2*d*Pi*Log[1 + (
Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*d*ArcCsc[c*x]
*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*d*A
rcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^
2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 2*d*Pi*Log[1 + (-Sqrt[e] + Sqrt[
c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*d*ArcCsc[c*x]*Log[1 + (-Sqrt
[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*d*ArcSin[Sqrt[1 +
 (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*
Sqrt[d]*E^(I*ArcCsc[c*x]))] - 2*d*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c
*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*d*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*
d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8*d*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c
*Sqrt[d])]/Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*Arc
Csc[c*x]))] - 2*d*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*Ar
cCsc[c*x]))] + 4*d*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[
d]*E^(I*ArcCsc[c*x]))] + 8*d*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[
2]]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*
d*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] + 2*d*Pi*Log[Sqrt[e] - (I*Sqrt
[d])/x] + 2*d*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] + (d*Sqrt[e]*Log[(2*Sqrt[d]*S
qrt[e]*(Sqrt[e] + c*((-I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^
2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/Sqrt[-(c^2*d) - e]
+ (d*Sqrt[e]*Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(
c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqr
t[e]*x))])/Sqrt[-(c^2*d) - e] + (4*I)*d*PolyLog[2, (Sqrt[e] - Sqrt[c^2*d +
e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*d*PolyLog[2, (-Sqrt[e] + Sqrt[c^
2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*d*PolyLog[2, -((Sqrt[e] +
Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x])))] + (4*I)*d*PolyLog[2, (Sqrt
[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*d*PolyLog[2,
E^((2*I)*ArcCsc[c*x])]))/e^3
```

fricas [F] time = 1.12, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^5 \operatorname{arccsc}(cx) + ax^5}{e^2x^4 + 2\,dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^5*arccsc(c*x) + a*x^5)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 1.83, size = 729, normalized size = 1.16

$$\frac{a\,x^2}{2e^2}-\frac{ad\ln\left(c^2e\,x^2+c^2d\right)}{e^3}-\frac{c^2a\,d^2}{2e^3\left(c^2e\,x^2+c^2d\right)}+\frac{c^2b\,\mathrm{arccsc}\left(cx\right)x^4}{2e\left(c^2e\,x^2+c^2d\right)}+\frac{c^2b\,\mathrm{arccsc}\left(cx\right)d\,x^2}{e^2\left(c^2e\,x^2+c^2d\right)}+\frac{cb\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^3}{2e\left(c^2e\,x^2+c^2d\right)}+\frac{cb}{2e^2(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^2,x)
```

```
[Out] 1/2*a*x^2/e^2-a/e^3*d*ln(c^2*e*x^2+c^2*d)-1/2*c^2*a/e^3*d^2/(c^2*e*x^2+c^2*
d)+1/2*c^2*b/e/(c^2*e*x^2+c^2*d)*arccsc(c*x)*x^4+c^2*b/e^2/(c^2*e*x^2+c^2*d
)*arccsc(c*x)*d*x^2+1/2*c*b/e/(c^2*e*x^2+c^2*d)*((c^2*x^2-1)/c^2/x^2)^(1/2)
*x^3+1/2*c*b/e^2/(c^2*e*x^2+c^2*d)*((c^2*x^2-1)/c^2/x^2)^(1/2)*x*d-1/2*I*b/
e/(c^2*e*x^2+c^2*d)*x^2-1/2*I*b/e^2/(c^2*e*x^2+c^2*d)*d+1/2*I*b*(e*(c^2*d+e
))^(1/2)/(c^2*d+e)/e^3*arctanh(1/4*(2*c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2-2
*c^2*d-4*e)/(c^2*d*e+e^2)^(1/2))*d+2*b/e^3*d*arccsc(c*x)*ln(1+I/c/x+(1-1/c^
2/x^2)^(1/2))-2*I*b/e^3*d*dilog(1+I/c/x+(1-1/c^2/x^2)^(1/2))+1/2*I*b/e^3*d*
sum((_R1^2*c^2*d-c^2*d-4*e)/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-
I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1))
,_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))+2*I*b/e^3*d*dilog(I/c/x+
(1-1/c^2/x^2)^(1/2))+1/2*I*c^2*b/e^3*d^2*sum((_R1^2-1)/(_R1^2*c^2*d-c^2*d-2
*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x
-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}a\left(\frac{d^2}{e^4x^2+de^3}-\frac{x^2}{e^2}+\frac{2\,d\log\left(ex^2+d\right)}{e^3}\right)+b\int\frac{x^5\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right)}{e^2x^4+2\,dex^2+d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*(d^2/(e^4*x^2 + d*e^3) - x^2/e^2 + 2*d*log(e*x^2 + d)/e^3) + b*integ
rate(x^5*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(e^2*x^4 + 2*d*e*x^2 + d^2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^5\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^2,x)
```

```
[Out] int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acsc(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

3.104 $\int \frac{x^3\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=593

$$\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}+\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}+\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic}{\ }\right.}{2e^2}$$

```
[Out] 1/2*(-a-b*arccsc(c*x))/e/(e+d/x^2)-(a+b*arccsc(c*x))*ln(1-(I/c/x+(1-1/c^2/x
^2)^(1/2))^2)/e^2+1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2)
)*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^2+1/2*(a+b*arccsc(c*x))*ln(1+I*c*
(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^2+1/2*(
a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(
c^2*d+e)^(1/2)))/e^2+1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1
/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^2+1/2*I*b*polylog(2,(I/c/x+(1-
1/c^2/x^2)^(1/2))^2)/e^2-1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))
*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^2-1/2*I*b*polylog(2,I*c*(I/c/x+(1-
1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^2-1/2*I*b*polylog
(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e
^2-1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c
^2*d+e)^(1/2)))/e^2+1/2*b*arctan((c^2*d+e)^(1/2)/c/x/e^(1/2)/(1-1/c^2/x^2)^
(1/2))/e^(3/2)/(c^2*d+e)^(1/2)
```

Rubi [A] time = 1.24, antiderivative size = 590, normalized size of antiderivative = 0.99, number of steps used = 29, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5241, 4733, 4625, 3717, 2190, 2279, 2391, 4729, 377, 205, 4741, 4519}

$$-\frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}-\frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}-\frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^2}-\frac{ib\,\text{PolyLog}\left(2,\frac{i}{\ }\right.}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -(a + b*ArcCsc[c*x])/(2*e*(e + d/x^2)) + (b*ArcTan[Sqrt[c^2*d + e]/(c*Sqrt[
e]*Sqrt[1 - 1/(c^2*x^2)]*x)])/(2*e^(3/2)*Sqrt[c^2*d + e]) + ((a + b*ArcCsc[
c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])]
)/(2*e^2) + ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(
Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^2) + ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sq
rt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^2) + ((a + b*A
rcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d
+ e])])/(2*e^2) - ((a + b*ArcCsc[c*x])*Log[1 - E^((2*I)*ArcCsc[c*x])])/e^2
- ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c
^2*d + e])])/e^2 - ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sq
rt[e] - Sqrt[c^2*d + e])])/e^2 - ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*
ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^2 - ((I/2)*b*PolyLog[2, (I*c*
Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^2 + ((I/2)*b*Po
lyLog[2, E^((2*I)*ArcCsc[c*x])])/e^2
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rule 4625

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n/Tan[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 4729

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x]))/(2*e*(p + 1)), x]
 - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/Sqrt[1 - c^2*x^2], x],
 x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 4733

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4741

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[((a + b*x)^n*Cos[x])/(c*d + e*Sin[x]), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5241

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSin[x/c])^n)/x^
(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^3\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = -\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{x\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{e^2x} - \frac{dx\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{e\left(e+dx^2\right)^2} - \frac{dx\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{e^2\left(e+dx^2\right)}\right) dx, x$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)}{e^2} + \frac{d\,\text{Subst}\left(\int \frac{x\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e^2} + \frac{d\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{a+b\csc^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} - \frac{\text{Subst}\left(\int (a+bx)\cot(x)\,dx, x, \csc^{-1}(cx)\right)}{e^2} + \frac{d\,\text{Subst}\left(\int\left(-\frac{\sqrt{-d}}{2a}\right.\right.}{}$$

$$= -\frac{a+b\csc^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2be^2} + \frac{(2i)\,\text{Subst}\left(\int \frac{e^{2ix}(a+bx)}{1-e^{2ix}}\,dx, x, \csc^{-1}(cx)\right)}{e^2}$$

$$= -\frac{a+b\csc^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2be^2} + \frac{b\tan^{-1}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{2e^{3/2}\sqrt{c^2d+e}} - \frac{\left(a+b\csc^{-1}(c\right.}{}$$

$$= -\frac{a+b\csc^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{b\tan^{-1}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{2e^{3/2}\sqrt{c^2d+e}} - \frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-e^{2i\csc^{-1}(cx)}\right.}{e^2}$$

$$= -\frac{a+b\csc^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{b\tan^{-1}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{2e^{3/2}\sqrt{c^2d+e}} + \frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-}}}{\sqrt{e}-\sqrt{c^2d}}\right.}{2e^2}$$

$$= -\frac{a+b\csc^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{b\tan^{-1}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{2e^{3/2}\sqrt{c^2d+e}} + \frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-}}}{\sqrt{e}-\sqrt{c^2d}}\right.}{2e^2}$$

$$= -\frac{a+b\csc^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{b\tan^{-1}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{2e^{3/2}\sqrt{c^2d+e}} + \frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-}}}{\sqrt{e}-\sqrt{c^2d}}\right.}{2e^2}$$

Mathematica [B] time = 2.26, size = 1442, normalized size = 2.43

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (I*b*Pi^2 + (4*a*d)/(d + e*x^2) - (4*I)*b*Pi*ArcCsc[c*x] + (2*b*Sqrt[d]*Arc
Csc[c*x])/(Sqrt[d] - I*Sqrt[e]*x) + (2*b*Sqrt[d]*ArcCsc[c*x])/(Sqrt[d] + I*
Sqrt[e]*x) + (8*I)*b*ArcCsc[c*x]^2 - 4*b*ArcSin[1/(c*x)] - (16*I)*b*ArcSin[
Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e
])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - (16*I)*b*ArcSin[Sqrt[1 +
 (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(Pi
+ 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - 2*b*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*
d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*b*ArcCsc[c*x]*Log[1 + (Sqrt[e] -
 Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*b*ArcSin[Sqrt[1 - (I*S
qrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d
]*E^(I*ArcCsc[c*x]))] - 2*b*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt
[d]*E^(I*ArcCsc[c*x]))] + 4*b*ArcCsc[c*x]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d +
e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqr
t[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc
[c*x]))] - 2*b*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCs
c[c*x]))] + 4*b*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*
E^(I*ArcCsc[c*x]))] + 8*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]
*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 2*b*P
i*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*b*
ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]
))] + 8*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e
] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*b*ArcCsc[c*x]*Log[1
 - E^((2*I)*ArcCsc[c*x])] + 2*b*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] + 2*b*Pi*Lo
g[Sqrt[e] + (I*Sqrt[d])/x] + (2*b*Sqrt[e]*Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e] +
 c*((-I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(
c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/Sqrt[-(c^2*d) - e] + (2*b*Sqrt[e]*Lo
g[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqr
t[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))])/Sqrt
[-(c^2*d) - e] + 4*a*Log[d + e*x^2] + (4*I)*b*PolyLog[2, (Sqrt[e] - Sqrt[c^
2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*b*PolyLog[2, (-Sqrt[e] + S
qrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*b*PolyLog[2, -((Sqrt
[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x])))] + (4*I)*b*PolyLog[2,
 (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*b*PolyL
og[2, E^((2*I)*ArcCsc[c*x])])/(8*e^2)
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^3 \operatorname{arccsc}(cx) + ax^3}{e^2x^4 + 2\,dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arccsc(c*x) + a*x^3)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 1.36, size = 541, normalized size = 0.91

$$\frac{c^{2}ad}{2e^{2}\left(c^{2}e\,x^{2}+c^{2}d\right)}+\frac{a\ln\left(c^{2}e\,x^{2}+c^{2}d\right)}{2e^{2}}-\frac{c^{2}b\,x^{2}\operatorname{arccsc}\left(cx\right)}{2\left(c^{2}e\,x^{2}+c^{2}d\right)e}-\frac{ib\sqrt{e\left(c^{2}d+e\right)}\,\operatorname{arctanh}\left(\frac{2c^{2}d\left(\frac{i}{cx}+\sqrt{1-\frac{1}{c^{2}x^{2}}}\right)^{2}-2c^{2}d-4e}{4\sqrt{c^{2}ed+e^{2}}}\right)}{2e^{2}\left(c^{2}d+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^2,x)
```

```
[Out] 1/2*c^2*a/e^2*d/(c^2*e*x^2+c^2*d)+1/2*a/e^2*ln(c^2*e*x^2+c^2*d)-1/2*c^2*b*x
^2*arccsc(c*x)/(c^2*e*x^2+c^2*d)/e-1/2*I*b*(e*(c^2*d+e))^(1/2)/e^2/(c^2*d+e
)*arctanh(1/4*(2*c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2-2*c^2*d-4*e)/(c^2*d*e+
e^2)^(1/2))-b/e^2*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))+I*b/e^2*dilog
(1+I/c/x+(1-1/c^2/x^2)^(1/2))-1/4*I*b/e^2*sum((_R1^2*c^2*d-c^2*d-4*e)/(_R1^
2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+d
ilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-
4*e)*_Z^2+c^2*d))-I*b/e^2*dilog(I/c/x+(1-1/c^2/x^2)^(1/2))-1/4*I*c^2*b/e^2*
sum((_R1^2-1)/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2
/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^
2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,a\left(\frac{d}{e^{3}x^{2}+de^{2}}+\frac{\log\left(ex^{2}+d\right)}{e^{2}}\right)+b\int\frac{x^{3}\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right)}{e^{2}x^{4}+2\,dex^{2}+d^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*(d/(e^3*x^2 + d*e^2) + log(e*x^2 + d)/e^2) + b*integrate(x^3*arctan2(
1, sqrt(c*x + 1)*sqrt(c*x - 1))/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^{3}\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^{2}+d\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^2,x)
```

```
[Out] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsc(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

3.105 $\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=134

$$\frac{-a-b\csc^{-1}(cx)}{2e\left(d+ex^2\right)}-\frac{bcx\tan^{-1}\left(\sqrt{c^2x^2-1}\right)}{2de\sqrt{c^2x^2}}+\frac{bcx\tan^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{\sqrt{c^2d+e}}\right)}{2d\sqrt{e}\sqrt{c^2x^2}\sqrt{c^2d+e}}$$

```
[Out] 1/2*(-a-b*arccsc(c*x))/e/(e*x^2+d)-1/2*b*c*x*arctan((c^2*x^2-1)^(1/2))/d/e/
(c^2*x^2)^(1/2)+1/2*b*c*x*arctan(e^(1/2)*(c^2*x^2-1)^(1/2)/(c^2*d+e)^(1/2))
/d/e^(1/2)/(c^2*d+e)^(1/2)/(c^2*x^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 131, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5237, 446, 86, 63, 205}

$$-\frac{a+b\csc^{-1}(cx)}{2e\left(d+ex^2\right)}-\frac{bcx\tan^{-1}\left(\sqrt{c^2x^2-1}\right)}{2de\sqrt{c^2x^2}}+\frac{bcx\tan^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{\sqrt{c^2d+e}}\right)}{2d\sqrt{e}\sqrt{c^2x^2}\sqrt{c^2d+e}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -(a + b*ArcCsc[c*x])/(2*e*(d + e*x^2)) - (b*c*x*ArcTan[Sqrt[-1 + c^2*x^2]])
/(2*d*e*Sqrt[c^2*x^2]) + (b*c*x*ArcTan[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/Sqrt[c^
2*d + e]])/(2*d*Sqrt[e]*Sqrt[c^2*d + e]*Sqrt[c^2*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5237

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCsc[c*x]))/(2*e*(p + 1)), x
] + Dist[(b*c*x)/(2*e*(p + 1)*Sqrt[c^2*x^2]), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = -\frac{a+b\csc^{-1}(cx)}{2e\left(d+ex^2\right)} - \frac{(bcx)\int \frac{1}{x\sqrt{-1+c^2x^2}\left(d+ex^2\right)}\,dx}{2e\sqrt{c^2x^2}}$$

$$= -\frac{a+b\csc^{-1}(cx)}{2e\left(d+ex^2\right)} - \frac{(bcx)\,\text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}\,(d+ex)}\,dx, x, x^2\right)}{4e\sqrt{c^2x^2}}$$

$$= -\frac{a+b\csc^{-1}(cx)}{2e\left(d+ex^2\right)} + \frac{(bcx)\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+c^2x}\,(d+ex)}\,dx, x, x^2\right)}{4d\sqrt{c^2x^2}} - \frac{(bcx)\,\text{Subst}\left(\int \frac{1}{x\sqrt{-1+}}\right.}{4de\sqrt{c^2x^2}}$$

$$= -\frac{a+b\csc^{-1}(cx)}{2e\left(d+ex^2\right)} + \frac{(bx)\,\text{Subst}\left(\int \frac{1}{d+\frac{e}{c^2}+\frac{ex^2}{c^2}}\,dx, x, \sqrt{-1+c^2x^2}\right)}{2cd\sqrt{c^2x^2}} - \frac{(bx)\,\text{Subst}\left(\int \frac{1}{\frac{1}{c^2}+}\right.}{2cd}$$

$$= -\frac{a+b\csc^{-1}(cx)}{2e\left(d+ex^2\right)} - \frac{bcx\tan^{-1}\left(\sqrt{-1+c^2x^2}\right)}{2de\sqrt{c^2x^2}} + \frac{bcx\tan^{-1}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{\sqrt{c^2d+e}}\right)}{2d\sqrt{e}\sqrt{c^2d+e}\sqrt{c^2x^2}}$$

Mathematica [C] time = 0.78, size = 286, normalized size = 2.13

$$-\frac{\frac{2a}{d+ex^2} + \frac{b\sqrt{e}\log\left(\frac{4ide-4cd\sqrt{e}x\left(c\sqrt{d}+i\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2(-d)-e}\right)}{b\sqrt{c^2(-d)-e}\left(\sqrt{d}-i\sqrt{e}x\right)}\right)}{d\sqrt{c^2(-d)-e}} + \frac{b\sqrt{e}\log\left(\frac{4i\left(-de+cd\sqrt{e}x\left(\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2(-d)-e}+ic\sqrt{d}\right)\right)}{b\sqrt{c^2(-d)-e}\left(\sqrt{d}+i\sqrt{e}x\right)}\right)}{d\sqrt{c^2(-d)-e}} + \frac{2b\csc^{-1}(cx)}{d+ex^2} - \frac{2b\sin^{-1}}{d}}{4e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/4*((2*a)/(d + e*x^2) + (2*b*ArcCsc[c*x])/(d + e*x^2) - (2*b*ArcSin[1/(c*
x)])/d + (b*Sqrt[e]*Log[((4*I)*d*e - 4*c*d*Sqrt[e]*(c*Sqrt[d] + I*Sqrt[-(c^
2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x)/(b*Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt
[e]*x))])/(d*Sqrt[-(c^2*d) - e]) + (b*Sqrt[e]*Log[((4*I)*(-(d*e) + c*d*Sqrt
[e]*(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(b*Sqrt[-(
c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/(d*Sqrt[-(c^2*d) - e]))/e
```

fricas [A] time = 0.61, size = 385, normalized size = 2.87

$$\left[-\frac{2\,ac^2d^2 + 2\,ade + \sqrt{-c^2de-e^2}\left(bex^2+bd\right)\log\left(\frac{c^2ex^2-c^2d-2\sqrt{-c^2de-e^2}\sqrt{c^2x^2-1}-2\,e}{ex^2+d}\right) + 2\left(bc^2d^2+bde\right)\operatorname{arccsc}(cx)}{4\left(c^2d^3e+d^2e^2+\left(c^2d^2e^2+de^3\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*c^2*d^2 + 2*a*d*e + sqrt(-c^2*d*e - e^2)*(b*e*x^2 + b*d)*log((c^
2*e*x^2 - c^2*d - 2*sqrt(-c^2*d*e - e^2)*sqrt(c^2*x^2 - 1) - 2*e)/(e*x^2 +
```

```
d)) + 2*(b*c^2*d^2 + b*d*e)*arccsc(c*x) + 4*(b*c^2*d^2 + b*d*e + (b*c^2*d*e
 + b*e^2)*x^2)*arctan(-c*x + sqrt(c^2*x^2 - 1)))/(c^2*d^3*e + d^2*e^2 + (c^
2*d^2*e^2 + d*e^3)*x^2), -1/2*(a*c^2*d^2 + a*d*e - sqrt(c^2*d*e + e^2)*(b*e
*x^2 + b*d)*arctan(sqrt(c^2*d*e + e^2)*sqrt(c^2*x^2 - 1)/(c^2*d + e)) + (b*
c^2*d^2 + b*d*e)*arccsc(c*x) + 2*(b*c^2*d^2 + b*d*e + (b*c^2*d*e + b*e^2)*x
^2)*arctan(-c*x + sqrt(c^2*x^2 - 1)))/(c^2*d^3*e + d^2*e^2 + (c^2*d^2*e^2 +
 d*e^3)*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sy
m(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Va
lue
```

maple [B] time = 0.08, size = 354, normalized size = 2.64

$$-\frac{c^2 a}{2e\left(c^2 e x^2+c^2 d\right)}-\frac{c^2 b \operatorname{arccsc}(cx)}{2e\left(c^2 e x^2+c^2 d\right)}+\frac{b\sqrt{c^2x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{2ce\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,xd}-\frac{b\sqrt{c^2x^2-1}\,\ln\left(\frac{2\sqrt{-\frac{c^2d+e}{e}}\,\sqrt{c^2x^2-1}\,e-2\sqrt{-c^2ed}\,cx}{cex+\sqrt{-c^2ed}}\right)}{4ce\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,xd\sqrt{-\frac{c^2d+e}{e}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsc(c*x))/(e*x^2+d)^2,x)
```

```
[Out] -1/2*c^2*a/e/(c^2*e*x^2+c^2*d)-1/2*c^2*b/e/(c^2*e*x^2+c^2*d)*arccsc(c*x)+1/
2/c*b/e*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/d*arctan(1/(c^2*x^2
-1)^(1/2))-1/4/c*b/e*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/d/(-(c
^2*d+e)/e)^(1/2)*ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e-(-c^2*e*d)^
(1/2)*c*x-e)/(c*e*x+(-c^2*e*d)^(1/2)))-1/4/c*b/e*(c^2*x^2-1)^(1/2)/((c^2*x^
2-1)/c^2/x^2)^(1/2)/x/d/(-(c^2*d+e)/e)^(1/2)*ln(-2*((-(c^2*d+e)/e)^(1/2)*(c
^2*x^2-1)^(1/2)*e+(-c^2*e*d)^(1/2)*c*x-e)/(-c*e*x+(-c^2*e*d)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(\left(c^2e^2x^2+c^2de\right)\int \frac{xe^{\left(\frac{1}{2}\log(cx+1)+\frac{1}{2}\log(cx-1)\right)}}{c^2e^2x^4+\left(c^2e^2x^4+\left(c^2de-e^2\right)x^2-de\right)(cx+1)(cx-1)+\left(c^2de-e^2\right)x^2-de}\,dx+\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right)\right)b}{2\left(e^2x^2+de\right)}-\frac{\cdots}{2\left(e^2x\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(c^2*e^2*x^2 + c^2*d*e)*integrate(1/2*x*e^(1/2*log(c*x + 1) + 1/2*l
og(c*x - 1))/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e + (c^2*e^2*x^4 + (c^2
*d*e - e^2)*x^2 - d*e)*e^(log(c*x + 1) + log(c*x - 1))), x) + arctan2(1, sq
rt(c*x + 1)*sqrt(c*x - 1)))*b/(e^2*x^2 + d*e) - 1/2*a/(e^2*x^2 + d*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^2,x)
```

```
[Out] int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(a+b\operatorname{acsc}\left(cx\right)\right)}{\left(d+ex^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsc(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Integral(x*(a + b*acsc(c*x))/(d + e*x**2)**2, x)
```

3.106 $\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=566

$$-\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}-\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}-\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\right.}{2d^2}$$

```
[Out] -1/2*e*(a+b*arccsc(c*x))/d^2/(e+d/x^2)+1/2*I*(a+b*arccsc(c*x))^2/b/d^2-1/2*
(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-
(c^2*d+e)^(1/2)))/d^2-1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(
1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^2-1/2*(a+b*arccsc(c*x))*ln(1-
I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^2-1
/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/
2)+(c^2*d+e)^(1/2)))/d^2+1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))
*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^2+1/2*I*b*polylog(2,I*c*(I/c/x+(1-
1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^2+1/2*I*b*polylog
(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d
^2+1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c
^2*d+e)^(1/2)))/d^2+1/2*b*arctan((c^2*d+e)^(1/2)/c/x/e^(1/2)/(1-1/c^2/x^2)^
(1/2))*e^(1/2)/d^2/(c^2*d+e)^(1/2)
```

Rubi [A] time = 1.16, antiderivative size = 566, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {5241, 4733, 4729, 377, 205, 4741, 4519, 2190, 2279, 2391}

$$\frac{ib\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}+\frac{ib\text{PolyLog}\left(2,\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}+\frac{ib\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^2}+\frac{ib\text{PolyLog}\left(2,\frac{ic}{}\right.}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^2),x]
```

```
[Out] -(e*(a + b*ArcCsc[c*x]))/(2*d^2*(e + d/x^2)) + ((I/2)*(a + b*ArcCsc[c*x])^2
)/(b*d^2) + (b*Sqrt[e]*ArcTan[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^
2)]*x)])/(2*d^2*Sqrt[c^2*d + e]) - ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-
d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^2) - ((a + b*ArcCs
c[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e]
)])/(2*d^2) - ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))
/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d^2) - ((a + b*ArcCsc[c*x])*Log[1 + (I*c*
Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d^2) + ((I/2)*
b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e]
)])/d^2 + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - S
qrt[c^2*d + e])])/d^2 + ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*
x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/d^2 + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*
E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/d^2
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rule 4729

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x]))/(2*e*(p + 1)), x]
 - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/Sqrt[1 - c^2*x^2], x],
 x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 4733

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4741

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[((a + b*x)^n*Cos[x])/(c*d + e*Sin[x]), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5241

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSin[x/c])^n)/x^
(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx = -\text{Subst}\left(\int \frac{x^3\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(-\frac{ex\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)^2}+\frac{x\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{x\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d}+\frac{e\,\text{Subst}\left(\int \frac{x\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{d}$$

$$= -\frac{e\left(a+b\csc^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)}-\frac{\text{Subst}\left(\int \left(-\frac{\sqrt{-d}\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}\,x\right)}+\frac{\sqrt{-d}\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}+\sqrt{-d}\,x\right)}\right)dx, x, \frac{1}{x}\right)}{d}+\frac{(be)\,\text{S}\cdots}{\cdots}$$

$$= -\frac{e\left(a+b\csc^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)}+\frac{\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}\,x}\,dx, x, \frac{1}{x}\right)}{2(-d)^{3/2}}-\frac{\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}\,x}\,dx, x, \frac{1}{x}\right)}{2(-d)^{3/2}}+\cdots$$

$$= -\frac{e\left(a+b\csc^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)}+\frac{b\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{2d^2\sqrt{c^2d+e}}+\frac{\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\,\sin(x)}\,dx, x, \csc^{-1}(c\cdots\right)}{2(-d)^{3/2}}\cdots$$

$$= -\frac{e\left(a+b\csc^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)}+\frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2bd^2}+\frac{b\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{2d^2\sqrt{c^2d+e}}-\frac{\text{Subst}\left(\int \frac{\cdots}{\frac{\sqrt{e}}{c}-\cdots}\cdots\right)}{\cdots}$$

$$= -\frac{e\left(a+b\csc^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)}+\frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2bd^2}+\frac{b\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{2d^2\sqrt{c^2d+e}}-\frac{\left(a+b\csc^{-1}(cx\cdots\right.}{\cdots}$$

$$= -\frac{e\left(a+b\csc^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)}+\frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2bd^2}+\frac{b\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{2d^2\sqrt{c^2d+e}}-\frac{\left(a+b\csc^{-1}(cx\cdots\right.}{\cdots}$$

$$= -\frac{e\left(a+b\csc^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)}+\frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2bd^2}+\frac{b\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{2d^2\sqrt{c^2d+e}}-\frac{\left(a+b\csc^{-1}(cx\cdots\right.}{\cdots}$$

Mathematica [F] time = 43.45, size = 0, normalized size = 0.00

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^2),x]
```

```
[Out] Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^2), x]
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b \operatorname{arccsc}(cx) + a}{e^2 x^5 + 2\, dex^3 + d^2 x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sy
m(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Va
lue
```

maple [C] time = 2.61, size = 3036, normalized size = 5.36

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x/(e*x^2+d)^2,x)
```

```
[Out] -1/2*a/d^2*ln(c^2*e*x^2+c^2*d)+a/d^2*ln(c*x)+1/8*I*b*c^2*(e*(c^2*d+e))^(1/2
)/d/e/(c^2*d+e)*polylog(2,c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d+2*(e*(
c^2*d+e))^(1/2)+2*e))-3/2*I*b/c^2*e*polylog(2,c^2*d*(I/c/x+(1-1/c^2/x^2)^(1
/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))/d^3/(c^2*d+e)*(e*(c^2*d+e))^(1/2)
-2*I*b/c^4*e^2*arccsc(c*x)^2/d^4/(c^2*d+e)*(e*(c^2*d+e))^(1/2)-3*I*b/c^2*ar
ccsc(c*x)^2/d^3/(c^2*d+e)*(e*(c^2*d+e))^(1/2)*e+3*b/c^2*e*ln(1-c^2*d*(I/c/x
+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)/d^3/
(c^2*d+e)*(e*(c^2*d+e))^(1/2)-1/4*b*c^2*(e*(c^2*d+e))^(1/2)/d/e/(c^2*d+e)*a
rccsc(c*x)*ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d+2*(e*(c^2*d+e))^
(1/2)+2*e))-1/8*I*b*c^2*polylog(2,c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*
d-2*(e*(c^2*d+e))^(1/2)+2*e))/d/e/(c^2*d+e)*(e*(c^2*d+e))^(1/2)+1/2*a*c^2/d
/(c^2*e*x^2+c^2*d)+2*b/c^4*e^3*ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^
2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)/d^4/(c^2*d+e)-2*I*b/c^2*e^2*pol
ylog(2,c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e
))/d^3/(c^2*d+e)-I*b/c^4*e^3*polylog(2,c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/
(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))/d^4/(c^2*d+e)-2*I*b/c^4*e^3*arccsc(c*x)^
2/d^4/(c^2*d+e)-4*I*b/c^2*arccsc(c*x)^2/d^3/(c^2*d+e)*e^2+2*I*b/c^4*arccsc(
c*x)^2*e/d^4*(e*(c^2*d+e))^(1/2)-1/2*b*c^2*x^2*arccsc(c*x)*e/(c^2*e*x^2+c^2
*d)/d^2+1/4*I*b*polylog(2,c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(
c^2*d+e))^(1/2)+2*e))/d^2-1/2*b*ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c
^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)/d^2+I*b*arccsc(c*x)^2/d^2+1/2*
I*b/d^2*sum((_R1^2*c^2*d-2*c^2*d-4*e)/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x
)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1
/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))+I*b/c^4*polylo
```

```
g(2,c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*
e/d^4*(e*(c^2*d+e))^(1/2)-2*b/c^4*ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/
(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)*e/d^4*(e*(c^2*d+e))^(1/2)+4*
b/c^2*e^2*ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(
1/2)+2*e))*arccsc(c*x)/d^3/(c^2*d+e)+3/2*b*ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^
(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)/d^2/(c^2*d+e)*(e*(c
^2*d+e))^(1/2)+5/2*b*ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(
c^2*d+e))^(1/2)+2*e))*arccsc(c*x)/d^2*e/(c^2*d+e)-1/2*b*(e*(c^2*d+e))^(1/2)
/d^2/(c^2*d+e)*arccsc(c*x)*ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d+
2*(e*(c^2*d+e))^(1/2)+2*e))+1/2*I*b/c^2*polylog(2,c^2*d*(I/c/x+(1-1/c^2/x^2
)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))/d^3*(e*(c^2*d+e))^(1/2)-1/4*I
*b*c^2*polylog(2,c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))
^(1/2)+2*e))/d/(c^2*d+e)+2*I*b/c^2*arccsc(c*x)^2*e/d^3+2*I*b/c^4*arccsc(c*x
)^2*e^2/d^4-1/2*I*b*c^2*arccsc(c*x)^2/d/(c^2*d+e)-5/4*I*b*polylog(2,c^2*d*(
I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))/d^2*e/(c^2*
d+e)-I*b*(e*(c^2*d+e))^(1/2)/d^2/(c^2*d+e)*arccsc(c*x)^2-3/4*I*b*polylog(2,
c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))/d^2/
(c^2*d+e)*(e*(c^2*d+e))^(1/2)+1/4*I*b*(e*(c^2*d+e))^(1/2)/d^2/(c^2*d+e)*pol
ylog(2,c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e
))-1/2*I*b*(e*(c^2*d+e))^(1/2)/d^2/(c^2*d+e)*arctanh(1/4*(2*c^2*d*(I/c/x+(1
-1/c^2/x^2)^(1/2))^2-2*c^2*d-4*e)/(c^2*d*e+e^2)^(1/2))-5/2*I*b*arccsc(c*x)^
2/d^2/(c^2*d+e)*e-2*b/c^4*ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2
*(e*(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)*e^2/d^4-b/c^2*ln(1-c^2*d*(I/c/x+(1-1
/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)/d^3*(e*(c
^2*d+e))^(1/2)+I*b/c^2*polylog(2,c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d
-2*(e*(c^2*d+e))^(1/2)+2*e))/d^3*e+I*b/c^2*arccsc(c*x)^2/d^3*(e*(c^2*d+e))^
(1/2)+1/2*b*c^2*ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d
+e))^(1/2)+2*e))*arccsc(c*x)/d/(c^2*d+e)+I*b/c^4*polylog(2,c^2*d*(I/c/x+(1-
1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*e^2/d^4-2*b/c^2*ln(1
-c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*arc
csc(c*x)*e/d^3+2*b/c^4*e^2*ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-
2*(e*(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)/d^4/(c^2*d+e)*(e*(c^2*d+e))^(1/2)+1
/4*b*c^2*ln(1-c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1
/2)+2*e))*arccsc(c*x)/d/e/(c^2*d+e)*(e*(c^2*d+e))^(1/2)-I*b/c^4*e^2*polylog
(2,c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))/d
^4/(c^2*d+e)*(e*(c^2*d+e))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a\left(\frac{1}{dex^2+d^2}-\frac{\log\left(ex^2+d\right)}{d^2}+\frac{2\log(x)}{d^2}\right)+b\int\frac{\arctan\left(1,\sqrt{cx+1}\sqrt{cx-1}\right)}{e^2x^5+2dex^3+d^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*(1/(d*e*x^2 + d^2) - log(e*x^2 + d)/d^2 + 2*log(x)/d^2) + b*integrate
(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x\,\left(e\,x^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^2),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

3.107 $\int \frac{x^4\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=803

$$\frac{x\left(a+b\csc^{-1}(cx)\right)}{e^2}-\frac{3\sqrt{-d}\,\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)\left(a+b\csc^{-1}(cx)\right)}{4e^{5/2}}+\frac{3\sqrt{-d}\,\log\left(\frac{i\sqrt{-d}\,e^{i\csc^{-1}(cx)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)\left(a+b\csc^{-1}(\right.}{4e^{5/2}}$$

```
[Out] x*(a+b*arccsc(c*x))/e^2+b*arctanh((1-1/c^2/x^2)^(1/2))/c/e^2-3/4*(a+b*arccs
c(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^
(1/2)))*(-d)^(1/2)/e^(5/2)+3/4*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x
^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)-3/4*(a+
b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^
2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)+3/4*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-
1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)-
3/4*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2
*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)+3/4*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)
^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)-3/4*I*b*po
lylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2
)))*(-d)^(1/2)/e^(5/2)+3/4*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-
d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)-1/4*d*(a+b*arccsc(c*
x))/e^2/(-d/x+(-d)^(1/2)*e^(1/2))+1/4*d*(a+b*arccsc(c*x))/e^2/(d/x+(-d)^(1/
2)*e^(1/2))+1/4*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^
(1/2)/(1-1/c^2/x^2)^(1/2))*d^(1/2)/e^2/(c^2*d+e)^(1/2)+1/4*b*arctanh((c^2*d
+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))*d^(1/
2)/e^2/(c^2*d+e)^(1/2)
```

Rubi [A] time = 2.40, antiderivative size = 803, normalized size of antiderivative = 1.00, number of steps used = 51, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5241, 4733, 4627, 266, 63, 208, 4667, 4743, 725, 206, 4741, 4519, 2190, 2279, 2391}

$$\frac{x\left(a+b\csc^{-1}(cx)\right)}{e^2}-\frac{3\sqrt{-d}\,\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)\left(a+b\csc^{-1}(cx)\right)}{4e^{5/2}}+\frac{3\sqrt{-d}\,\log\left(\frac{i\sqrt{-d}\,e^{i\csc^{-1}(cx)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)\left(a+b\csc^{-1}(\right.}{4e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -(d*(a + b*ArcCsc[c*x]))/(4*e^2*(Sqrt[-d]*Sqrt[e] - d/x)) + (d*(a + b*ArcCs
c[c*x]))/(4*e^2*(Sqrt[-d]*Sqrt[e] + d/x)) + (x*(a + b*ArcCsc[c*x]))/e^2 + (
b*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/(c*e^2) + (b*Sqrt[d]*ArcTanh[(c^2*d - (Sq
rt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*e
^2*Sqrt[c^2*d + e]) + (b*Sqrt[d]*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*
Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*e^2*Sqrt[c^2*d + e]) -
(3*Sqrt[-d]*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(S
qrt[e] - Sqrt[c^2*d + e])])/(4*e^(5/2)) + (3*Sqrt[-d]*(a + b*ArcCsc[c*x])*L
og[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*e^
(5/2)) - (3*Sqrt[-d]*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[
c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*e^(5/2)) + (3*Sqrt[-d]*(a + b*ArcCs
c[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e]
)])/(4*e^(5/2)) - (((3*I)/4)*b*Sqrt[-d]*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*Ar
cCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/e^(5/2) + (((3*I)/4)*b*Sqrt[-d]*P
```

```
olyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/e^
(5/2) - (((3*I)/4)*b*Sqrt[-d]*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x])
)/(Sqrt[e] + Sqrt[c^2*d + e])])/e^(5/2) + (((3*I)/4)*b*Sqrt[-d]*PolyLog[2,
(I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^(5/2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
```

```
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rule 4627

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4667

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4733

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4741

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[((a + b*x)^n*Cos[x])/(c*d + e*Sin[x]), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4743

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSin[c*x])^n)/(e*(m + 1)), x] -
Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1
))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
&& NeQ[m, -1]
```

Rule 5241

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSin[x/c])^n)/x^
(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^4 \left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = -\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{x^2\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{e^2x^2} - \frac{d\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{e\left(e+dx^2\right)^2} - \frac{d\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{e^2\left(e+dx^2\right)}\right) dx, x, \frac{1}{x}$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{x^2}\,dx, x, \frac{1}{x}\right)}{e^2} + \frac{d\,\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e^2} + \frac{d\,\text{Subst}\left(\int \frac{a\text{-}}{}\right.}{}$$

$$= \frac{x\left(a+b\csc^{-1}(cx)\right)}{e^2} - \frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{ce^2} + \frac{d\,\text{Subst}\left(\int \left(\frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}\,x\right)} + \right.\right.}{e^2}$$

$$= \frac{x\left(a+b\csc^{-1}(cx)\right)}{e^2} + \frac{d\,\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}\,x}\,dx, x, \frac{1}{x}\right)}{2e^{5/2}} + \frac{d\,\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}\,x}\,dx\right.}{2e^{5/2}}$$

$$= -\frac{d\left(a+b\csc^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{d\left(a+b\csc^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{x\left(a+b\csc^{-1}(cx)\right)}{e^2} + \frac{d\,\text{Subst}\left(\int \frac{(a}{\frac{\sqrt{e}}{c}}\right.}{}$$

$$= -\frac{d\left(a+b\csc^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{d\left(a+b\csc^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{x\left(a+b\csc^{-1}(cx)\right)}{e^2} + \frac{b\tanh^{-1}\left(\sqrt{1}\right.}{ce^2}$$

$$= -\frac{d\left(a+b\csc^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{d\left(a+b\csc^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{x\left(a+b\csc^{-1}(cx)\right)}{e^2} + \frac{b\tanh^{-1}\left(\sqrt{1}\right.}{ce^2}$$

$$= -\frac{d\left(a+b\csc^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{d\left(a+b\csc^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{x\left(a+b\csc^{-1}(cx)\right)}{e^2} + \frac{b\tanh^{-1}\left(\sqrt{1}\right.}{ce^2}$$

$$= -\frac{d\left(a+b\csc^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{d\left(a+b\csc^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{x\left(a+b\csc^{-1}(cx)\right)}{e^2} + \frac{b\tanh^{-1}\left(\sqrt{1}\right.}{ce^2}$$

$$= -\frac{d\left(a+b\csc^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{d\left(a+b\csc^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{x\left(a+b\csc^{-1}(cx)\right)}{e^2} + \frac{b\tanh^{-1}\left(\sqrt{1}\right.}{ce^2}$$

$$= -\frac{d\left(a+b\csc^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{d\left(a+b\csc^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{x\left(a+b\csc^{-1}(cx)\right)}{e^2} + \frac{b\tanh^{-1}\left(\sqrt{1}\right.}{ce^2}$$

Mathematica [B] time = 6.17, size = 1634, normalized size = 2.03

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (a*x)/e^2 + (a*d*x)/(2*e^2*(d + e*x^2)) - (3*a*Sqrt[d]*ArcTan[(Sqrt[e]*x)/S
qrt[d]])/(2*e^(5/2)) + b*(-1/4*(d*(-(ArcCsc[c*x]/((-I)*Sqrt[d]*Sqrt[e] + e*
x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*((-
I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d)
 - e]*(Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/e^2 - (d*(-(
ArcCsc[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[
(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqrt[
1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))]/Sqrt[-(
c^2*d) - e]))/Sqrt[d]))/(4*e^2) + (3*Sqrt[d]*(Pi^2 - 4*Pi*ArcCsc[c*x] + 8*A
rcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[
(((-I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] +
 (4*I)*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))
] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*A
rcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Lo
g[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*Pi
*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)
*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x
]))] - (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sq
rt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (8*I)*ArcCsc[c*x]
*Log[1 - E^((2*I)*ArcCsc[c*x])] - (4*I)*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] + 8
*PolyLog[2, (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8
*PolyLog[2, -((Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x])))] +
 4*PolyLog[2, E^((2*I)*ArcCsc[c*x])]))/(32*e^(5/2)) - (3*Sqrt[d]*(Pi^2 - 4*
Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d
])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/S
qrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*
E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e]
)/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sq
rt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCs
c[c*x]))] + (4*I)*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*Ar
cCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqr
t[d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/
Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))]
 + (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] - (4*I)*Pi*Log[Sqrt[e]
- (I*Sqrt[d])/x] + 8*PolyLog[2, (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I
*ArcCsc[c*x]))] + 8*PolyLog[2, (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*
ArcCsc[c*x]))] + 4*PolyLog[2, E^((2*I)*ArcCsc[c*x])]))/(32*e^(5/2)) + ((Arc
Csc[c*x]*Cot[ArcCsc[c*x]/2])/2 + Log[Cos[ArcCsc[c*x]/2]] - Log[Sin[ArcCsc[c
*x]/2]] + (ArcCsc[c*x]*Tan[ArcCsc[c*x]/2])/2)/(c*e^2))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^4\operatorname{arccsc}(cx) + ax^4}{e^2x^4 + 2\,dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arccsc(c*x) + a*x^4)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 19.15, size = 1888, normalized size = 2.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^2,x)
```

```
[Out] a*x/e^2+1/2*c^2*a/e^2*d*x/(c^2*e*x^2+c^2*d)-3/2*a/e^2*d/(d*e)^(1/2)*arctan(
e*x/(d*e)^(1/2))+c^2*b*x^3*arccsc(c*x)/e/(c^2*e*x^2+c^2*d)+3/2*c^2*b*x*arcc
sc(c*x)/e^2/(c^2*e*x^2+c^2*d)*d+1/c*b/e^2*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))+1
/2/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(I/c/x+(1-
1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/e^2/d+1/c^4
*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(I/c/x+(1-1/c^2/
x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/e^2/d^2*(e*(c^2*d
+e))^(1/2)+1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*
(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/e
/d^2-1/2/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(I/c
/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/e^2/(
c^2*d+e)/d*(e*(c^2*d+e))^(1/2)-1/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*
d)^(1/2)*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1
/2)-2*e)*d)^(1/2))/e/(c^2*d+e)/d-1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e
)*d)^(1/2)*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^
(1/2)-2*e)*d)^(1/2))/e/(c^2*d+e)/d^2*(e*(c^2*d+e))^(1/2)-1/c^4*b*(-(c^2*d-2
*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/(
(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)/d^2+1/2/c^2*b*((c^2*
d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2
))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e^2/d-1/c^4*b*((c^2*d+2*(e*
(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^
2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e^2/d^2*(e*(c^2*d+e))^(1/2)+1/c^4*
b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(I/c/x+(1-1/c^2/x
^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e/d^2+1/2/c^2*b*((c
^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(
1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e^2/(c^2*d+e)/d*(e*(c^2*
d+e))^(1/2)-1/c^2*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d
*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e
/(c^2*d+e)/d+1/c^4*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*
d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/
e/(c^2*d+e)/d^2*(e*(c^2*d+e))^(1/2)-1/c^4*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2
*e)*d)^(1/2)*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e)
)^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)/d^2-1/c*b/e^2*ln(I/c/x+(1-1/c^2/x^2)^(1/2)
-1)-3/16*c*b/e^3*d*sum((_R1^2*c^2*d-c^2*d-4*e)/_R1/(_R1^2*c^2*d-c^2*d-2*e)*
(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-
1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))+3
/16*c*b/e^3*d*sum((_R1^2*c^2*d+4*_R1^2*e-c^2*d)/_R1/(_R1^2*c^2*d-c^2*d-2*e)
*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1
-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,a\left(\frac{dx}{e^3x^2+de^2}-\frac{3\,d\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,e^2}+\frac{2\,x}{e^2}\right)+b\int\frac{x^4\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right)}{e^2x^4+2\,dex^2+d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*(d*x/(e^3*x^2 + d*e^2) - 3*d*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^2) +
2*x/e^2) + b*integrate(x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(e^2*x^4
 + 2*d*e*x^2 + d^2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \left(a + b \operatorname{asin}\left(\frac{1}{c x}\right)\right)}{\left(e x^2 + d\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asin(1/(c*x))))/(d + e*x^2)^2,x)
```

```
[Out] int((x^4*(a + b*asin(1/(c*x))))/(d + e*x^2)^2, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acsc(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

3.108 $\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=765

$$-\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}\,e^{3/2}}+\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}\,e^{3/2}}-\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1\right.}{4\sqrt{-d}\,e^{3}}$$

```
[Out] -1/4*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(
1/2)-(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*(a+b*arccsc(c*x))*ln(1+I*c*(I
/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(3/2)/(-d
)^(1/2)-1/4*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/
2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*(a+b*arccsc(c*x))*ln(1
+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(3
/2)/(-d)^(1/2)-1/4*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2
)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*I*b*polylog(2,I*c*(I/c/
x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(
1/2)-1/4*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)
+(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2
/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*(
a+b*arccsc(c*x))/e/(-d/x+(-d)^(1/2)*e^(1/2))+1/4*(-a-b*arccsc(c*x))/e/(d/x+
(-d)^(1/2)*e^(1/2))-1/4*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c
^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/e/d^(1/2)/(c^2*d+e)^(1/2)-1/4*b*arctanh(
(c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))
/e/d^(1/2)/(c^2*d+e)^(1/2)
```

Rubi [A] time = 1.25, antiderivative size = 765, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {5241, 4667, 4743, 725, 206, 4741, 4519, 2190, 2279, 2391}

$$-\frac{ib\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}\,e^{3/2}}+\frac{ib\text{PolyLog}\left(2,\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}\,e^{3/2}}-\frac{ib\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4\sqrt{-d}\,e^{3/2}}+\frac{ib\text{PolyLog}\left(\right.}{4\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (a + b*ArcCsc[c*x])/(4*e*(Sqrt[-d]*Sqrt[e] - d/x)) - (a + b*ArcCsc[c*x])/(4
*e*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c
*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*Sqrt[d]*e*Sqrt[c^2*d +
 e]) - (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]
*Sqrt[1 - 1/(c^2*x^2)])])/(4*Sqrt[d]*e*Sqrt[c^2*d + e]) - ((a + b*ArcCsc[c*
x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/
(4*Sqrt[-d]*e^(3/2)) + ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcC
sc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) - ((a + b*ArcC
sc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e
])])/(4*Sqrt[-d]*e^(3/2)) + ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I
*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) - ((I/4)*
b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e]
)])/(Sqrt[-d]*e^(3/2)) + ((I/4)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]
))/(Sqrt[e] - Sqrt[c^2*d + e])])/(Sqrt[-d]*e^(3/2)) - ((I/4)*b*PolyLog[2, (
(-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(Sqrt[-d]*
```

```
e^(3/2)) + ((I/4)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] +
Sqrt[c^2*d + e])])/(Sqrt[-d]*e^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rule 4667

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4741

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[((a + b*x)^n*Cos[x])/(c*d + e*Sin[x]), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4743

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSin[c*x])^n)/(e*(m + 1)), x] -
```

```
Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1
))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
&& NeQ[m, -1]
```

Rule 5241

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSin[x/c])^n)/x^
(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = -\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int\left(-\frac{d\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-dx\right)^2}-\frac{d\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}+dx\right)^2}-\frac{d\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{2e\left(-de-d^2x^2\right)}\right)dx,\right.$$

$$= \frac{d\,\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{4e}+\frac{d\,\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{4e}+\frac{d\,\text{Subst}\left(\int \right.}{}$$

$$= \frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}-\frac{b\,\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}-dx\right)\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{4ce}+$$

$$= \frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}-\frac{\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}\,x}\,dx, x, \frac{1}{x}\right)}{4e^{3/2}}-\frac{\text{Subst}\left(\int \right.}{}$$

$$= \frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}-\frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}\,e\sqrt{c^2d+e}}-\frac{b\tanh^{-1}\left(\right.}{4\sqrt{}}$$

$$= \frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}-\frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}\,e\sqrt{c^2d+e}}-\frac{b\tanh^{-1}\left(\right.}{4\sqrt{}}$$

$$= \frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}-\frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}\,e\sqrt{c^2d+e}}-\frac{b\tanh^{-1}\left(\right.}{4\sqrt{}}$$

$$= \frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}-\frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}\,e\sqrt{c^2d+e}}-\frac{b\tanh^{-1}\left(\right.}{4\sqrt{}}$$

$$= \frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}-\frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}\,e\sqrt{c^2d+e}}-\frac{b\tanh^{-1}\left(\right.}{4\sqrt{}}$$

Mathematica [A] time = 2.80, size = 1482, normalized size = 1.94

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] ((-4*a*Sqrt[e]*x)/(d + e*x^2) + (4*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] +
 b*((2*ArcCsc[c*x])/(I*Sqrt[d] - Sqrt[e]*x) - (2*ArcCsc[c*x])/(I*Sqrt[d] +
Sqrt[e]*x) + (8*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((
(-I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]])/Sq
rt[d] - (8*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*S
qrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]])/Sqrt[d] -
(I*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/S
qrt[d] + ((2*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*
E^(I*ArcCsc[c*x]))])/Sqrt[d] - ((4*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d
])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x
]))])/Sqrt[d] + (I*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*
ArcCsc[c*x]))])/Sqrt[d] - ((2*I)*ArcCsc[c*x]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d
 + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] + ((4*I)*ArcSin[Sqrt[1 + (I*
Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt
[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] + (I*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e]
)/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] - ((2*I)*ArcCsc[c*x]*Log[1 - (Sqr
t[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] - ((4*I)*Ar
cSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2
*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] - (I*Pi*Log[1 + (Sqrt[e] +
 Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] + ((2*I)*ArcCsc[c
*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqr
t[d] + ((4*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sq
rt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] - (I*Pi*Lo
g[Sqrt[e] - (I*Sqrt[d])/x])/Sqrt[d] + (I*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x])/S
qrt[d] - ((2*I)*Sqrt[e]*Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*((-I)*c*Sqrt[d]
 - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[
d] + I*Sqrt[e]*x))])/(Sqrt[d]*Sqrt[-(c^2*d) - e]) + ((2*I)*Sqrt[e]*Log[(2*S
qrt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqrt[1 -
1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))])/(Sqrt[d]*S
qrt[-(c^2*d) - e]) + (2*PolyLog[2, (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E
^(I*ArcCsc[c*x]))])/Sqrt[d] - (2*PolyLog[2, (-Sqrt[e] + Sqrt[c^2*d + e])/(c
*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] - (2*PolyLog[2, -((Sqrt[e] + Sqrt[c^2
*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x])))])/Sqrt[d] + (2*PolyLog[2, (Sqrt[e]
+ Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d]))/(8*e^(3/2))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^2\,\text{arccsc}\,(cx) + ax^2}{e^2x^4 + 2\,dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccsc(c*x) + a*x^2)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sy
m(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Va
lue
```

maple [C] time = 4.79, size = 1722, normalized size = 2.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^2,x)
```

```
[Out] -1/2*c^2*a/e*x/(c^2*e*x^2+c^2*d)+1/2*a/e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)
)-1/2*c^2*b*arccsc(c*x)/e*x/(c^2*e*x^2+c^2*d)-1/2/c^2*b*(-(c^2*d-2*(e*(c^2*
d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2
*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/e/d^2-1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^
(1/2)+2*e)*d)^(1/2)*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c
^2*d+e))^(1/2)-2*e)*d)^(1/2))/e/d^3*(e*(c^2*d+e))^(1/2)-1/c^4*b*(-(c^2*d-2*
(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((
-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^3+1/2/c^2*b*(-(c^2*d-2*(e*(c^
2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d
+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/e/(c^2*d+e)/d^2*(e*(c^2*d+e))^(1/2)+1
/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(I/c/x+(1-1/
c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)/d^2
+1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(I/c/x+(1-
1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)/d
^3*(e*(c^2*d+e))^(1/2)+1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)
*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)
*d)^(1/2))*e/(c^2*d+e)/d^3-1/2/c^2*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^
(1/2)*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)
+2*e)*d)^(1/2))/e/d^2+1/c^4*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*a
rctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d
)^(1/2))/e/d^3*(e*(c^2*d+e))^(1/2)-1/c^4*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*
e)*d)^(1/2)*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))
^(1/2)+2*e)*d)^(1/2))/d^3-1/2/c^2*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(
1/2)*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+
2*e)*d)^(1/2))/e/(c^2*d+e)/d^2*(e*(c^2*d+e))^(1/2)+1/c^2*b*((c^2*d+2*(e*(c^
2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d
+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)/d^2-1/c^4*b*((c^2*d+2*(e*(c
^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*
d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)/d^3*(e*(c^2*d+e))^(1/2)+1/
c^4*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(I/c/x+(1-1/c
^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))*e/(c^2*d+e)/d^3
-1/4*c*b/e*sum(1/_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(
1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=Ro
otOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-1/4*c*b/e*sum(_R1/(_R1^2*c^2*d-
c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R
1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^
2+c^2*d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}a\left(\frac{x}{e^2x^2+de}-\frac{\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,e}\right)+b\int\frac{x^2\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right)}{e^2x^4+2\,dex^2+d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*(x/(e^2*x^2 + d*e) - arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e)) + b*integr
ate(x^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(e^2*x^4 + 2*d*e*x^2 + d^2)
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^2,x)
[Out] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsc(c*x))/(e*x**2+d)**2,x)
[Out] Timed out
```

3.109 $\int \frac{a+b\csc^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=762

$$\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic}{\cdots}\right.}{4(-d)^{3/2}\sqrt{e}}$$

```
[Out] 1/4*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1
/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*(a+b*arccsc(c*x))*ln(1+I*c*(I/
c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e
^(1/2)+1/4*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2
)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*(a+b*arccsc(c*x))*ln(1+
I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^
(3/2)/e^(1/2)+1/4*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)
/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*I*b*polylog(2,I*c*(I/c/x
+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1
/2)+1/4*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+
(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/
x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*(-
a-b*arccsc(c*x))/d/(-d/x+(-d)^(1/2)*e^(1/2))+1/4*(a+b*arccsc(c*x))/d/(d/x+(
-d)^(1/2)*e^(1/2))+1/4*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^
2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(3/2)/(c^2*d+e)^(1/2)+1/4*b*arctanh((c^
2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^
(3/2)/(c^2*d+e)^(1/2)
```

Rubi [A] time = 2.29, antiderivative size = 759, normalized size of antiderivative = 1.00, number of steps used = 47, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {5231, 4733, 4667, 4743, 725, 206, 4741, 4519, 2190, 2279, 2391}

$$\frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{ib\,\text{PolyLog}\left(2,\frac{ic}{\cdots}\right.}{4(-d)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(d + e*x^2)^2,x]
```

```
[Out] -(a + b*ArcCsc[c*x])/(4*d*(Sqrt[-d]*Sqrt[e] - d/x)) + (a + b*ArcCsc[c*x])/(
4*d*(Sqrt[-d]*Sqrt[e] + d/x)) + (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(
c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*d^(3/2)*Sqrt[c^2*d +
e]) + (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*
Sqrt[1 - 1/(c^2*x^2)])])/(4*d^(3/2)*Sqrt[c^2*d + e]) + ((a + b*ArcCsc[c*x])
*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*
(-d)^(3/2)*Sqrt[e]) - ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCs
c[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) + ((a + b*Arc
Csc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d +
e])])/(4*(-d)^(3/2)*Sqrt[e]) - ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E
^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) + ((
I/4)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d
 + e])])/((-d)^(3/2)*Sqrt[e]) - ((I/4)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcC
sc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/((-d)^(3/2)*Sqrt[e]) + ((I/4)*b*Pol
yLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(
```

```
(-d)^(3/2)*Sqrt[e]) - ((I/4)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/
(Sqrt[e] + Sqrt[c^2*d + e])])/((-d)^(3/2)*Sqrt[e])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rule 4667

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4733

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4741

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[((a + b*x)^n*Cos[x])/(c*d + e*Sin[x]), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4743

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSin[c*x])^n)/(e*(m + 1)), x] -
Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1
))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
&& NeQ[m, -1]
```

Rule 5231

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSin[x/c])^n)/x^(2*(p + 1))
, x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx = -\text{Subst}\left(\int \frac{x^2\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int\left(-\frac{e\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)^2}+\frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{d\left(e+dx^2\right)}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d} + \frac{e\,\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int\left(\frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}\,x\right)}+\frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}\,x\right)}\right)dx, x, \frac{1}{x}\right)}{d} + \frac{e\,\text{Subst}\left(\int\left(-\frac{d\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-dx\right)^2} - \right.\right.}{}$$

$$= -\left(\frac{1}{4}\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)\right) - \frac{1}{4}\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right) -$$

$$= -\frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{1}{2}\text{Subst}\left(\int\left(-\frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}-\sqrt{-d}\,x\right)} - \frac{a}{2d\sqrt{}}\right.\right.$$

$$= -\frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\,\text{Subst}\left(\int \frac{1}{d^2+\frac{de}{c^2}-x^2}\,dx, x, \frac{-d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4cd} + \frac{b\,\text{S}}{}$$

$$= -\frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d+e}} + \frac{b\tanh^{-1}\left(\right.}{4d}$$

$$= -\frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d+e}} + \frac{b\tanh^{-1}\left(\right.}{4d}$$

$$= -\frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d+e}} + \frac{b\tanh^{-1}\left(\right.}{4d}$$

$$= -\frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d+e}} + \frac{b\tanh^{-1}\left(\right.}{4d}$$

$$= -\frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d+e}} + \frac{b\tanh^{-1}\left(\right.}{4d}$$

Mathematica [A] time = 2.87, size = 1477, normalized size = 1.94

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(d + e*x^2)^2,x]
```

```
[Out] ((a*x)/(d^2 + d*e*x^2) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*Sqrt[e])
+ (b*((2*Sqrt[d]*ArcCsc[c*x])/((-I)*Sqrt[d]*Sqrt[e] + e*x) + (2*Sqrt[d]*Arc
Csc[c*x])/(I*Sqrt[d]*Sqrt[e] + e*x) + (8*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqr
t[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])
/4])/Sqrt[c^2*d + e]])/Sqrt[e] - (8*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])
]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqr
t[c^2*d + e]])/Sqrt[e] - (I*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[
d]*E^(I*ArcCsc[c*x]))])/Sqrt[e] + ((2*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] - Sqr
t[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[e] - ((4*I)*ArcSin[Sqrt[
1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(
c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[e] + (I*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2
*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[e] - ((2*I)*ArcCsc[c*x]*Log[1
 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[e] + (
(4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] +
 Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[e] + (I*Pi*Log[1 - (
Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[e] - ((2*I)
*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x
]))])/Sqrt[e] - ((4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Lo
g[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[e] -
 (I*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/
Sqrt[e] + ((2*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]
*E^(I*ArcCsc[c*x]))])/Sqrt[e] + ((4*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[
d])]/Sqrt[2]]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*
x]))])/Sqrt[e] - (I*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x])/Sqrt[e] + (I*Pi*Log[Sq
rt[e] + (I*Sqrt[d])/x])/Sqrt[e] + ((2*I)*Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e] +
c*((-I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c
^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/Sqrt[-(c^2*d) - e] - ((2*I)*Log[(2*Sq
rt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqrt[1 - 1
/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))])/Sqrt[-(c^2*
d) - e] + (2*PolyLog[2, (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[
c*x]))])/Sqrt[e] - (2*PolyLog[2, (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^
(I*ArcCsc[c*x]))])/Sqrt[e] - (2*PolyLog[2, -((Sqrt[e] + Sqrt[c^2*d + e])/(c
*Sqrt[d]*E^(I*ArcCsc[c*x])))])/Sqrt[e] + (2*PolyLog[2, (Sqrt[e] + Sqrt[c^2*
d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[e]))/(4*d^(3/2)))/2
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arccsc}\,(cx) + a}{e^2x^4 + 2\,dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sy
m(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Va
lue
```

maple [C] time = 5.05, size = 1716, normalized size = 2.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/(e*x^2+d)^2,x)
```

```
[Out] 1/2*c^2*a*x/d/(c^2*e*x^2+c^2*d)+1/2*a/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))
+1/2*c^2*b*arccsc(c*x)*x/d/(c^2*e*x^2+c^2*d)-1/4*c*b/d*sum(1/_R1/(_R1^2*c^2
*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog(
(_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*
_Z^2+c^2*d))-1/4*c*b/d*sum(_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_
R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R
1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))+1/2/c^2*b*(-(c^2*d-2*
(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((
-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^3+1/c^4*b*(-(c^2*d-2*(e*(c^2*
d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2
*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^4*(e*(c^2*d+e))^(1/2)+1/c^4*b*(-(c^2*
d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2)
)/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^4*e-1/2/c^2*b*(-(c^2*d-2*
(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((
-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)/d^3*(e*(c^2*d+e))^(1/
2)-1/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(I/c/x+(
1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))*e/(c^2*d+
e)/d^3-1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(I/c
/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^4/(
c^2*d+e)*(e*(c^2*d+e))^(1/2)*e-1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*
d)^(1/2)*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1
/2)-2*e)*d)^(1/2))/d^4/(c^2*d+e)*e^2+1/2/c^2*b*((c^2*d+2*(e*(c^2*d+e))^(1/2
)+2*e)*d)^(1/2)*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d
+e))^(1/2)+2*e)*d)^(1/2))/d^3-1/c^4*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)
^(1/2)*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2
)+2*e)*d)^(1/2))/d^4*(e*(c^2*d+e))^(1/2)+1/c^4*b*((c^2*d+2*(e*(c^2*d+e))^(1
/2)+2*e)*d)^(1/2)*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2
*d+e))^(1/2)+2*e)*d)^(1/2))/d^4*e+1/2/c^2*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2
*e)*d)^(1/2)*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e)
)^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)/d^3*(e*(c^2*d+e))^(1/2)-1/c^2*b*((c^2*d+2*
(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/(
(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))*e/(c^2*d+e)/d^3+1/c^4*b*((c^2*d
+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2)
)/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^4/(c^2*d+e)*(e*(c^2*d+e))^
(1/2)*e-1/c^4*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(I/
c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^4/(
c^2*d+e)*e^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a\left(\frac{x}{dex^2+d^2}+\frac{\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,d}\right)+b\int\frac{\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right)}{e^2x^4+2\,dex^2+d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*(x/(d*e*x^2 + d^2) + arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d)) + b*integra
te(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\,\mathrm{asin}\left(\frac{1}{cx}\right)}{\left(ex^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x^2)^2,x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(d + e*x^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

3.110 $\int \frac{a+b\csc^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=806

$$-\frac{a}{d^2x}-\frac{b\csc^{-1}(cx)}{d^2x}+\frac{e\left(a+b\csc^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{e\left(a+b\csc^{-1}(cx)\right)}{4d^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}-\frac{be\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{dc^2+e}}-\frac{be\tanh^{-1}\left(\frac{d\cdots}{c\sqrt{d}\sqrt{\cdots}}\right)}{4d^{5/2}\sqrt{d\cdots}}$$

```
[Out] -a/d^2/x-b*arccsc(c*x)/d^2/x+3/4*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2
/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)-3/4*(
a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(
c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)+3/4*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(
1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2
)-3/4*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^
(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)+3/4*I*b*polylog(2,-I*c*(I/c/x+(1
-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)
-3/4*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2
*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)+3/4*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2
)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)-3/4*I*b*p
olylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2
)))*e^(1/2)/(-d)^(5/2)+1/4*e*(a+b*arccsc(c*x))/d^2/(-d/x+(-d)^(1/2)*e^(1/2)
)-1/4*e*(a+b*arccsc(c*x))/d^2/(d/x+(-d)^(1/2)*e^(1/2))-1/4*b*e*arctanh((c^2
*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(
5/2)/(c^2*d+e)^(1/2)-1/4*b*e*arctanh((c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)
/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(5/2)/(c^2*d+e)^(1/2)-b*c*(1-1/c^2/
x^2)^(1/2)/d^2
```

Rubi [A] time = 2.36, antiderivative size = 806, normalized size of antiderivative = 1.00, number of steps used = 50, number of rules used = 13, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {5241, 4733, 4619, 261, 4667, 4743, 725, 206, 4741, 4519, 2190, 2279, 2391}

$$-\frac{a}{d^2x}-\frac{b\csc^{-1}(cx)}{d^2x}+\frac{e\left(a+b\csc^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{e\left(a+b\csc^{-1}(cx)\right)}{4d^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}-\frac{be\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{dc^2+e}}-\frac{be\tanh^{-1}\left(\frac{d\cdots}{c\sqrt{d}\sqrt{\cdots}}\right)}{4d^{5/2}\sqrt{d\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(x^2*(d + e*x^2)^2),x]
```

```
[Out] -((b*c*Sqrt[1 - 1/(c^2*x^2)])/d^2) - a/(d^2*x) - (b*ArcCsc[c*x])/(d^2*x) +
(e*(a + b*ArcCsc[c*x]))/(4*d^2*(Sqrt[-d]*Sqrt[e] - d/x)) - (e*(a + b*ArcCsc
[c*x]))/(4*d^2*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*e*ArcTanh[(c^2*d - (Sqrt[-d]*
Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*d^(5/2)*
Sqrt[c^2*d + e]) - (b*e*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*S
qrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*d^(5/2)*Sqrt[c^2*d + e]) + (3*Sq
rt[e]*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e]
 - Sqrt[c^2*d + e])])/(4*(-d)^(5/2)) - (3*Sqrt[e]*(a + b*ArcCsc[c*x])*Log[1
 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*(-d)^(
5/2)) + (3*Sqrt[e]*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*
x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*(-d)^(5/2)) - (3*Sqrt[e]*(a + b*ArcCs
c[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e]
)])/(4*(-d)^(5/2)) + (((3*I)/4)*b*Sqrt[e]*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*
ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(-d)^(5/2) - (((3*I)/4)*b*Sqrt[
```

```
e]*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])]
)/(-d)^(5/2) + (((3*I)/4)*b*Sqrt[e]*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc
[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(-d)^(5/2) - (((3*I)/4)*b*Sqrt[e]*Pol
yLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(-d)
^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rule 4619

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4667

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4733

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4741

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[((a + b*x)^n*Cos[x])/(c*d + e*Sin[x]), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4743

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSin[c*x])^n)/(e*(m + 1)), x] -
Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1
))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
&& NeQ[m, -1]
```

Rule 5241

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSin[x/c])^n)/x^
(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx = -\text{Subst}\left(\int \frac{x^4\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{d^2} + \frac{e^2\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^2} - \frac{2e\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)}\right) dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right) dx, x, \frac{1}{x}\right)}{d^2} + \frac{(2e)\,\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d^2} - \frac{e^2\,\text{Subst}\Big(\int}{}$$

$$= -\frac{a}{d^2x} - \frac{b\,\text{Subst}\left(\int \sin^{-1}\left(\frac{x}{c}\right) dx, x, \frac{1}{x}\right)}{d^2} + \frac{(2e)\,\text{Subst}\left(\int\left(\frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}\,x\right)} + \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}\,x\right)}\right)\right.}{d^2}$$

$$= -\frac{a}{d^2x} - \frac{b\csc^{-1}(cx)}{d^2x} + \frac{b\,\text{Subst}\left(\int \frac{x}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{cd^2} + \frac{\sqrt{e}\,\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}\,x}\,dx, x, \frac{1}{x}\right)}{d^2} + $$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\csc^{-1}(cx)}{d^2x} + \frac{e\left(a+b\csc^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{e\left(a+b\csc^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{e}\ \text{Sub}}{}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\csc^{-1}(cx)}{d^2x} + \frac{e\left(a+b\csc^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{e\left(a+b\csc^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{\sqrt{e}\ \text{Sub}}{}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\csc^{-1}(cx)}{d^2x} + \frac{e\left(a+b\csc^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{e\left(a+b\csc^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{be\tanh}{}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\csc^{-1}(cx)}{d^2x} + \frac{e\left(a+b\csc^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{e\left(a+b\csc^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{be\tanh}{}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\csc^{-1}(cx)}{d^2x} + \frac{e\left(a+b\csc^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{e\left(a+b\csc^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{be\tanh}{}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\csc^{-1}(cx)}{d^2x} + \frac{e\left(a+b\csc^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{e\left(a+b\csc^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{be\tanh}{}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\csc^{-1}(cx)}{d^2x} + \frac{e\left(a+b\csc^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{e\left(a+b\csc^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{be\tanh}{}$$

Mathematica [A] time = 2.71, size = 1525, normalized size = 1.89

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x^2*(d + e*x^2)^2),x]
```

```
[Out] ((-8*a*Sqrt[d])/x - (4*a*Sqrt[d]*e*x)/(d + e*x^2) - 12*a*Sqrt[e]*ArcTan[(Sq
rt[e]*x)/Sqrt[d]] + b*(-8*c*Sqrt[d]*Sqrt[1 - 1/(c^2*x^2)] - (8*Sqrt[d]*ArcC
sc[c*x])/x - (2*Sqrt[d]*e*ArcCsc[c*x])/((-I)*Sqrt[d]*Sqrt[e] + e*x) - (2*Sq
rt[d]*e*ArcCsc[c*x])/(I*Sqrt[d]*Sqrt[e] + e*x) - 24*Sqrt[e]*ArcSin[Sqrt[1 -
 (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Cot[(
Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] + 24*Sqrt[e]*ArcSin[Sqrt[1 + (I*Sq
rt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*Ar
cCsc[c*x])/4])/Sqrt[c^2*d + e]] + (3*I)*Sqrt[e]*Pi*Log[1 + (Sqrt[e] - Sqrt[
c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (6*I)*Sqrt[e]*ArcCsc[c*x]*Log[
1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (12*I)*Sqr
t[e]*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - S
qrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (3*I)*Sqrt[e]*Pi*Log[1 + (
-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (6*I)*Sqrt[e]*
ArcCsc[c*x]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x
]))] - (12*I)*Sqrt[e]*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log
[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (3*I)*Sq
rt[e]*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))]
 + (6*I)*Sqrt[e]*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]
*E^(I*ArcCsc[c*x]))] + (12*I)*Sqrt[e]*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d
])]/Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x
]))] + (3*I)*Sqrt[e]*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I
*ArcCsc[c*x]))] - (6*I)*Sqrt[e]*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d +
 e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (12*I)*Sqrt[e]*ArcSin[Sqrt[1 - (I*Sqr
t[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*
E^(I*ArcCsc[c*x]))] + (3*I)*Sqrt[e]*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] - (3*I)
*Sqrt[e]*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] - ((2*I)*e*Log[(2*Sqrt[d]*Sqrt[e]*
(Sqrt[e] + c*((-I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x)
)/(Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/Sqrt[-(c^2*d) - e] + ((2*I
)*e*Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) -
e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))]
)/Sqrt[-(c^2*d) - e] - 6*Sqrt[e]*PolyLog[2, (Sqrt[e] - Sqrt[c^2*d + e])/(c*
Sqrt[d]*E^(I*ArcCsc[c*x]))] + 6*Sqrt[e]*PolyLog[2, (-Sqrt[e] + Sqrt[c^2*d +
 e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 6*Sqrt[e]*PolyLog[2, -((Sqrt[e] + Sqr
t[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x])))] - 6*Sqrt[e]*PolyLog[2, (Sqrt[
e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))]))/(8*d^(5/2))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b \operatorname{arccsc}(cx) + a}{e^2x^6 + 2\,dex^4 + d^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)/(e^2*x^6 + 2*d*e*x^4 + d^2*x^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 19.92, size = 1784, normalized size = 2.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x^2/(e*x^2+d)^2,x)
```

```
[Out] -1/2*a/d^2*e*c^2*x/(c^2*e*x^2+c^2*d)-3/2*a/d^2*e/(d*e)^(1/2)*arctan(e*x/(d*
e)^(1/2))-a/d^2/x-b*arccsc(c*x)/d^2/x+b/c^2*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2
*e)*d)^(1/2)*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e)
)^(1/2)+2*e)*d)^(1/2))/d^4/(c^2*d+e)*e^2+b/c^4*((c^2*d+2*(e*(c^2*d+e))^(1/2
)+2*e)*d)^(1/2)*e^3*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c
^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^5/(c^2*d+e)+3/4*c*b/d^2*e*sum(1/_R1/(_R1^2*
c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dil
og((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*
e)*_Z^2+c^2*d))+3/4*c*b/d^2*e*sum(_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x
)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1
/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-b/c^4*(-(c^2*d
-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2
))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^5*(e*(c^2*d+e))^(1/2)+b/
c^2*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(I/c/x+(1-1/c^2
/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^4/(c^2*d+e)*e^
2+b/c^4*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e^3*arctan(c*d*(I/c/x+
(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^5/(c^2
*d+e)+b/c^4*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e*arctanh(c*d*(I/c/
x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^5*(e*
(c^2*d+e))^(1/2)+1/2*b/c^2*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arc
tan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^
(1/2))/d^4/(c^2*d+e)*(e*(c^2*d+e))^(1/2)*e+b/c^4*(-(c^2*d-2*(e*(c^2*d+e))^(
1/2)+2*e)*d)^(1/2)*e^2*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e
*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^5/(c^2*d+e)*(e*(c^2*d+e))^(1/2)-1/2*b/c^
2*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(I/c/x+(1-1/c^2/x
^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^4/(c^2*d+e)*(e*(c
^2*d+e))^(1/2)*e-b/c^4*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e^2*arct
anh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(
1/2))/d^5/(c^2*d+e)*(e*(c^2*d+e))^(1/2)-1/2*b/c^2*(-(c^2*d-2*(e*(c^2*d+e))^
(1/2)+2*e)*d)^(1/2)*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c
^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^4*e-b/c^4*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*
e)*d)^(1/2)*e^2*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d
+e))^(1/2)-2*e)*d)^(1/2))/d^5-1/2*b/c^2*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*
d)^(1/2)*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1
/2)+2*e)*d)^(1/2))/d^4*e-b/c^4*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*
e^2*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2
*e)*d)^(1/2))/d^5-c*b/d^2*((c^2*x^2-1)/c^2/x^2)^(1/2)-1/2*b*c^2*x*e*arccsc(
c*x)/(c^2*e*x^2+c^2*d)/d^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\,a\left(\frac{3\,ex^2+2\,d}{d^2ex^3+d^3x}+\frac{3\,e\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,d^2}\right)+b\int\frac{\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right)}{e^2x^6+2\,dex^4+d^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*((3*e*x^2 + 2*d)/(d^2*e*x^3 + d^3*x) + 3*e*arctan(e*x/sqrt(d*e))/(sq
rt(d*e)*d^2)) + b*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(e^2*x^
6 + 2*d*e*x^4 + d^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^2\,\left(e\,x^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x^2)^2),x)
[Out] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x^2)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x**2/(e*x**2+d)**2,x)
[Out] Timed out
```

3.111 $\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx$

Optimal. Leaf size=727

$$\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}+\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}+\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic}{}\right.}{2e^3}$$

```
[Out] 1/4*(-a-b*arccsc(c*x))/e/(e+d/x^2)^2+1/2*(-a-b*arccsc(c*x))/e^2/(e+d/x^2)+1
/8*b*(c^2*d+2*e)*arctan((c^2*d+e)^(1/2)/c/x/e^(1/2)/(1-1/c^2/x^2)^(1/2))/e^
(5/2)/(c^2*d+e)^(3/2)-(a+b*arccsc(c*x))*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)
/e^3+1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/
(e^(1/2)-(c^2*d+e)^(1/2)))/e^3+1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c
^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3+1/2*(a+b*arccsc(c*
x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2
)))/e^3+1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/
2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3+1/2*I*b*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1
/2))^2)/e^3-1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(
e^(1/2)-(c^2*d+e)^(1/2)))/e^3-1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1
/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3-1/2*I*b*polylog(2,-I*c*(I/c/
x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3-1/2*I*b*po
lylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)
))/e^3+1/2*b*arctan((c^2*d+e)^(1/2)/c/x/e^(1/2)/(1-1/c^2/x^2)^(1/2))/e^(5/2
)/(c^2*d+e)^(1/2)+1/8*b*c*d*(1-1/c^2/x^2)^(1/2)/e^2/(c^2*d+e)/(e+d/x^2)/x
```

Rubi [A] time = 1.44, antiderivative size = 727, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 13, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {5241, 4733, 4625, 3717, 2190, 2279, 2391, 4729, 382, 377, 205, 4741, 4519}

$$-\frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}-\frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}-\frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^3}-\frac{ib\,\text{PolyLog}\left(2,\frac{i}{}\right.}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (b*c*d*Sqrt[1 - 1/(c^2*x^2)])/(8*e^2*(c^2*d + e)*(e + d/x^2)*x) - (a + b*Ar
cCsc[c*x])/(4*e*(e + d/x^2)^2) - (a + b*ArcCsc[c*x])/(2*e^2*(e + d/x^2)) +
(b*ArcTan[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)])/(2*e^(5/2)*
Sqrt[c^2*d + e]) + (b*(c^2*d + 2*e)*ArcTan[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[
1 - 1/(c^2*x^2)]*x)])/(8*e^(5/2)*(c^2*d + e)^(3/2)) + ((a + b*ArcCsc[c*x])*
Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e
^3) + ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e
] - Sqrt[c^2*d + e])])/(2*e^3) + ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]
*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^3) + ((a + b*ArcCsc[
c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])]
)/(2*e^3) - ((a + b*ArcCsc[c*x])*Log[1 - E^((2*I)*ArcCsc[c*x])])/e^3 - ((I/
2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d +
 e])])/e^3 - ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e]
- Sqrt[c^2*d + e])])/e^3 - ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc
[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3 - ((I/2)*b*PolyLog[2, (I*c*Sqrt[-
```

```
d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3 + ((I/2)*b*PolyLog[
2, E^((2*I)*ArcCsc[c*x])])/e^3
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rule 4625

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n/Tan[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 4729

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x]))/(2*e*(p + 1)), x]
 - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/Sqrt[1 - c^2*x^2], x],
 x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 4733

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4741

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[((a + b*x)^n*Cos[x])/(c*d + e*Sin[x]), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5241

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSin[x/c])^n)/x^
(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^5\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = -\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{x\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int\left(\frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{e^3x} - \frac{dx\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{e\left(e+dx^2\right)^3} - \frac{dx\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{e^2\left(e+dx^2\right)^2} - \frac{dx\left(\cdots\right.}{\cdots}\right.\right.$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)}{e^3} + \frac{d\,\text{Subst}\left(\int \frac{x\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e^3} + \frac{d\,\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{a+b\csc^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\csc^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} - \frac{\text{Subst}\left(\int (a+bx)\cot(x)\,dx, x, \csc^{-1}(cx)\right)}{e^3} + \cdots$$

$$= \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{8e^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\csc^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\csc^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2be^3}\cdots$$

$$= \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{8e^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\csc^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\csc^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2be^3}\cdots$$

$$= \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{8e^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\csc^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\csc^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{b\tan^{-1}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\cdots}\right.}{2e^{5/2}\sqrt{c^2d+e}}\cdots$$

$$= \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{8e^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\csc^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\csc^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{b\tan^{-1}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\cdots}\right.}{2e^{5/2}\sqrt{c^2d+e}}\cdots$$

$$= \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{8e^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\csc^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\csc^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{b\tan^{-1}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\cdots}\right.}{2e^{5/2}\sqrt{c^2d+e}}\cdots$$

$$= \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{8e^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\csc^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\csc^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{b\tan^{-1}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\cdots}\right.}{2e^{5/2}\sqrt{c^2d+e}}\cdots$$

Mathematica [B] time = 7.76, size = 2053, normalized size = 2.82

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^5*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/4*(a*d^2)/(e^3*(d + e*x^2)^2) + (a*d)/(e^3*(d + e*x^2)) + (a*Log[d + e*x
^2])/(2*e^3) + b*((((7*I)/16)*Sqrt[d]*(-(ArcCsc[c*x]/((-I)*Sqrt[d]*Sqrt[e]
+ e*x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c
*((-I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^
2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/e^(5/2)
- (((7*I)/16)*Sqrt[d]*(-(ArcCsc[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSi
n[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[d] +
 Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d]
 - I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/e^(5/2) - (d*((I*c*Sqrt[e]
*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x))
- ArcCsc[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) - ArcSin[1/(c*x)]/(d*S
qrt[e]) + (I*(2*c^2*d + e)*Log[(4*d*Sqrt[e]*Sqrt[c^2*d + e]*(I*Sqrt[e] + c*
(c*Sqrt[d] - Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d + e)*((-I
)*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/(16*e^(5/2)) - (d*(((-I)*
c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*(I*Sqrt[d] + Sqrt[e
]*x)) - ArcCsc[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - ArcSin[1/(c*x)]/(
d*Sqrt[e]) + (I*(2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*((-I)*Sqrt[
e] + c*(c*Sqrt[d] + Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d +
e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/(16*e^(5/2)) + ((I/16
)*(Pi^2 - 4*Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 - (I*Sqrt[e
])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*Arc
Csc[c*x])/4])/Sqrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e
])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] - Sq
rt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1 - (I*S
qrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d
]*E^(I*ArcCsc[c*x]))] + (4*I)*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqr
t[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d
+ e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(
c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*Ar
cCsc[c*x]))] + (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] - (4*I)*Pi*
Log[Sqrt[e] + (I*Sqrt[d])/x] + 8*PolyLog[2, (-Sqrt[e] + Sqrt[c^2*d + e])/(c
*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8*PolyLog[2, -((Sqrt[e] + Sqrt[c^2*d + e])/(
c*Sqrt[d]*E^(I*ArcCsc[c*x])))] + 4*PolyLog[2, E^((2*I)*ArcCsc[c*x])]))/e^3
+ ((I/16)*(Pi^2 - 4*Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 + (
I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(Pi +
2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2
*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (-Sqrt[
e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1
 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(
c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e]
)/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqr
t[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 + (I*Sq
rt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]
*E^(I*ArcCsc[c*x]))] + (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] - (
4*I)*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] + 8*PolyLog[2, (Sqrt[e] - Sqrt[c^2*d +
 e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8*PolyLog[2, (Sqrt[e] + Sqrt[c^2*d +
e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*PolyLog[2, E^((2*I)*ArcCsc[c*x])]))/
e^3)
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^5 \operatorname{arccsc}(cx) + ax^5}{e^3x^6 + 3\,de^2x^4 + 3\,d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^5*arccsc(c*x) + a*x^5)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 +
 d^3), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 2.03, size = 1498, normalized size = 2.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^3,x)
```

```
[Out] 1/2*a/e^3*ln(c^2*e*x^2+c^2*d)-1/4*c^4*a*d^2/e^3/(c^2*e*x^2+c^2*d)^2+c^2*a/e
^3*d/(c^2*e*x^2+c^2*d)-3/4*c^6*b/e/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*arccsc(c*x
)*d*x^4-1/2*c^6*b/e^2/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*arccsc(c*x)*d^2*x^2+1/8
*c^5*b/e/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*((c^2*x^2-1)/c^2/x^2)^(1/2)*x^3*d+1/
8*c^5*b/e^2/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*((c^2*x^2-1)/c^2/x^2)^(1/2)*x*d^2
-1/4*I*c^2*b/e^3/(c^2*d+e)*d*sum((_R1^2*c^2*d-c^2*d-4*e)/(_R1^2*c^2*d-c^2*d
-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c
/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2
*d))-1/4*I*c^4*b/e/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*x^2*d-1/4*I*c^2*b/e^2/(c^2
*d+e)*sum((_R1^2-1)/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1
-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=Roo
tOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))*d-3/4*c^4*b/(c^2*e*x^2+c^2*d)^2/
(c^2*d+e)*arccsc(c*x)*x^4-1/2*c^4*b/e/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*arccsc(
c*x)*d*x^2-I*b/e^2/(c^2*d+e)*dilog(I/c/x+(1-1/c^2/x^2)^(1/2))-b/e^2/(c^2*d+
e)*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))-1/8*I*c^4*b/(c^2*e*x^2+c^2*d
)^2/(c^2*d+e)*x^4+I*b/e^2/(c^2*d+e)*dilog(1+I/c/x+(1-1/c^2/x^2)^(1/2))-1/4*
I*b/e^2/(c^2*d+e)*sum((_R1^2*c^2*d-c^2*d-4*e)/(_R1^2*c^2*d-c^2*d-2*e)*(I*ar
ccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2
/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-1/8*I*
c^4*b/e^2/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*d^2-3/4*I*b*(e*(c^2*d+e))^(1/2)/e^2
/(c^2*d+e)^2*arctanh(1/4*(2*c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2-2*c^2*d-4*e
)/(c^2*d*e+e^2)^(1/2))-c^2*b/e^3/(c^2*d+e)*d*arccsc(c*x)*ln(1+I/c/x+(1-1/c^
2/x^2)^(1/2))+I*c^2*b/e^3/(c^2*d+e)*d*dilog(1+I/c/x+(1-1/c^2/x^2)^(1/2))-1/
4*I*c^4*b/e^3/(c^2*d+e)*d^2*sum((_R1^2-1)/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc
(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2
)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-5/8*I*c^2*
b*(e*(c^2*d+e))^(1/2)/e^3/(c^2*d+e)^2*arctanh(1/4*(2*c^2*d*(I/c/x+(1-1/c^2/
x^2)^(1/2))^2-2*c^2*d-4*e)/(c^2*d*e+e^2)^(1/2))*d-I*c^2*b/e^3/(c^2*d+e)*d*d
ilog(I/c/x+(1-1/c^2/x^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\,a\left(\frac{4\,dex^2+3\,d^2}{e^5x^4+2\,de^4x^2+d^2e^3}+\frac{2\,\log\left(ex^2+d\right)}{e^3}\right)+b\int\frac{x^5\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right)}{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/4*a*((4*d*e*x^2 + 3*d^2)/(e^5*x^4 + 2*d*e^4*x^2 + d^2*e^3) + 2*log(e*x^2
+ d)/e^3) + b*integrate(x^5*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(e^3*x^
6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5 \left(a + b \operatorname{asin}\left(\frac{1}{c x}\right)\right)}{\left(e x^2 + d\right)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^3,x)
[Out] int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acsc(c*x))/(e*x**2+d)**3,x)
[Out] Timed out
```

3.112 $\int \frac{x^3\left(a+b \csc^{-1}(cx)\right)}{\left(d+ex^2\right)^3} \, dx$

Optimal. Leaf size=157

$$\frac{x^4\left(a+b\csc^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}+\frac{bcx\left(c^2d+2e\right)\tan^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{\sqrt{c^2d+e}}\right)}{8de^{3/2}\sqrt{c^2x^2}\left(c^2d+e\right)^{3/2}}-\frac{bcx\sqrt{c^2x^2-1}}{8e\sqrt{c^2x^2}\left(c^2d+e\right)\left(d+ex^2\right)}$$

```
[Out] 1/4*x^4*(a+b*arccsc(c*x))/d/(e*x^2+d)^2+1/8*b*c*(c^2*d+2*e)*x*arctan(e^(1/2
)*(c^2*x^2-1)^(1/2)/(c^2*d+e)^(1/2))/d/e^(3/2)/(c^2*d+e)^(3/2)/(c^2*x^2)^(1
/2)-1/8*b*c*x*(c^2*x^2-1)^(1/2)/e/(c^2*d+e)/(e*x^2+d)/(c^2*x^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {264, 5239, 12, 446, 78, 63, 205}

$$\frac{x^4\left(a+b\csc^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}+\frac{bcx\left(c^2d+2e\right)\tan^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{\sqrt{c^2d+e}}\right)}{8de^{3/2}\sqrt{c^2x^2}\left(c^2d+e\right)^{3/2}}-\frac{bcx\sqrt{c^2x^2-1}}{8e\sqrt{c^2x^2}\left(c^2d+e\right)\left(d+ex^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -(b*c*x*Sqrt[-1 + c^2*x^2])/(8*e*(c^2*d + e)*Sqrt[c^2*x^2]*(d + e*x^2)) + (
x^4*(a + b*ArcCsc[c*x]))/(4*d*(d + e*x^2)^2) + (b*c*(c^2*d + 2*e)*x*ArcTan[
(Sqrt[e]*Sqrt[-1 + c^2*x^2])/Sqrt[c^2*d + e]])/(8*d*e^(3/2)*(c^2*d + e)^(3/
2)*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{x^3\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \frac{x^4\left(a+b\csc^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}+\frac{(bcx)\int \frac{x^3}{4d\sqrt{-1+c^2x^2}\left(d+ex^2\right)^2}\,dx}{\sqrt{c^2x^2}}$$

$$= \frac{x^4\left(a+b\csc^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}+\frac{(bcx)\int \frac{x^3}{\sqrt{-1+c^2x^2}\left(d+ex^2\right)^2}\,dx}{4d\sqrt{c^2x^2}}$$

$$= \frac{x^4\left(a+b\csc^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}+\frac{(bcx)\,\text{Subst}\left(\int \frac{x}{\sqrt{-1+c^2x}\,(d+ex)^2}\,dx,x,x^2\right)}{8d\sqrt{c^2x^2}}$$

$$= -\frac{bcx\sqrt{-1+c^2x^2}}{8e\left(c^2d+e\right)\sqrt{c^2x^2}\left(d+ex^2\right)}+\frac{x^4\left(a+b\csc^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}+\frac{\left(bc\left(c^2d+2e\right)x\right)\text{Subst}\left(\int -\right.}{16de\left(c^2d+e\right)}$$

$$= -\frac{bcx\sqrt{-1+c^2x^2}}{8e\left(c^2d+e\right)\sqrt{c^2x^2}\left(d+ex^2\right)}+\frac{x^4\left(a+b\csc^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}+\frac{\left(b\left(c^2d+2e\right)x\right)\text{Subst}\left(\int \frac{}{d+}\right.}{8cde\left(c^2d+\right.}$$

$$= -\frac{bcx\sqrt{-1+c^2x^2}}{8e\left(c^2d+e\right)\sqrt{c^2x^2}\left(d+ex^2\right)}+\frac{x^4\left(a+b\csc^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}+\frac{bc\left(c^2d+2e\right)x\tan^{-1}\left(\frac{\sqrt{e}\sqrt{-}}{\sqrt{c^2}}\right.}{8de^{3/2}\left(c^2d+e\right)^{3/2}\sqrt{c^2x}}$$

Mathematica [C] time = 1.12, size = 390, normalized size = 2.48

$$\frac{-\frac{8a}{d+ex^2}+\frac{4ad}{\left(d+ex^2\right)^2}+\frac{b\sqrt{e}\left(c^2d+2e\right)\log\left(\frac{16de^{3/2}\sqrt{c^2(-d)-e}\left(cx\left(c\sqrt{d}-i\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2(-d)-e}\right)+i\sqrt{e}\right)}{b\left(c^2d+2e\right)\left(\sqrt{d}+i\sqrt{e}x\right)}\right)}{d\left(c^2(-d)-e\right)^{3/2}}+\frac{b\sqrt{e}\left(c^2d+2e\right)\log\left(-\frac{16de^{3/2}\sqrt{c^2(-d)-e}\left(-\sqrt{e}+cx\left(\sqrt{1}\right.\right.}{b\left(c^2d+2e\right)\left(\sqrt{e}\right.}\right.}{d\left(c^2(-d)-e\right)^{3/2}}}{16e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] ((4*a*d)/(d + e*x^2)^2 - (8*a)/(d + e*x^2) - (2*b*c*e*Sqrt[1 - 1/(c^2*x^2)]
*x)/((c^2*d + e)*(d + e*x^2)) - (4*b*(d + 2*e*x^2)*ArcCsc[c*x])/(d + e*x^2)
^2 + (4*b*ArcSin[1/(c*x)])/d + (b*Sqrt[e]*(c^2*d + 2*e)*Log[(16*d*Sqrt[-(c^
2*d) - e]*e^(3/2)*(I*Sqrt[e] + c*(c*Sqrt[d] - I*Sqrt[-(c^2*d) - e]*Sqrt[1 -
 1/(c^2*x^2)])*x))/(b*(c^2*d + 2*e)*(Sqrt[d] + I*Sqrt[e]*x))])/(d*(-(c^2*d)
 - e)^(3/2)) + (b*Sqrt[e]*(c^2*d + 2*e)*Log[(-16*d*Sqrt[-(c^2*d) - e]*e^(3/
2)*(-Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)]
)*x))/(b*(c^2*d + 2*e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(-(c^2*d) - e)^(3/2)))
/(16*e^2)
```

fricas [B] time = 1.39, size = 1015, normalized size = 6.46

$$\left[-\frac{4\,ac^4d^4+8\,ac^2d^3e+4\,ad^2e^2+8\left(ac^4d^3e+2\,ac^2d^2e^2+ade^3\right)x^2+\left(bc^2d^3+\left(bc^2de^2+2\,be^3\right)x^4+2\,bd^2e+2\,(\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*a*c^4*d^4 + 8*a*c^2*d^3*e + 4*a*d^2*e^2 + 8*(a*c^4*d^3*e + 2*a*c^
2*d^2*e^2 + a*d*e^3)*x^2 + (b*c^2*d^3 + (b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b*d
^2*e + 2*(b*c^2*d^2*e + 2*b*d*e^2)*x^2)*sqrt(-c^2*d*e - e^2)*log((c^2*e*x^2
 - c^2*d - 2*sqrt(-c^2*d*e - e^2)*sqrt(c^2*x^2 - 1) - 2*e)/(e*x^2 + d)) + 4
*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2
+ b*d*e^3)*x^2)*arccsc(c*x) + 8*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b
*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^
2 + b*d*e^3)*x^2)*arctan(-c*x + sqrt(c^2*x^2 - 1)) + 2*(b*c^2*d^3*e + b*d^2
*e^2 + (b*c^2*d^2*e^2 + b*d*e^3)*x^2)*sqrt(c^2*x^2 - 1))/(c^4*d^5*e^2 + 2*c
^2*d^4*e^3 + d^3*e^4 + (c^4*d^3*e^4 + 2*c^2*d^2*e^5 + d*e^6)*x^4 + 2*(c^4*d
^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^2), -1/8*(2*a*c^4*d^4 + 4*a*c^2*d^3*e +
 2*a*d^2*e^2 + 4*(a*c^4*d^3*e + 2*a*c^2*d^2*e^2 + a*d*e^3)*x^2 - (b*c^2*d^3
 + (b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b*d^2*e + 2*(b*c^2*d^2*e + 2*b*d*e^2)*x^
2)*sqrt(c^2*d*e + e^2)*arctan(sqrt(c^2*d*e + e^2)*sqrt(c^2*x^2 - 1)/(c^2*d
+ e)) + 2*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + 2*(b*c^4*d^3*e + 2*b*c^2
*d^2*e^2 + b*d*e^3)*x^2)*arccsc(c*x) + 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2
*e^2 + (b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c
^2*d^2*e^2 + b*d*e^3)*x^2)*arctan(-c*x + sqrt(c^2*x^2 - 1)) + (b*c^2*d^3*e
+ b*d^2*e^2 + (b*c^2*d^2*e^2 + b*d*e^3)*x^2)*sqrt(c^2*x^2 - 1))/(c^4*d^5*e^
2 + 2*c^2*d^4*e^3 + d^3*e^4 + (c^4*d^3*e^4 + 2*c^2*d^2*e^5 + d*e^6)*x^4 + 2
*(c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sy
m(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Va
lue
```

maple [B] time = 0.09, size = 1870, normalized size = 11.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^3,x)
```

```
[Out] 1/4*c^4*a/e^2*d/(c^2*e*x^2+c^2*d)^2-1/2*c^2*a/e^2/(c^2*e*x^2+c^2*d)+1/4*c^4
*b*arccsc(c*x)/e^2*d/(c^2*e*x^2+c^2*d)^2-1/2*c^2*b*arccsc(c*x)/e^2/(c^2*e*x
^2+c^2*d)-1/4*c^3*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x/(c^2*d+
e)/(c*e*x+(-c^2*e*d)^(1/2))/(-c*e*x+(-c^2*e*d)^(1/2))*arctan(1/(c^2*x^2-1)^
(1/2))-1/4*c^3*b*(c^2*x^2-1)^(1/2)/e/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d/(c^2*d
+e)/(c*e*x+(-c^2*e*d)^(1/2))/(-c*e*x+(-c^2*e*d)^(1/2))*arctan(1/(c^2*x^2-1)
^(1/2))+1/16*c^3*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x/(-(c^2*d
+e)/e)^(1/2)/(c^2*d+e)/(c*e*x+(-c^2*e*d)^(1/2))/(-c*e*x+(-c^2*e*d)^(1/2))*l
n(2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e-(-c^2*e*d)^(1/2)*c*x-e)/(c*e*
x+(-c^2*e*d)^(1/2)))+1/16*c^3*b*(c^2*x^2-1)^(1/2)/e/((c^2*x^2-1)/c^2/x^2)^(
1/2)/x*d/(-(c^2*d+e)/e)^(1/2)/(c^2*d+e)/(c*e*x+(-c^2*e*d)^(1/2))/(-c*e*x+(-
c^2*e*d)^(1/2))*ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e-(-c^2*e*d)^(
1/2)*c*x-e)/(c*e*x+(-c^2*e*d)^(1/2)))+1/16*c^3*b*(c^2*x^2-1)^(1/2)/((c^2*x^
2-1)/c^2/x^2)^(1/2)*x/(-(c^2*d+e)/e)^(1/2)/(c^2*d+e)/(c*e*x+(-c^2*e*d)^(1/2
))/(-c*e*x+(-c^2*e*d)^(1/2))*ln(-2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*
e+(-c^2*e*d)^(1/2)*c*x-e)/(-c*e*x+(-c^2*e*d)^(1/2)))+1/16*c^3*b*(c^2*x^2-1)
^(1/2)/e/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d/(-(c^2*d+e)/e)^(1/2)/(c^2*d+e)/(c*
e*x+(-c^2*e*d)^(1/2))/(-c*e*x+(-c^2*e*d)^(1/2))*ln(-2*((-(c^2*d+e)/e)^(1/2)
*(c^2*x^2-1)^(1/2)*e+(-c^2*e*d)^(1/2)*c*x-e)/(-c*e*x+(-c^2*e*d)^(1/2)))-1/4
*c*b*(c^2*x^2-1)^(1/2)*e/((c^2*x^2-1)/c^2/x^2)^(1/2)*x/d/(c^2*d+e)/(c*e*x+(
-c^2*e*d)^(1/2))/(-c*e*x+(-c^2*e*d)^(1/2))*arctan(1/(c^2*x^2-1)^(1/2))-1/4*
c*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/(c^2*d+e)/(c*e*x+(-c^2*
e*d)^(1/2))/(-c*e*x+(-c^2*e*d)^(1/2))*arctan(1/(c^2*x^2-1)^(1/2))+1/8*c^3*b
/((c^2*x^2-1)/c^2/x^2)^(1/2)*x/(c^2*d+e)/(c*e*x+(-c^2*e*d)^(1/2))/(-c*e*x+(
-c^2*e*d)^(1/2))-1/8*c*b/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/(c^2*d+e)/(c*e*x+(-c
^2*e*d)^(1/2))/(-c*e*x+(-c^2*e*d)^(1/2))+1/8*c*b*(c^2*x^2-1)^(1/2)*e/((c^2*
x^2-1)/c^2/x^2)^(1/2)*x/d/(-(c^2*d+e)/e)^(1/2)/(c^2*d+e)/(c*e*x+(-c^2*e*d)^
(1/2))/(-c*e*x+(-c^2*e*d)^(1/2))*ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/
2)*e-(-c^2*e*d)^(1/2)*c*x-e)/(c*e*x+(-c^2*e*d)^(1/2)))+1/8*c*b*(c^2*x^2-1)^
(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/(-(c^2*d+e)/e)^(1/2)/(c^2*d+e)/(c*e*x+(
-c^2*e*d)^(1/2))/(-c*e*x+(-c^2*e*d)^(1/2))*ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*
x^2-1)^(1/2)*e-(-c^2*e*d)^(1/2)*c*x-e)/(c*e*x+(-c^2*e*d)^(1/2)))+1/8*c*b*(c
^2*x^2-1)^(1/2)*e/((c^2*x^2-1)/c^2/x^2)^(1/2)*x/d/(-(c^2*d+e)/e)^(1/2)/(c^2
*d+e)/(c*e*x+(-c^2*e*d)^(1/2))/(-c*e*x+(-c^2*e*d)^(1/2))*ln(-2*((-(c^2*d+e)
/e)^(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*e*d)^(1/2)*c*x-e)/(-c*e*x+(-c^2*e*d)^(1
/2)))+1/8*c*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/(-(c^2*d+e)/e
)^(1/2)/(c^2*d+e)/(c*e*x+(-c^2*e*d)^(1/2))/(-c*e*x+(-c^2*e*d)^(1/2))*ln(-2*
((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*e*d)^(1/2)*c*x-e)/(-c*e*x+(
-c^2*e*d)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(2\,ex^2+d\right)a}{4\left(e^4x^4+2\,de^3x^2+d^2e^2\right)}-\frac{\left(2\,ex^2\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right)+d\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right)+\left(e^4x^4+2\,de^3x\right.\right.}{4\left(e^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*e*x^2 + d)*a/(e^4*x^4 + 2*d*e^3*x^2 + d^2*e^2) - 1/4*(2*e*x^2*arcta
n2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + d*arctan2(1, sqrt(c*x + 1)*sqrt(c*x -
1)) + 4*(e^4*x^4 + 2*d*e^3*x^2 + d^2*e^2)*integrate(1/4*(2*c^2*e*x^3 + c^2*
d*x)*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1))/(c^2*e^4*x^6 + (2*c^2*d*e^3 -
e^4)*x^4 - d^2*e^2 + (c^2*d^2*e^2 - 2*d*e^3)*x^2 + (c^2*e^4*x^6 + (2*c^2*d*
```

```
e^3 - e^4)*x^4 - d^2*e^2 + (c^2*d^2*e^2 - 2*d*e^3)*x^2)*e^(log(c*x + 1) + l
og(c*x - 1))), x))*b/(e^4*x^4 + 2*d*e^3*x^2 + d^2*e^2)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^3,x)
```

```
[Out] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^3, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsc(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

3.113 $\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=193

$$-\frac{a+b\csc^{-1}(cx)}{4e\left(d+ex^2\right)^2}-\frac{bcx\tan^{-1}\left(\sqrt{c^2x^2-1}\right)}{4d^2e\sqrt{c^2x^2}}+\frac{bcx\left(3c^2d+2e\right)\tan^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{\sqrt{c^2d+e}}\right)}{8d^2\sqrt{e}\sqrt{c^2x^2}\left(c^2d+e\right)^{3/2}}+\frac{bcx\sqrt{c^2x^2-1}}{8d\sqrt{c^2x^2}\left(c^2d+e\right)\left(d+ex^2\right)}$$

```
[Out] 1/4*(-a-b*arccsc(c*x))/e/(e*x^2+d)^2-1/4*b*c*x*arctan((c^2*x^2-1)^(1/2))/d^
2/e/(c^2*x^2)^(1/2)+1/8*b*c*(3*c^2*d+2*e)*x*arctan(e^(1/2)*(c^2*x^2-1)^(1/2
)/(c^2*d+e)^(1/2))/d^2/(c^2*d+e)^(3/2)/e^(1/2)/(c^2*x^2)^(1/2)+1/8*b*c*x*(c
^2*x^2-1)^(1/2)/d/(c^2*d+e)/(e*x^2+d)/(c^2*x^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5237, 446, 103, 156, 63, 205}

$$-\frac{a+b\csc^{-1}(cx)}{4e\left(d+ex^2\right)^2}-\frac{bcx\tan^{-1}\left(\sqrt{c^2x^2-1}\right)}{4d^2e\sqrt{c^2x^2}}+\frac{bcx\left(3c^2d+2e\right)\tan^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{\sqrt{c^2d+e}}\right)}{8d^2\sqrt{e}\sqrt{c^2x^2}\left(c^2d+e\right)^{3/2}}+\frac{bcx\sqrt{c^2x^2-1}}{8d\sqrt{c^2x^2}\left(c^2d+e\right)\left(d+ex^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (b*c*x*Sqrt[-1 + c^2*x^2])/(8*d*(c^2*d + e)*Sqrt[c^2*x^2]*(d + e*x^2)) - (a
 + b*ArcCsc[c*x])/(4*e*(d + e*x^2)^2) - (b*c*x*ArcTan[Sqrt[-1 + c^2*x^2]])/
(4*d^2*e*Sqrt[c^2*x^2]) + (b*c*(3*c^2*d + 2*e)*x*ArcTan[(Sqrt[e]*Sqrt[-1 +
c^2*x^2])/Sqrt[c^2*d + e]])/(8*d^2*Sqrt[e]*(c^2*d + e)^(3/2)*Sqrt[c^2*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5237

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCsc[c*x]))/(2*e*(p + 1)), x
] + Dist[(b*c*x)/(2*e*(p + 1)*Sqrt[c^2*x^2]), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = -\frac{a+b\csc^{-1}(cx)}{4e\left(d+ex^2\right)^2} - \frac{(bcx)\int \frac{1}{x\sqrt{-1+c^2x^2}\left(d+ex^2\right)^2}\,dx}{4e\sqrt{c^2x^2}}$$

$$= -\frac{a+b\csc^{-1}(cx)}{4e\left(d+ex^2\right)^2} - \frac{(bcx)\,\text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}\,(d+ex)^2}\,dx, x, x^2\right)}{8e\sqrt{c^2x^2}}$$

$$= \frac{bcx\sqrt{-1+c^2x^2}}{8d\left(c^2d+e\right)\sqrt{c^2x^2}\left(d+ex^2\right)} - \frac{a+b\csc^{-1}(cx)}{4e\left(d+ex^2\right)^2} + \frac{(bcx)\,\text{Subst}\left(\int \frac{-c^2d-e+\frac{1}{2}c^2ex}{x\sqrt{-1+c^2x}\,(d+ex)}\,dx \cdots\right.}{8de\left(c^2d+e\right)\sqrt{c^2x^2}}$$

$$= \frac{bcx\sqrt{-1+c^2x^2}}{8d\left(c^2d+e\right)\sqrt{c^2x^2}\left(d+ex^2\right)} - \frac{a+b\csc^{-1}(cx)}{4e\left(d+ex^2\right)^2} - \frac{(bcx)\,\text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}}\,dx, x, x^{\cdots}\right.}{8d^2e\sqrt{c^2x^2}}$$

$$= \frac{bcx\sqrt{-1+c^2x^2}}{8d\left(c^2d+e\right)\sqrt{c^2x^2}\left(d+ex^2\right)} - \frac{a+b\csc^{-1}(cx)}{4e\left(d+ex^2\right)^2} - \frac{(bx)\,\text{Subst}\left(\int \frac{1}{\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx, x, \sqrt{-1+\cdots}\right.}{4cd^2e\sqrt{c^2x^2}}$$

$$= \frac{bcx\sqrt{-1+c^2x^2}}{8d\left(c^2d+e\right)\sqrt{c^2x^2}\left(d+ex^2\right)} - \frac{a+b\csc^{-1}(cx)}{4e\left(d+ex^2\right)^2} - \frac{bcx\tan^{-1}\left(\sqrt{-1+c^2x^2}\right)}{4d^2e\sqrt{c^2x^2}} + \frac{bc\,(3\cdots}{\cdots}$$

Mathematica [C] time = 0.94, size = 385, normalized size = 1.99

$$\frac{1}{16}\left(-\frac{4a}{e\left(d+ex^2\right)^2} + \frac{b\left(3c^2d+2e\right)\log\left(\frac{16d^2\sqrt{e}\sqrt{c^2(-d)-e}\left(cx\left(c\sqrt{d}-i\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2(-d)-e}\right)+i\sqrt{e}\right)}{b\left(3c^2d+2e\right)\left(\sqrt{d}+i\sqrt{e}\,x\right)}\right)}{d^2\sqrt{e}\left(c^2(-d)-e\right)^{3/2}} + \frac{b\left(3c^2d+2e\right)\log\left(-\cdots\right.}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] ((-4*a)/(e*(d + e*x^2)^2) + (2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x)/(d*(c^2*d + e)*
(d + e*x^2)) - (4*b*ArcCsc[c*x])/(e*(d + e*x^2)^2) + (4*b*ArcSin[1/(c*x)])/
(d^2*e) + (b*(3*c^2*d + 2*e)*Log[(16*d^2*Sqrt[-(c^2*d) - e]*Sqrt[e]*(I*Sqrt
[e] + c*(c*Sqrt[d] - I*Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(b*(3*
c^2*d + 2*e)*(Sqrt[d] + I*Sqrt[e]*x))])/(d^2*(-(c^2*d) - e)^(3/2)*Sqrt[e])
+ (b*(3*c^2*d + 2*e)*Log[(-16*d^2*Sqrt[-(c^2*d) - e]*Sqrt[e]*(-Sqrt[e] + c*
((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(b*(3*c^2*d
 + 2*e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d^2*(-(c^2*d) - e)^(3/2)*Sqrt[e]))/16
```

fricas [B] time = 1.08, size = 890, normalized size = 4.61

$$\left[-\frac{4\,ac^4d^4 + 8\,ac^2d^3e + 4\,ad^2e^2 + \left(3\,bc^2d^3 + \left(3\,bc^2de^2 + 2\,be^3\right)x^4 + 2\,bd^2e + 2\left(3\,bc^2d^2e + 2\,bde^2\right)x^2\right)\sqrt{-c^2de - e^2}\,\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*a*c^4*d^4 + 8*a*c^2*d^3*e + 4*a*d^2*e^2 + (3*b*c^2*d^3 + (3*b*c^2
*d*e^2 + 2*b*e^3)*x^4 + 2*b*d^2*e + 2*(3*b*c^2*d^2*e + 2*b*d*e^2)*x^2)*sqrt
(-c^2*d*e - e^2)*log((c^2*e*x^2 - c^2*d - 2*sqrt(-c^2*d*e - e^2)*sqrt(c^2*x
^2 - 1) - 2*e)/(e*x^2 + d)) + 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2)*arc
csc(c*x) + 8*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 + 2*b*
c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*a
rctan(-c*x + sqrt(c^2*x^2 - 1)) - 2*(b*c^2*d^3*e + b*d^2*e^2 + (b*c^2*d^2*e
^2 + b*d*e^3)*x^2)*sqrt(c^2*x^2 - 1))/(c^4*d^6*e + 2*c^2*d^5*e^2 + d^4*e^3
+ (c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^4 + 2*(c^4*d^5*e^2 + 2*c^2*d^4*
e^3 + d^3*e^4)*x^2), -1/8*(2*a*c^4*d^4 + 4*a*c^2*d^3*e + 2*a*d^2*e^2 - (3*b
*c^2*d^3 + (3*b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b*d^2*e + 2*(3*b*c^2*d^2*e + 2
*b*d*e^2)*x^2)*sqrt(c^2*d*e + e^2)*arctan(sqrt(c^2*d*e + e^2)*sqrt(c^2*x^2
- 1)/(c^2*d + e)) + 2*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2)*arccsc(c*x) +
 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 + 2*b*c^2*d*e^3
+ b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*arctan(-c*x
 + sqrt(c^2*x^2 - 1)) - (b*c^2*d^3*e + b*d^2*e^2 + (b*c^2*d^2*e^2 + b*d*e^3
)*x^2)*sqrt(c^2*x^2 - 1))/(c^4*d^6*e + 2*c^2*d^5*e^2 + d^4*e^3 + (c^4*d^4*e
^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^4 + 2*(c^4*d^5*e^2 + 2*c^2*d^4*e^3 + d^3*e^
4)*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sy
m(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Va
lue
```

maple [B] time = 0.08, size = 1840, normalized size = 9.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsc(c*x))/(e*x^2+d)^3,x)
```

```
[Out] -1/4*c^4*a/e/(c^2*e*x^2+c^2*d)^2-1/4*c^4*b/e/(c^2*e*x^2+c^2*d)^2*arccsc(c*x
)-1/4*c^3*b*(c^2*x^2-1)^(1/2)*e/((c^2*x^2-1)/c^2/x^2)^(1/2)*x/d/(c^2*d+e)/(
-c*e*x+(-c^2*e*d)^(1/2))/(c*e*x+(-c^2*e*d)^(1/2))*arctan(1/(c^2*x^2-1)^(1/2
))-1/4*c^3*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/(c^2*d+e)/(-c*
e*x+(-c^2*e*d)^(1/2))/(c*e*x+(-c^2*e*d)^(1/2))*arctan(1/(c^2*x^2-1)^(1/2))+
3/16*c^3*b*(c^2*x^2-1)^(1/2)*e/((c^2*x^2-1)/c^2/x^2)^(1/2)*x/d/(-(c^2*d+e)/
e)^(1/2)/(c^2*d+e)/(-c*e*x+(-c^2*e*d)^(1/2))/(c*e*x+(-c^2*e*d)^(1/2))*ln(-2
*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*e*d)^(1/2)*c*x-e)/(-c*e*x+
(-c^2*e*d)^(1/2)))+3/16*c^3*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)
/x/(-(c^2*d+e)/e)^(1/2)/(c^2*d+e)/(-c*e*x+(-c^2*e*d)^(1/2))/(c*e*x+(-c^2*e*
d)^(1/2))*ln(-2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*e*d)^(1/2)*
c*x-e)/(-c*e*x+(-c^2*e*d)^(1/2)))+3/16*c^3*b*(c^2*x^2-1)^(1/2)*e/((c^2*x^2-
1)/c^2/x^2)^(1/2)*x/d/(-(c^2*d+e)/e)^(1/2)/(c^2*d+e)/(c*e*x+(-c^2*e*d)^(1/2
))/(-c*e*x+(-c^2*e*d)^(1/2))*ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e
-(-c^2*e*d)^(1/2)*c*x-e)/(c*e*x+(-c^2*e*d)^(1/2)))+3/16*c^3*b*(c^2*x^2-1)^(
1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/(-(c^2*d+e)/e)^(1/2)/(c^2*d+e)/(c*e*x+(-
c^2*e*d)^(1/2))/(-c*e*x+(-c^2*e*d)^(1/2))*ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*x
^2-1)^(1/2)*e-(-c^2*e*d)^(1/2)*c*x-e)/(c*e*x+(-c^2*e*d)^(1/2)))-1/8*c^3*b/(
(c^2*x^2-1)/c^2/x^2)^(1/2)*x/d/(c^2*d+e)/(-c*e*x+(-c^2*e*d)^(1/2))/(c*e*x+(
-c^2*e*d)^(1/2))*e+1/8*c*b/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/d/(c^2*d+e)/(-c*e*
x+(-c^2*e*d)^(1/2))/(c*e*x+(-c^2*e*d)^(1/2))*e-1/4*c*b*(c^2*x^2-1)^(1/2)/((
c^2*x^2-1)/c^2/x^2)^(1/2)*x/d^2/(c^2*d+e)/(-c*e*x+(-c^2*e*d)^(1/2))/(c*e*x+
(-c^2*e*d)^(1/2))*arctan(1/(c^2*x^2-1)^(1/2))*e^2-1/4*c*b*(c^2*x^2-1)^(1/2)
/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/d/(c^2*d+e)/(-c*e*x+(-c^2*e*d)^(1/2))/(c*e*x
+(-c^2*e*d)^(1/2))*arctan(1/(c^2*x^2-1)^(1/2))*e+1/8*c*b*(c^2*x^2-1)^(1/2)/
((c^2*x^2-1)/c^2/x^2)^(1/2)*x/d^2/(-(c^2*d+e)/e)^(1/2)/(c^2*d+e)/(-c*e*x+(-
c^2*e*d)^(1/2))/(c*e*x+(-c^2*e*d)^(1/2))*ln(-2*((-(c^2*d+e)/e)^(1/2)*(c^2*x
^2-1)^(1/2)*e+(-c^2*e*d)^(1/2)*c*x-e)/(-c*e*x+(-c^2*e*d)^(1/2)))*e^2+1/8*c*
b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/d/(-(c^2*d+e)/e)^(1/2)/(c
^2*d+e)/(-c*e*x+(-c^2*e*d)^(1/2))/(c*e*x+(-c^2*e*d)^(1/2))*ln(-2*((-(c^2*d+
e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*e*d)^(1/2)*c*x-e)/(-c*e*x+(-c^2*e*d)^
(1/2)))*e+1/8*c*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x/d^2/(c^2*
d+e)/(-c*e*x+(-c^2*e*d)^(1/2))/(-(c^2*d+e)/e)^(1/2)/(c*e*x+(-c^2*e*d)^(1/2)
)*ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e-(-c^2*e*d)^(1/2)*c*x-e)/(c
*e*x+(-c^2*e*d)^(1/2)))*e^2+1/8*c*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)
^(1/2)/x/d/(c^2*d+e)/(-c*e*x+(-c^2*e*d)^(1/2))/(-(c^2*d+e)/e)^(1/2)/(c*e*x+
(-c^2*e*d)^(1/2))*ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e-(-c^2*e*d)
^(1/2)*c*x-e)/(c*e*x+(-c^2*e*d)^(1/2)))*e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(\left(c^2e^3x^4 + 2\,c^2de^2x^2 + c^2d^2e\right)\int \frac{xe^{\left(\frac{1}{2}\,\log(cx+1)+\frac{1}{2}\,\log(cx-1)\right)}}{c^2e^3x^6+\left(2\,c^2de^2-e^3\right)x^4+\left(c^2e^3x^6+\left(2\,c^2de^2-e^3\right)x^4-d^2e+\left(c^2d^2e-2\,de^2\right)x^2\right)(cx+1)(cx-1)-d^2e+\left(c^2d^2e-2\,d\right.}}{4\left(e^3x^4 + 2\,de^2x^2 + d^2e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(4*(c^2*e^3*x^4 + 2*c^2*d*e^2*x^2 + c^2*d^2*e)*integrate(1/4*x*e^(1/2*
log(c*x + 1) + 1/2*log(c*x - 1))/(c^2*e^3*x^6 + (2*c^2*d*e^2 - e^3)*x^4 - d
^2*e + (c^2*d^2*e - 2*d*e^2)*x^2 + (c^2*e^3*x^6 + (2*c^2*d*e^2 - e^3)*x^4 -
 d^2*e + (c^2*d^2*e - 2*d*e^2)*x^2)*e^(log(c*x + 1) + log(c*x - 1))), x) +
arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*b/(e^3*x^4 + 2*d*e^2*x^2 + d^2*e)
- 1/4*a/(e^3*x^4 + 2*d*e^2*x^2 + d^2*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^3,x)
```

```
[Out] int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsc(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

3.114 $\int \frac{a+b\csc^{-1}(cx)}{x(d+ex^2)^3}\,dx$

Optimal. Leaf size=704

$$-\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}-\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}-\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1\right.}{2d^3}$$

```
[Out] 1/4*e^2*(a+b*arccsc(c*x))/d^3/(e+d/x^2)^2-e*(a+b*arccsc(c*x))/d^3/(e+d/x^2)
+1/2*I*(a+b*arccsc(c*x))^2/b/d^3-1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1
/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^3-1/2*(a+b*arccsc(
c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1
/2)))/d^3-1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(
1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^3-1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(
1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^3+1/2*I*b*polyl
og(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))
/d^3+1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-
(c^2*d+e)^(1/2)))/d^3+1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-
d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^3+1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c
^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^3-1/8*b*(c^2*d+2*e)*
arctan((c^2*d+e)^(1/2)/c/x/e^(1/2)/(1-1/c^2/x^2)^(1/2))*e^(1/2)/d^3/(c^2*d+
e)^(3/2)+b*arctan((c^2*d+e)^(1/2)/c/x/e^(1/2)/(1-1/c^2/x^2)^(1/2))*e^(1/2)/
d^3/(c^2*d+e)^(1/2)-1/8*b*c*e*(1-1/c^2/x^2)^(1/2)/d^2/(c^2*d+e)/(e+d/x^2)/x
```

Rubi [A] time = 1.32, antiderivative size = 704, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {5241, 4733, 4729, 382, 377, 205, 4741, 4519, 2190, 2279, 2391}

$$\frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}+\frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}+\frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^3}+\frac{ib\,\text{PolyLog}\left(2\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^3),x]
```

```
[Out] -(b*c*e*Sqrt[1 - 1/(c^2*x^2)])/(8*d^2*(c^2*d + e)*(e + d/x^2)*x) + (e^2*(a
+ b*ArcCsc[c*x]))/(4*d^3*(e + d/x^2)^2) - (e*(a + b*ArcCsc[c*x]))/(d^3*(e +
 d/x^2)) + ((I/2)*(a + b*ArcCsc[c*x])^2)/(b*d^3) + (b*Sqrt[e]*ArcTan[Sqrt[c
^2*d + e]/(c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)])/(d^3*Sqrt[c^2*d + e]) - (b*
Sqrt[e]*(c^2*d + 2*e)*ArcTan[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2
)]*x)])/(8*d^3*(c^2*d + e)^(3/2)) - ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[
-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^3) - ((a + b*ArcC
sc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e
])])/(2*d^3) - ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x])
)/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d^3) - ((a + b*ArcCsc[c*x])*Log[1 + (I*c
*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d^3) + ((I/2)
*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e
])])/d^3 + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] -
Sqrt[c^2*d + e])])/d^3 + ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c
*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/d^3 + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]
*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/d^3
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rule 4729

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x]))/(2*e*(p + 1)), x]
 - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/Sqrt[1 - c^2*x^2], x],
 x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 4733

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
```

```
e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4741

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[((a + b*x)^n*Cos[x])/(c*d + e*Sin[x]), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5241

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSin[x/c])^n)/x^
(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx = -\text{Subst}\left(\int \frac{x^5\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int\left(\frac{e^2x\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^3} - \frac{2ex\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^2} + \frac{x\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)}\right) dx, x, \frac{1}{x}$$

$$= -\frac{\text{Subst}\left(\int \frac{x\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d^2} + \frac{(2e)\,\text{Subst}\left(\int \frac{x\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{d^2} - \frac{e^2\,\text{Subst}\left(\int \frac{x(}{}\right.}{}$$

$$= \frac{e^2\left(a+b\csc^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\csc^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)} - \frac{\text{Subst}\left(\int\left(-\frac{\sqrt{-d}\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}\,x\right)} + \frac{\sqrt{-d}\left(a+b\sin^{-1}\left(\frac{x}{c}\right.\right.}{2d\left(\sqrt{e}+\sqrt{-d}\,x\right)}\right.\right.}{d^2}$$

$$= -\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{8d^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2\left(a+b\csc^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\csc^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)} - \frac{\text{Subst}\left(\int \frac{a+b\sin^{-1}}{\sqrt{e}-\sqrt{-}}\right.}{2(-d)^5}$$

$$= -\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{8d^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2\left(a+b\csc^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\csc^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)} + \frac{b\sqrt{e}\tan^{-1}\left(\frac{\sqrt{}}{c\sqrt{e}\sqrt{}}\right.}{d^3\sqrt{c^2d+}}$$

$$= -\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{8d^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2\left(a+b\csc^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\csc^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)} + \frac{i\left(a+b\csc^{-1}(cx\right.}{2bd^3}$$

$$= -\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{8d^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2\left(a+b\csc^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\csc^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)} + \frac{i\left(a+b\csc^{-1}(cx\right.}{2bd^3}$$

$$= -\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{8d^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2\left(a+b\csc^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\csc^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)} + \frac{i\left(a+b\csc^{-1}(cx\right.}{2bd^3}$$

$$= -\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{8d^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2\left(a+b\csc^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\csc^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)} + \frac{i\left(a+b\csc^{-1}(cx\right.}{2bd^3}$$

Mathematica [F] time = 64.96, size = 0, normalized size = 0.00

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^3),x]
```

```
[Out] Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^3), x]
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b \operatorname{arccsc}(cx) + a}{e^3x^7 + 3\,de^2x^5 + 3\,d^2ex^3 + d^3x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x),
 x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sy
m(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Va
lue
```

maple [C] time = 7.10, size = 5294, normalized size = 7.52

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x/(e*x^2+d)^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\,a\left(\frac{2\,ex^2 + 3\,d}{d^2e^2x^4 + 2\,d^3ex^2 + d^4} - \frac{2\,\log\left(ex^2 + d\right)}{d^3} + \frac{4\,\log(x)}{d^3}\right) + b\int \frac{\arctan\left(1, \sqrt{cx+1}\,\sqrt{cx-1}\right)}{e^3x^7 + 3\,de^2x^5 + 3\,d^2ex^3 + d^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/4*a*((2*e*x^2 + 3*d)/(d^2*e^2*x^4 + 2*d^3*e*x^2 + d^4) - 2*log(e*x^2 + d)
/d^3 + 4*log(x)/d^3) + b*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/
(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x\,\left(e\,x^2 + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^3),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

3.115 $\int \frac{x^4\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=1144

$$-\frac{b\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}\,c}{16e^{3/2}\left(dc^2+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{b\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}\,c}{16e^{3/2}\left(dc^2+e\right)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{3\left(a+b\csc^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{3\left(a+b\csc^{-1}(cx)\right)}{16e^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\cdots$$

```
[Out] -3/16*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^
(1/2)-(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)+3/16*(a+b*arccsc(c*x))*ln(1+I*c*
(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(5/2)/(
-d)^(1/2)-3/16*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^
(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)+3/16*(a+b*arccsc(c*x))*
ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/
e^(5/2)/(-d)^(1/2)-3/16*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)
^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)+3/16*I*b*polylog(2,I*c
*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(5/2)/
(-d)^(1/2)+3/16*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e
^(1/2)-(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)-3/16*I*b*polylog(2,-I*c*(I/c/x+
(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/
2)-1/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1
-1/c^2/x^2)^(1/2))/e/(c^2*d+e)^(3/2)/d^(1/2)-1/16*b*arctanh((c^2*d+(-d)^(1/
2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/e/(c^2*d+e)^(3
/2)/d^(1/2)+1/16*(a+b*arccsc(c*x))*(-d)^(1/2)/e^(3/2)/(-d/x+(-d)^(1/2)*e^(1
/2))^2+3/16*(a+b*arccsc(c*x))/e^2/(-d/x+(-d)^(1/2)*e^(1/2))-1/16*(a+b*arccs
c(c*x))*(-d)^(1/2)/e^(3/2)/(d/x+(-d)^(1/2)*e^(1/2))^2-3/16*(a+b*arccsc(c*x)
)/e^2/(d/x+(-d)^(1/2)*e^(1/2))-3/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/
c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/e^2/d^(1/2)/(c^2*d+e)^(1/2)-
3/16*b*arctanh((c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/
c^2/x^2)^(1/2))/e^2/d^(1/2)/(c^2*d+e)^(1/2)-1/16*b*c*(-d)^(1/2)*(1-1/c^2/x^
2)^(1/2)/e^(3/2)/(c^2*d+e)/(-d/x+(-d)^(1/2)*e^(1/2))-1/16*b*c*(-d)^(1/2)*(1
-1/c^2/x^2)^(1/2)/e^(3/2)/(c^2*d+e)/(d/x+(-d)^(1/2)*e^(1/2))
```

Rubi [A] time = 1.60, antiderivative size = 1144, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {5241, 4667, 4743, 731, 725, 206, 4741, 4519, 2190, 2279, 2391}

$$-\frac{b\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}\,c}{16e^{3/2}\left(dc^2+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{b\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}\,c}{16e^{3/2}\left(dc^2+e\right)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{3\left(a+b\csc^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{3\left(a+b\csc^{-1}(cx)\right)}{16e^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -(b*c*Sqrt[-d]*Sqrt[1 - 1/(c^2*x^2)])/(16*e^(3/2)*(c^2*d + e)*(Sqrt[-d]*Sqr
t[e] - d/x)) - (b*c*Sqrt[-d]*Sqrt[1 - 1/(c^2*x^2)])/(16*e^(3/2)*(c^2*d + e)
*(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[-d]*(a + b*ArcCsc[c*x]))/(16*e^(3/2)*(Sq
rt[-d]*Sqrt[e] - d/x)^2) + (3*(a + b*ArcCsc[c*x]))/(16*e^2*(Sqrt[-d]*Sqrt[e
] - d/x)) - (Sqrt[-d]*(a + b*ArcCsc[c*x]))/(16*e^(3/2)*(Sqrt[-d]*Sqrt[e] +
d/x)^2) - (3*(a + b*ArcCsc[c*x]))/(16*e^2*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*Ar
cTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/
```

```
(c^2*x^2)])])/(16*Sqrt[d]*e*(c^2*d + e)^(3/2)) - (3*b*ArcTanh[(c^2*d - (Sqr
t[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*S
qrt[d]*e^2*Sqrt[c^2*d + e]) - (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*
Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*Sqrt[d]*e*(c^2*d + e)^
(3/2)) - (3*b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d
+ e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*Sqrt[d]*e^2*Sqrt[c^2*d + e]) - (3*(a + b*
ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d
 + e])])/(16*Sqrt[-d]*e^(5/2)) + (3*(a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-
d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2)) -
 (3*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] +
 Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2)) + (3*(a + b*ArcCsc[c*x])*Log[1 +
(I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*Sqrt[-d]
*e^(5/2)) - (((3*I)/16)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(S
qrt[e] - Sqrt[c^2*d + e])])/(Sqrt[-d]*e^(5/2)) + (((3*I)/16)*b*PolyLog[2, (
I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(Sqrt[-d]*e^(
5/2)) - (((3*I)/16)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[
e] + Sqrt[c^2*d + e])])/(Sqrt[-d]*e^(5/2)) + (((3*I)/16)*b*PolyLog[2, (I*c*
Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(Sqrt[-d]*e^(5/2)
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rule 4667

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4741

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[((a + b*x)^n*Cos[x])/(c*d + e*Sin[x]), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4743

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSin[c*x])^n)/(e*(m + 1)), x] -
Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1
))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
&& NeQ[m, -1]
```

Rule 5241

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSin[x/c])^n)/x^
(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^4 \left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = -\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(-\frac{d^3\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{8(-d)^{3/2}e^{3/2}\left(\sqrt{-d}\sqrt{e}-dx\right)^3} - \frac{3d\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-dx\right)^2} - \frac{d^3\left(a+b\right.}{8(-d)^{3/2}e^{3/2}\left(\right.}\right.\right.$$

$$= \frac{(3d)\,\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}dx, x, \frac{1}{x}\right)}{16e^2} + \frac{(3d)\,\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}dx, x, \frac{1}{x}\right)}{16e^2} + \frac{(3d)\,\text{Su}}{}$$

$$= \frac{\sqrt{-d}\left(a+b\csc^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{3\left(a+b\csc^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{-d}\left(a+b\csc^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} - \frac{3\left(a+b\,\text{cs}\right.}{16e^2\left(\sqrt{-d}\right.}$$

$$= -\frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{-d}\left(a+b\,\text{csc}\right.}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}\right.}$$

$$= -\frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{-d}\left(a+b\,\text{csc}\right.}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}\right.}$$

$$= -\frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{-d}\left(a+b\,\text{csc}\right.}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}\right.}$$

$$= -\frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{-d}\left(a+b\,\text{csc}\right.}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}\right.}$$

$$= -\frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{-d}\left(a+b\,\text{csc}\right.}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}\right.}$$

$$= -\frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{-d}\left(a+b\,\text{csc}\right.}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}\right.}$$

Mathematica [A] time = 6.20, size = 2067, normalized size = 1.81

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (a*d*x)/(4*e^2*(d + e*x^2)^2) - (5*a*x)/(8*e^2*(d + e*x^2)) + (3*a*ArcTan[(
Sqrt[e]*x)/Sqrt[d]])/(8*Sqrt[d]*e^(5/2)) + b*((5*(-(ArcCsc[c*x]/((-I)*Sqrt[
d]*Sqrt[e] + e*x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(
Sqrt[e] + c*((-I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))
/(Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]
))/(16*e^2) + (5*(-(ArcCsc[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSin[1/(
c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt
[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*
Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/(16*e^2) + ((I/16)*Sqrt[d]*((I*
c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*((-I)*Sqrt[d] + Sqr
t[e]*x)) - ArcCsc[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) - ArcSin[1/(c
*x)]/(d*Sqrt[e]) + (I*(2*c^2*d + e)*Log[(4*d*Sqrt[e]*Sqrt[c^2*d + e]*(I*Sqr
t[e] + c*(c*Sqrt[d] - Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d
+ e)*((-I)*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/e^2 - ((I/16)*Sq
rt[d]*(((-I)*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*(I*Sqr
t[d] + Sqrt[e]*x)) - ArcCsc[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - ArcS
in[1/(c*x)]/(d*Sqrt[e]) + (I*(2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e
]*((-I)*Sqrt[e] + c*(c*Sqrt[d] + Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))
/((2*c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/e^2 - (3
*(Pi^2 - 4*Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 - (I*Sqrt[e]
)/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcC
sc[c*x])/4])/Sqrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e]
)/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] - Sqr
t[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1 - (I*Sq
rt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]
*E^(I*ArcCsc[c*x]))] + (4*I)*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt
[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d +
 e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c
*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*Arc
Csc[c*x]))] + (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] - (4*I)*Pi*L
og[Sqrt[e] + (I*Sqrt[d])/x] + 8*PolyLog[2, (-Sqrt[e] + Sqrt[c^2*d + e])/(c*
Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8*PolyLog[2, -((Sqrt[e] + Sqrt[c^2*d + e])/(c
*Sqrt[d]*E^(I*ArcCsc[c*x])))] + 4*PolyLog[2, E^((2*I)*ArcCsc[c*x])]))/(128*
Sqrt[d]*e^(5/2)) + (3*(Pi^2 - 4*Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSi
n[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e]
)*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (-Sqrt[e
] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log
[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*A
rcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c
^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*Pi*Log[1 - (Sqrt[e] + Sqr
t[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 - (S
qrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sq
rt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e]
)/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCs
c[c*x])] - (4*I)*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] + 8*PolyLog[2, (Sqrt[e] -
Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8*PolyLog[2, (Sqrt[e] + S
qrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*PolyLog[2, E^((2*I)*ArcC
sc[c*x])]))/(128*Sqrt[d]*e^(5/2)))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^4 \operatorname{arccsc}(cx) + ax^4}{e^3x^6 + 3\,de^2x^4 + 3\,d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arccsc(c*x) + a*x^4)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 +
 d^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sy
m(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Va
lue
```

maple [C] time = 5.01, size = 3175, normalized size = 2.78

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^3,x)
```

```
[Out] -3/8*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(I/c/x+(1-1/
c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/e^2/(c^2*d+e)
/d-1/8*c^5*b*x^4/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*((c^2*x^2-1)/c^2/x^2)^(1/2)-
5/8*c^4*b*x^3/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*arccsc(c*x)-3/16*c^3*b/e^2/(c^2
*d+e)*d*sum(1/_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1
/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootO
f(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-3/16*c^3*b/e^2/(c^2*d+e)*d*sum(_R1
/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/
_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*
c^2*d-4*e)*_Z^2+c^2*d))-1/c^4*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)
*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)
*d)^(1/2))/(c^2*d+e)/d^3+7/4/c^2*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1
/2)*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2
*e)*d)^(1/2))/(c^2*d+e)^2/d^2-3/8*c^6*b*x/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)/e^2
*arccsc(c*x)*d^2-1/8*c^5*b*x^2/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)/e*((c^2*x^2-1)
/c^2/x^2)^(1/2)*d-3/8*c^4*b*x/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)/e*arccsc(c*x)*d
+1/c^4*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(I/c/x+(1-
1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e/(c^2*d+e)/
d^3*(e*(c^2*d+e))^(1/2)-5/4/c^2*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/
2)*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*
e)*d)^(1/2))/(c^2*d+e)^2/e/d^2*(e*(c^2*d+e))^(1/2)-3/4/c^2*b*(-(c^2*d-2*(e*
(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^
2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/e^2/(c^2*d+e)/d^2*(e*(c^2*d+e))^(1
/2)+3/4/c^2*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(I/c/
x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e^2/(c^
2*d+e)/d^2*(e*(c^2*d+e))^(1/2)-5/8*c^6*b*x^3/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)/
e*arccsc(c*x)*d-1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan
(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/
2))/e/(c^2*d+e)/d^3*(e*(c^2*d+e))^(1/2)+5/4/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^
(1/2)+2*e)*d)^(1/2)*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c
^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)^2/e/d^2*(e*(c^2*d+e))^(1/2)+3/8*b*(
-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(I/c/x+(1-1/c^2/x^2)
^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/e^2/(c^2*d+e)^2/d*(e*
(c^2*d+e))^(1/2)-3/8*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(
c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)
)/e^2/(c^2*d+e)^2/d*(e*(c^2*d+e))^(1/2)-5/4/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^
(1/2)+2*e)*d)^(1/2)*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c
^2*d+e))^(1/2)-2*e)*d)^(1/2))/e/(c^2*d+e)/d^2-1/c^4*b*((c^2*d+2*(e*(c^2*d+e
))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e
```

```
*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)^2/d^3*(e*(c^2*d+e))^(1/2)+1/c^4*
b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(I/c/x+(1-1/c^2/x
^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)^2/d^3*(e
*(c^2*d+e))^(1/2)-1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arct
an(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(
1/2))/(c^2*d+e)/d^3+7/4/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*
arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*
d)^(1/2))/(c^2*d+e)^2/d^2-3/8*c^4*a/(c^2*e*x^2+c^2*d)^2/e^2*d*x+3/4*b*(-(c^
2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/
2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/e/(c^2*d+e)^2/d-3/8*b*((c
^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(
1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e^2/(c^2*d+e)/d+3/4*b*((
c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^
(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e/(c^2*d+e)^2/d+3/8*a/e
^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/
2)+2*e)*d)^(1/2)*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*
d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)^2*e/d^3-5/4/c^2*b*((c^2*d+2*(e*(c^2*d+
e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(
e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e/(c^2*d+e)/d^2+1/c^4*b*((c^2*d+2*(e*(c^2
*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+
2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)^2*e/d^3-5/8*c^4*a/(c^2*e*x^2
+c^2*d)^2/e*x^3-3/16*c*b/e/(c^2*d+e)*sum(1/_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*a
rccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^
2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-3/16*
c*b/e/(c^2*d+e)*sum(_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/
x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1
=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{8}\,a\left(\frac{5\,ex^3+3\,dx}{e^4x^4+2\,de^3x^2+d^2e^2}-\frac{3\,\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,e^2}\right)+b\int\frac{x^4\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right)}{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*a*((5*e*x^3 + 3*d*x)/(e^4*x^4 + 2*d*e^3*x^2 + d^2*e^2) - 3*arctan(e*x/
sqrt(d*e))/(sqrt(d*e)*e^2)) + b*integrate(x^4*arctan2(1, sqrt(c*x + 1)*sqrt
(c*x - 1))/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^4\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asin(1/(c*x))))/(d + e*x^2)^3,x)
```

```
[Out] int((x^4*(a + b*asin(1/(c*x))))/(d + e*x^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acsc(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

3.116 $\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=1144

$$-\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,c}{16\sqrt{-d}\,\sqrt{e}\,\left(dc^2+e\right)\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)}-\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,c}{16\sqrt{-d}\,\sqrt{e}\,\left(dc^2+e\right)\left(\frac{d}{x}+\sqrt{-d}\,\sqrt{e}\right)}+\frac{a+b\csc^{-1}(cx)}{16de\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\csc^{-1}(c}{16de\left(\frac{d}{x}+\sqrt{-d}\right.}$$

```
[Out] 1/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/
c^2/x^2)^(1/2))/d^(3/2)/(c^2*d+e)^(3/2)+1/16*b*arctanh((c^2*d+(-d)^(1/2)*e^
(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(3/2)/(c^2*d+e)^(
3/2)+1/16*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)
/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)-1/16*(a+b*arccsc(c*x))*ln(1+
I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^
(3/2)/e^(3/2)+1/16*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(
-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)-1/16*(a+b*arccsc(c*
x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2
)))/(-d)^(3/2)/e^(3/2)+1/16*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*
(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)-1/16*I*b*polylog(2
,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)
^(3/2)/e^(3/2)-1/16*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2
)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*I*b*polylog(2,-I*c*(I/
c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e
^(3/2)+1/16*(a+b*arccsc(c*x))/(-d)^(1/2)/e^(1/2)/(-d/x+(-d)^(1/2)*e^(1/2))^
2+1/16*(a+b*arccsc(c*x))/d/e/(-d/x+(-d)^(1/2)*e^(1/2))+1/16*(-a-b*arccsc(c*
x))/(-d)^(1/2)/e^(1/2)/(d/x+(-d)^(1/2)*e^(1/2))^2+1/16*(-a-b*arccsc(c*x))/d
/e/(d/x+(-d)^(1/2)*e^(1/2))-1/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d
^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(3/2)/e/(c^2*d+e)^(1/2)-1/16*
b*arctanh((c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x
^2)^(1/2))/d^(3/2)/e/(c^2*d+e)^(1/2)-1/16*b*c*(1-1/c^2/x^2)^(1/2)/(c^2*d+e)
/(-d)^(1/2)/e^(1/2)/(-d/x+(-d)^(1/2)*e^(1/2))-1/16*b*c*(1-1/c^2/x^2)^(1/2)/
(c^2*d+e)/(-d)^(1/2)/e^(1/2)/(d/x+(-d)^(1/2)*e^(1/2))
```

Rubi [A] time = 3.07, antiderivative size = 1144, normalized size of antiderivative = 1.00, number of steps used = 63, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5241, 4733, 4667, 4743, 731, 725, 206, 4741, 4519, 2190, 2279, 2391}

$$-\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,c}{16\sqrt{-d}\,\sqrt{e}\,\left(dc^2+e\right)\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)}-\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,c}{16\sqrt{-d}\,\sqrt{e}\,\left(dc^2+e\right)\left(\frac{d}{x}+\sqrt{-d}\,\sqrt{e}\right)}+\frac{a+b\csc^{-1}(cx)}{16de\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\csc^{-1}(c}{16de\left(\frac{d}{x}+\sqrt{-d}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -(b*c*Sqrt[1 - 1/(c^2*x^2)])/(16*Sqrt[-d]*Sqrt[e]*(c^2*d + e)*(Sqrt[-d]*Sqr
t[e] - d/x)) - (b*c*Sqrt[1 - 1/(c^2*x^2)])/(16*Sqrt[-d]*Sqrt[e]*(c^2*d + e)
*(Sqrt[-d]*Sqrt[e] + d/x)) + (a + b*ArcCsc[c*x])/(16*Sqrt[-d]*Sqrt[e]*(Sqrt
[-d]*Sqrt[e] - d/x)^2) + (a + b*ArcCsc[c*x])/(16*d*e*(Sqrt[-d]*Sqrt[e] - d/
x)) - (a + b*ArcCsc[c*x])/(16*Sqrt[-d]*Sqrt[e]*(Sqrt[-d]*Sqrt[e] + d/x)^2)
- (a + b*ArcCsc[c*x])/(16*d*e*(Sqrt[-d]*Sqrt[e] + d/x)) + (b*ArcTanh[(c^2*d
 - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])]
```

```
)/(16*d^(3/2)*(c^2*d + e)^(3/2)) - (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x
)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(3/2)*e*Sqrt[c^
2*d + e]) + (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d
 + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(3/2)*(c^2*d + e)^(3/2)) - (b*ArcTanh[
(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x
^2)])])/(16*d^(3/2)*e*Sqrt[c^2*d + e]) + ((a + b*ArcCsc[c*x])*Log[1 - (I*c*
Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^
(3/2)) - ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqr
t[e] - Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) + ((a + b*ArcCsc[c*x])*Lo
g[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-
d)^(3/2)*e^(3/2)) - ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[
c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) + ((I/16)*b*Po
lyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/
((-d)^(3/2)*e^(3/2)) - ((I/16)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x])
)/(Sqrt[e] - Sqrt[c^2*d + e])])/((-d)^(3/2)*e^(3/2)) + ((I/16)*b*PolyLog[2,
 ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/((-d)^(3
/2)*e^(3/2)) - ((I/16)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[
e] + Sqrt[c^2*d + e])])/((-d)^(3/2)*e^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
```

```
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rule 4667

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4733

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4741

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[((a + b*x)^n*Cos[x])/(c*d + e*Sin[x]), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4743

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSin[c*x])^n)/(e*(m + 1)), x] -
Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1
))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
&& NeQ[m, -1]
```

Rule 5241

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSin[x/c])^n)/x^
(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = -\text{Subst}\left(\int \frac{x^2\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int\left(-\frac{e\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)^3}+\frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{d\left(e+dx^2\right)^2}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{d}+\frac{e\,\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int\left(-\frac{d\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\,\sqrt{e}-dx\right)^2}-\frac{d\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\,\sqrt{e}+dx\right)^2}-\frac{d\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{2e\left(-de-d^2x^2\right)}\right)dx, x, \frac{1}{x}\right)}{d}+\frac{e\,\text{Subst}\cdots}{\cdots}$$

$$= -\frac{3\,\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\,\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16e}-\frac{3\,\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\,\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16e}+\frac{\text{Subst}\left(\int\cdots\right.}{\cdots}$$

$$= \frac{a+b\csc^{-1}(cx)}{16\sqrt{-d}\,\sqrt{e}\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)^2}+\frac{a+b\csc^{-1}(cx)}{16de\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\csc^{-1}(cx)}{16\sqrt{-d}\,\sqrt{e}\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)^2}\cdots$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\,\sqrt{e}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)}-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\,\sqrt{e}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)}+\frac{\cdots}{16\cdots}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\,\sqrt{e}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)}-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\,\sqrt{e}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)}+\frac{\cdots}{16\cdots}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\,\sqrt{e}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)}-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\,\sqrt{e}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)}+\frac{\cdots}{16\cdots}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\,\sqrt{e}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)}-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\,\sqrt{e}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)}+\frac{\cdots}{16\cdots}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\,\sqrt{e}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)}-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\,\sqrt{e}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)}+\frac{\cdots}{16\cdots}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\,\sqrt{e}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)}-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\,\sqrt{e}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)}+\frac{\cdots}{16\cdots}$$

Mathematica [A] time = 6.14, size = 2075, normalized size = 1.81

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/4*(a*x)/(e*(d + e*x^2)^2) + (a*x)/(8*d*e*(d + e*x^2)) + (a*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(3/2)) + b*(-1/16*(-(ArcCsc[c*x]/((-I)*Sqrt[d]
*Sqrt[e] + e*x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(Sq
rt[e] + c*((-I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(
Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d])/
(d*e) - (-(ArcCsc[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSin[1/(c*x)]/Sqr
t[e] - Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(c^2*d)
 - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x
))]/Sqrt[-(c^2*d) - e]))/Sqrt[d])/(16*d*e) - ((I/16)*((I*c*Sqrt[e]*Sqrt[1 -
 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcCsc[
c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) - ArcSin[1/(c*x)]/(d*Sqrt[e]) +
 (I*(2*c^2*d + e)*Log[(4*d*Sqrt[e]*Sqrt[c^2*d + e]*(I*Sqrt[e] + c*(c*Sqrt[d
] - Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d + e)*((-I)*Sqrt[d]
 + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/(Sqrt[d]*e) + ((I/16)*(((-I)*c*Sqr
t[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x))
 - ArcCsc[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - ArcSin[1/(c*x)]/(d*Sqr
t[e]) + (I*(2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*((-I)*Sqrt[e] +
c*(c*Sqrt[d] + Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d + e)*(I
*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/(Sqrt[d]*e) - (Pi^2 - 4*Pi
*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])
]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/
Sqrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*
E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])
/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqr
t[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[
c*x]))] + (4*I)*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcC
sc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[
d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sq
rt[2]]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] +
 (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] - (4*I)*Pi*Log[Sqrt[e] +
(I*Sqrt[d])/x] + 8*PolyLog[2, (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*
ArcCsc[c*x]))] + 8*PolyLog[2, -((Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I
*ArcCsc[c*x])))] + 4*PolyLog[2, E^((2*I)*ArcCsc[c*x])])/(128*d^(3/2)*e^(3/2
)) + (Pi^2 - 4*Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 + (I*Sqr
t[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*Arc
Csc[c*x])/4])/Sqrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2*d +
e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (-Sqrt[e] +
Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1 + (I
*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqr
t[d]*E^(I*ArcCsc[c*x]))] + (4*I)*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*
Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2
*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 + (I*Sqrt[e]
)/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I
*ArcCsc[c*x]))] + (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] - (4*I)*
Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] + 8*PolyLog[2, (Sqrt[e] - Sqrt[c^2*d + e])/
(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8*PolyLog[2, (Sqrt[e] + Sqrt[c^2*d + e])/(
c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*PolyLog[2, E^((2*I)*ArcCsc[c*x])])/(128*d
^(3/2)*e^(3/2)))
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^2 \operatorname{arccsc}(cx) + ax^2}{e^3x^6 + 3\,de^2x^4 + 3\,d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccsc(c*x) + a*x^2)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 +
 d^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sy
m(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Va
lue
```

maple [C] time = 7.79, size = 2329, normalized size = 2.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^3,x)
```

```
[Out] 1/8*c^5*b*x^4/(c^2*e*x^2+c^2*d)^2*e/(c^2*d+e)/d*((c^2*x^2-1)/c^2/x^2)^(1/2)
+1/8*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(I/c/x+(1-1/
c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)^2/e
/d^2*(e*(c^2*d+e))^(1/2)-1/8*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*
arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*
d)^(1/2))/(c^2*d+e)^2/e/d^2*(e*(c^2*d+e))^(1/2)-1/4/c^2*b*((c^2*d+2*(e*(c^2
*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+
2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)^2/d^3*(e*(c^2*d+e))^(1/2)+1/
4/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(I/c/x+(1-1
/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)^2/
d^3*(e*(c^2*d+e))^(1/2)+1/4/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1
/2)*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2
*e)*d)^(1/2))/(c^2*d+e)^2*e/d^3+1/4/c^2*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e
)*d)^(1/2)*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^
(1/2)+2*e)*d)^(1/2))/(c^2*d+e)^2*e/d^3+1/8*c^5*b*x^2/(c^2*e*x^2+c^2*d)^2/(c
^2*d+e)*((c^2*x^2-1)/c^2/x^2)^(1/2)-1/16*c*b/d/(c^2*d+e)*sum(1/_R1/(_R1^2*c
^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilo
g((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e
)*_Z^2+c^2*d))+1/8*c^4*a/(c^2*e*x^2+c^2*d)^2/d*x^3-1/8*c^4*a/(c^2*e*x^2+c^2
*d)^2/e*x+1/8*a/d/e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-1/8*b*(-(c^2*d-2*(e
*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c
^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/e/(c^2*d+e)/d^2-1/8*b*((c^2*d+2*(
e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((
c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e/(c^2*d+e)/d^2-1/4/c^2*b*((c^2*
d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2
))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)/d^3-1/4/c^2*b*(-(
c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(
1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)/d^3-1/8*c^4*b
*x/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*arccsc(c*x)+1/8*c^6*b*x^3/(c^2*e*x^2+c^2*d
)^2/(c^2*d+e)*arccsc(c*x)-1/4/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^
(1/2)*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)
-2*e)*d)^(1/2))/e/(c^2*d+e)/d^3*(e*(c^2*d+e))^(1/2)+1/4/c^2*b*((c^2*d+2*(e*
(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^
```

```
2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e/(c^2*d+e)/d^3*(e*(c^2*d+e))^(1/2
)-1/8*c^6*b*x/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)/e*arccsc(c*x)*d+1/8*c^4*b*x^3/(
c^2*e*x^2+c^2*d)^2*e/(c^2*d+e)/d*arccsc(c*x)-1/16*c*b/d/(c^2*d+e)*sum(_R1/(
_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R
1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^
2*d-4*e)*_Z^2+c^2*d))-1/16*c^3*b/e/(c^2*d+e)*sum(1/_R1/(_R1^2*c^2*d-c^2*d-2
*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x
-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d
))-1/16*c^3*b/e/(c^2*d+e)*sum(_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln
((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))
/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))+1/4*b*((c^2*d+2*(e
*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c
^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)^2/d^2+1/4*b*(-(c^2*d-2*
(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((
-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)^2/d^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}\,a\left(\frac{ex^3-dx}{de^3x^4+2\,d^2e^2x^2+d^3e}+\frac{\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,de}\right)+b\int\frac{x^2\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right)}{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a*((e*x^3 - d*x)/(d*e^3*x^4 + 2*d^2*e^2*x^2 + d^3*e) + arctan(e*x/sqrt(
d*e))/(sqrt(d*e)*d*e)) + b*integrate(x^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x
- 1))/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^3,x)
```

```
[Out] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsc(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

3.117 $\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^3}\,dx$

Optimal. Leaf size=1134

$$-\frac{b\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,c}{16(-d)^{3/2}\left(dc^2+e\right)\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)}-\frac{b\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,c}{16(-d)^{3/2}\left(dc^2+e\right)\left(\frac{d}{x}+\sqrt{-d}\,\sqrt{e}\right)}-\frac{5\left(a+b\csc^{-1}(cx)\right)}{16d^2\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)}+\frac{5\left(a+b\csc^{-1}(\right.}{16d^2\left(\frac{d}{x}+\sqrt{-d}\right.}$$

```
[Out] -1/16*b*e*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1
-1/c^2/x^2)^(1/2))/d^(5/2)/(c^2*d+e)^(3/2)-1/16*b*e*arctanh((c^2*d+(-d)^(1/
2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(5/2)/(c^2*d
+e)^(3/2)-3/16*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^
(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*(a+b*arccsc(c*x))*
ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/
(-d)^(5/2)/e^(1/2)-3/16*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/
2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*(a+b*arcc
sc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)
^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1
/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*I*b*poly
log(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))
/(-d)^(5/2)/e^(1/2)+3/16*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)
^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*I*b*polylog(2,-I*
c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(5
/2)/e^(1/2)+1/16*(a+b*arccsc(c*x))*e^(1/2)/(-d)^(3/2)/(-d/x+(-d)^(1/2)*e^(1
/2))^2-5/16*(a+b*arccsc(c*x))/d^2/(-d/x+(-d)^(1/2)*e^(1/2))-1/16*(a+b*arccs
c(c*x))*e^(1/2)/(-d)^(3/2)/(d/x+(-d)^(1/2)*e^(1/2))^2+5/16*(a+b*arccsc(c*x)
)/d^2/(d/x+(-d)^(1/2)*e^(1/2))+5/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/
c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(5/2)/(c^2*d+e)^(1/2)+5/16
*b*arctanh((c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/
x^2)^(1/2))/d^(5/2)/(c^2*d+e)^(1/2)-1/16*b*c*e^(1/2)*(1-1/c^2/x^2)^(1/2)/(-
d)^(3/2)/(c^2*d+e)/(-d/x+(-d)^(1/2)*e^(1/2))-1/16*b*c*e^(1/2)*(1-1/c^2/x^2)
^(1/2)/(-d)^(3/2)/(c^2*d+e)/(d/x+(-d)^(1/2)*e^(1/2))
```

Rubi [A] time = 4.05, antiderivative size = 1134, normalized size of antiderivative = 1.00, number of steps used = 81, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5231, 4733, 4667, 4743, 731, 725, 206, 4741, 4519, 2190, 2279, 2391}

$$-\frac{b\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,c}{16(-d)^{3/2}\left(dc^2+e\right)\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)}-\frac{b\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,c}{16(-d)^{3/2}\left(dc^2+e\right)\left(\frac{d}{x}+\sqrt{-d}\,\sqrt{e}\right)}-\frac{5\left(a+b\csc^{-1}(cx)\right)}{16d^2\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)}+\frac{5\left(a+b\csc^{-1}(\right.}{16d^2\left(\frac{d}{x}+\sqrt{-d}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(d + e*x^2)^3,x]
```

```
[Out] -(b*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)])/(16*(-d)^(3/2)*(c^2*d + e)*(Sqrt[-d]*S
qrt[e] - d/x)) - (b*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)])/(16*(-d)^(3/2)*(c^2*d
+ e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[e]*(a + b*ArcCsc[c*x]))/(16*(-d)^(3/
2)*(Sqrt[-d]*Sqrt[e] - d/x)^2) - (5*(a + b*ArcCsc[c*x]))/(16*d^2*(Sqrt[-d]*
Sqrt[e] - d/x)) - (Sqrt[e]*(a + b*ArcCsc[c*x]))/(16*(-d)^(3/2)*(Sqrt[-d]*Sq
rt[e] + d/x)^2) + (5*(a + b*ArcCsc[c*x]))/(16*d^2*(Sqrt[-d]*Sqrt[e] + d/x))
 - (b*e*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*S
qrt[1 - 1/(c^2*x^2)])])/(16*d^(5/2)*(c^2*d + e)^(3/2)) + (5*b*ArcTanh[(c^2*
```

```
d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])
])/(16*d^(5/2)*Sqrt[c^2*d + e]) - (b*e*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/
x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(5/2)*(c^2*d +
 e)^(3/2)) + (5*b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^
2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(5/2)*Sqrt[c^2*d + e]) - (3*(a + b*
ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d
 + e])])/(16*(-d)^(5/2)*Sqrt[e]) + (3*(a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt
[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e
]) - (3*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[
e] + Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) + (3*(a + b*ArcCsc[c*x])*Lo
g[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-
d)^(5/2)*Sqrt[e]) - (((3*I)/16)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c
*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/((-d)^(5/2)*Sqrt[e]) + (((3*I)/16)*b*Po
lyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/((-
d)^(5/2)*Sqrt[e]) - (((3*I)/16)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c
*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/((-d)^(5/2)*Sqrt[e]) + (((3*I)/16)*b*Po
lyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/((-
d)^(5/2)*Sqrt[e])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
```

```
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rule 4667

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4733

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4741

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[((a + b*x)^n*Cos[x])/(c*d + e*Sin[x]), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4743

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSin[c*x])^n)/(e*(m + 1)), x] -
Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1
))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
&& NeQ[m, -1]
```

Rule 5231

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSin[x/c])^n)/x^(2*(p + 1))
, x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{\left(d+ex^2\right)^3}\,dx = -\text{Subst}\left(\int \frac{x^4\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int\left(\frac{e^2\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^3} - \frac{2e\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^2} + \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{d^2\left(e+dx^2\right)}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d^2} + \frac{(2e)\,\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{d^2} - \frac{e^2\,\text{Subst}\left(\int \frac{a+b\sin^{-1}}{(e+dx}\right.}{d^2}$$

$$= -\frac{\text{Subst}\left(\int\left(\frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}\,x\right)} + \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}\,x\right)}\right)dx, x, \frac{1}{x}\right)}{d^2} + \frac{(2e)\,\text{Subst}\left(\int\left(-\frac{d\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\,\sqrt{e}-dx\right)^2} - \right.\right.}{}$$

$$= \frac{3\,\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\,\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16d} + \frac{3\,\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\,\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16d} + \frac{3\,\text{Subst}\left(\int \frac{a+b\sin^{-1}}{-de-d}\right.}{8d}$$

$$= \frac{\sqrt{e}\left(a+b\csc^{-1}(cx)\right)}{16(-d)^{3/2}\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)^2} - \frac{5\left(a+b\csc^{-1}(cx)\right)}{16d^2\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{e}\left(a+b\csc^{-1}(cx)\right)}{16(-d)^{3/2}\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)^2} + \frac{5\left(a+b\right.}{16d^2\left(\sqrt{-}\right.}$$

$$= -\frac{bc\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{e}\left(a+b\,c\right.}{16(-d)^{3/2}\left(\sqrt{-}\right.}$$

$$= -\frac{bc\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{e}\left(a+b\,c\right.}{16(-d)^{3/2}\left(\sqrt{-}\right.}$$

$$= -\frac{bc\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{e}\left(a+b\,c\right.}{16(-d)^{3/2}\left(\sqrt{-}\right.}$$

$$= -\frac{bc\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{e}\left(a+b\,c\right.}{16(-d)^{3/2}\left(\sqrt{-}\right.}$$

$$= -\frac{bc\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{e}\left(a+b\,c\right.}{16(-d)^{3/2}\left(\sqrt{-}\right.}$$

$$= -\frac{bc\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{e}\left(a+b\,c\right.}{16(-d)^{3/2}\left(\sqrt{-}\right.}$$

Mathematica [A] time = 6.06, size = 2060, normalized size = 1.82

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(d + e*x^2)^3,x]
```

```
[Out] (a*x)/(4*d*(d + e*x^2)^2) + (3*a*x)/(8*d^2*(d + e*x^2)) + (3*a*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(8*d^(5/2)*Sqrt[e]) + b*((-3*(-(ArcCsc[c*x]/((-I)*Sqrt[d]*
Sqrt[e] + e*x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(Sqr
t[e] + c*((-I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(S
qrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/
(16*d^2) - (3*(-(ArcCsc[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSin[1/(c*x
)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(
c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqr
t[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/(16*d^2) + ((I/16)*((I*c*Sqrt[e]*S
qrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x)) -
ArcCsc[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) - ArcSin[1/(c*x)]/(d*Sqr
t[e]) + (I*(2*c^2*d + e)*Log[(4*d*Sqrt[e]*Sqrt[c^2*d + e]*(I*Sqrt[e] + c*(c
*Sqrt[d] - Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d + e)*((-I)*
Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/d^(3/2) - ((I/16)*(((-I)*c*
Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*
x)) - ArcCsc[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - ArcSin[1/(c*x)]/(d*
Sqrt[e]) + (I*(2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*((-I)*Sqrt[e]
 + c*(c*Sqrt[d] + Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d + e)
*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/d^(3/2) - (3*(Pi^2 - 4*
Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d
])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4]
)/Sqrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d
]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e
])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*S
qrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCs
c[c*x]))] + (4*I)*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*Ar
cCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqr
t[d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/
Sqrt[2]]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))]
 + (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] - (4*I)*Pi*Log[Sqrt[e]
+ (I*Sqrt[d])/x] + 8*PolyLog[2, (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(
I*ArcCsc[c*x]))] + 8*PolyLog[2, -((Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^
(I*ArcCsc[c*x])))] + 4*PolyLog[2, E^((2*I)*ArcCsc[c*x])]))/(128*d^(5/2)*Sqr
t[e]) + (3*(Pi^2 - 4*Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 +
(I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(Pi +
 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^
2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (-Sqrt
[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[
1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/
(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e
])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sq
rt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 + (I*S
qrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d
]*E^(I*ArcCsc[c*x]))] + (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] -
(4*I)*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] + 8*PolyLog[2, (Sqrt[e] - Sqrt[c^2*d
+ e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8*PolyLog[2, (Sqrt[e] + Sqrt[c^2*d +
 e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*PolyLog[2, E^((2*I)*ArcCsc[c*x])]))
/(128*d^(5/2)*Sqrt[e]))
```

fricas [F] time = 1.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b \operatorname{arccsc}(cx) + a}{e^3x^6 + 3\,de^2x^4 + 3\,d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x
)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sy
m(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Va
lue
```

maple [C] time = 10.04, size = 3166, normalized size = 2.79

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/(e*x^2+d)^3,x)
```

```
[Out] -5/8*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(I/c/x+(1-1/
c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)^2/d
^3*(e*(c^2*d+e))^(1/2)-5/4*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*a
rctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d
)^(1/2))/(c^2*d+e)^2*e/d^3+5/8*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2
)*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e
)*d)^(1/2))/(c^2*d+e)^2/d^3*(e*(c^2*d+e))^(1/2)+5/8*c^6*b*x/(c^2*e*x^2+c^2*
d)^2/(c^2*d+e)*arccsc(c*x)-3/16*c*b/d^2/(c^2*d+e)*e*sum(1/_R1/(_R1^2*c^2*d-
c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R
1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^
2+c^2*d))-3/16*c*b/d^2/(c^2*d+e)*e*sum(_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccs
c(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^
2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-5/4*b*((c
^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(
1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)^2*e/d^3+5/8*b*
(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(I/c/x+(1-1/c^2/x^2
)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)/d^3+5/8*b*
((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh(c*d*(I/c/x+(1-1/c^2/x^2
)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)/d^3-3/16*c^
3*b/d/(c^2*d+e)*sum(1/_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/
c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_
R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-3/16*c^3*b/d/(c^2*d+e)*sum
(_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/
2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+
(-2*c^2*d-4*e)*_Z^2+c^2*d))+1/4*c^4*a*x/d/(c^2*e*x^2+c^2*d)^2+3/8*c^2*a/d^2
*x/(c^2*e*x^2+c^2*d)+3/8*a/d^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+5/4/c^2*
b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(I/c/x+(1-1/c^2/x
^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^4/(c^2*d+e)*(e*(
c^2*d+e))^(1/2)-5/4/c^2*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arcta
nh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1
/2))/d^4/(c^2*d+e)*(e*(c^2*d+e))^(1/2)+7/4/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^(
1/2)+2*e)*d)^(1/2)*e*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(
c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^4/(c^2*d+e)+1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e
```

```
))^(1/2)+2*e)*d)^(1/2)*e^2*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+
2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^5/(c^2*d+e)-9/4/c^2*b*(-(c^2*d-2*(e*
(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e^2*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/(
(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^4/(c^2*d+e)^2-1/c^4*b*(-(c^2
*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e^3*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^
(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^5/(c^2*d+e)^2+7/4/c^
2*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e*arctanh(c*d*(I/c/x+(1-1/c
^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^4/(c^2*d+e)+1
/c^4*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e^2*arctanh(c*d*(I/c/x+(
1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^5/(c^2*d
+e)-9/4/c^2*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e^2*arctanh(c*d*(
I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^4
/(c^2*d+e)^2-1/c^4*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e^3*arctan
h(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/
2))/d^5/(c^2*d+e)^2+7/4/c^2*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e
*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)
*d)^(1/2))/d^4/(c^2*d+e)^2*(e*(c^2*d+e))^(1/2)+1/c^4*b*((c^2*d+2*(e*(c^2*d+
e))^(1/2)+2*e)*d)^(1/2)*e^2*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d
+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^5/(c^2*d+e)^2*(e*(c^2*d+e))^(1/2)+1
/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e*arctan(c*d*(I/c/x+(1-
1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^5/(c^2*d+
e)*(e*(c^2*d+e))^(1/2)-7/4/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/
2)*e*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-
2*e)*d)^(1/2))/d^4/(c^2*d+e)^2*(e*(c^2*d+e))^(1/2)-1/c^4*b*(-(c^2*d-2*(e*(c
^2*d+e))^(1/2)+2*e)*d)^(1/2)*e^2*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-
c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^5/(c^2*d+e)^2*(e*(c^2*d+e))^(1
/2)+3/8*c^6*b*x^3/(c^2*e*x^2+c^2*d)^2*e/(c^2*d+e)/d*arccsc(c*x)+3/8*c^4*b*x
^3/(c^2*e*x^2+c^2*d)^2/d^2/(c^2*d+e)*arccsc(c*x)*e^2+5/8*c^4*b*x/(c^2*e*x^2
+c^2*d)^2/d/(c^2*d+e)*arccsc(c*x)*e-1/c^4*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2
*e)*d)^(1/2)*e*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+
e))^(1/2)+2*e)*d)^(1/2))/d^5/(c^2*d+e)*(e*(c^2*d+e))^(1/2)-1/8*c^5*b*x^4/(c
^2*e*x^2+c^2*d)^2/d^2/(c^2*d+e)*((c^2*x^2-1)/c^2/x^2)^(1/2)*e^2-1/8*c^5*b*x
^2/(c^2*e*x^2+c^2*d)^2/d/(c^2*d+e)*((c^2*x^2-1)/c^2/x^2)^(1/2)*e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}\,a\left(\frac{3\,ex^3+5\,dx}{d^2e^2x^4+2\,d^3ex^2+d^4}+\frac{3\,\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,d^2}\right)+b\int\frac{\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right)}{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a*((3*e*x^3 + 5*d*x)/(d^2*e^2*x^4 + 2*d^3*e*x^2 + d^4) + 3*arctan(e*x/s
qrt(d*e))/(sqrt(d*e)*d^2)) + b*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x
- 1))/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{{\left(e\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x^2)^3,x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(d + e*x^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

3.118 $\int x^5\sqrt{d+ex^2}\,\left(a+b\csc^{-1}(cx)\right)\,dx$

Optimal. Leaf size=403

$$\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{8bcd^{7/2}x\,\mathrm{ta}}{105}$$

```
[Out] 1/3*d^2*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/e^3-2/5*d*(e*x^2+d)^(5/2)*(a+b*ar
ccsc(c*x))/e^3+1/7*(e*x^2+d)^(7/2)*(a+b*arccsc(c*x))/e^3-8/105*b*c*d^(7/2)*
x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e^3/(c^2*x^2)^(1/2)+1/1
680*b*(105*c^6*d^3-35*c^4*d^2*e+63*c^2*d*e^2+75*e^3)*x*arctanh(e^(1/2)*(c^2
*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^6/e^(5/2)/(c^2*x^2)^(1/2)-1/840*b*(29*c^
2*d-25*e)*x*(e*x^2+d)^(3/2)*(c^2*x^2-1)^(1/2)/c^3/e^2/(c^2*x^2)^(1/2)+1/42*
b*x*(e*x^2+d)^(5/2)*(c^2*x^2-1)^(1/2)/c/e^2/(c^2*x^2)^(1/2)-1/1680*b*(23*c^
4*d^2+12*c^2*d*e-75*e^2)*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c^5/e^2/(c^2*x
^2)^(1/2)
```

Rubi [A] time = 1.31, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {266, 43, 5239, 12, 1615, 154, 157, 63, 217, 206, 93, 204}

$$\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{bx\sqrt{c^2x^2-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] -(b*(23*c^4*d^2 + 12*c^2*d*e - 75*e^2)*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2]
)/(1680*c^5*e^2*Sqrt[c^2*x^2]) - (b*(29*c^2*d - 25*e)*x*Sqrt[-1 + c^2*x^2]*
(d + e*x^2)^(3/2))/(840*c^3*e^2*Sqrt[c^2*x^2]) + (b*x*Sqrt[-1 + c^2*x^2]*(d
 + e*x^2)^(5/2))/(42*c*e^2*Sqrt[c^2*x^2]) + (d^2*(d + e*x^2)^(3/2)*(a + b*A
rcCsc[c*x]))/(3*e^3) - (2*d*(d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]))/(5*e^3)
+ ((d + e*x^2)^(7/2)*(a + b*ArcCsc[c*x]))/(7*e^3) - (8*b*c*d^(7/2)*x*ArcTan
[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(105*e^3*Sqrt[c^2*x^2]) + (
b*(105*c^6*d^3 - 35*c^4*d^2*e + 63*c^2*d*e^2 + 75*e^3)*x*ArcTanh[(Sqrt[e]*S
qrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(1680*c^6*e^(5/2)*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
```

```
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int x^5\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)dx = \frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} - \frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3} + \ldots$$

$$= \frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} - \frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3} + \ldots$$

$$= \frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} - \frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3} + \ldots$$

$$= \frac{bx\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{5/2}}{42ce^2\sqrt{c^2x^2}} + \frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} - \frac{2d\left(d+\ldots\right.}{\ldots}$$

$$= -\frac{b\left(29c^2d-25e\right)x\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}{840c^3e^2\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{5/\ldots}}{42ce^2\sqrt{c^2x^2}}\ldots$$

$$= -\frac{b\left(23c^4d^2+12c^2de-75e^2\right)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{c^2x^2}} - \frac{b\left(29c^2d-25e\right)x\ldots}{840\ldots}$$

$$= -\frac{b\left(23c^4d^2+12c^2de-75e^2\right)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{c^2x^2}} - \frac{b\left(29c^2d-25e\right)x\ldots}{840\ldots}$$

$$= -\frac{b\left(23c^4d^2+12c^2de-75e^2\right)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{c^2x^2}} - \frac{b\left(29c^2d-25e\right)x\ldots}{840\ldots}$$

$$= -\frac{b\left(23c^4d^2+12c^2de-75e^2\right)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{c^2x^2}} - \frac{b\left(29c^2d-25e\right)x\ldots}{840\ldots}$$

$$= -\frac{b\left(23c^4d^2+12c^2de-75e^2\right)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{c^2x^2}} - \frac{b\left(29c^2d-25e\right)x\ldots}{840\ldots}$$

Mathematica [C] time = 0.64, size = 365, normalized size = 0.91

$$\frac{\sqrt{d+ex^2}\left(16ac^5\left(8d^3-4d^2ex^2+3de^2x^4+15e^3x^6\right)+16bc^5\csc^{-1}(cx)\left(8d^3-4d^2ex^2+3de^2x^4+15e^3x^6\right)+bex\sqrt{1}\right.}{1680c^5e^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*(-128*c^4*d^4*Sqrt[1 + d/(e*x^2)]*(-1 + c^2*x^2)*AppellF1[1, 1/2, 1/2, 2
, 1/(c^2*x^2), -(d/(e*x^2))] + e*(105*c^6*d^3 - 35*c^4*d^2*e + 63*c^2*d*e^2
 + 75*e^3)*Sqrt[1 - 1/(c^2*x^2)]*x^4*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*
AppellF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/d)]))/(3360*c^5*e^3*x*(-1 + c^2
*x^2)*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(16*a*c^5*(8*d^3 - 4*d^2*e*x^2 +
3*d*e^2*x^4 + 15*e^3*x^6) + b*e*Sqrt[1 - 1/(c^2*x^2)]*x*(75*e^2 + 2*c^2*e*(
19*d + 25*e*x^2) + c^4*(-41*d^2 + 22*d*e*x^2 + 40*e^2*x^4)) + 16*b*c^5*(8*d
^3 - 4*d^2*e*x^2 + 3*d*e^2*x^4 + 15*e^3*x^6)*ArcCsc[c*x]))/(1680*c^5*e^3)
```

fricas [A] time = 7.33, size = 1699, normalized size = 4.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6720*(128*b*c^7*sqrt(-d)*d^3*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c
^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2
+ d)*sqrt(-d) + 8*d^2)/x^4) + (105*b*c^6*d^3 - 35*b*c^4*d^2*e + 63*b*c^2*d*
e^2 + 75*b*e^3)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*
e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x
^2 + d)*sqrt(e) + e^2) + 4*(240*a*c^7*e^3*x^6 + 48*a*c^7*d*e^2*x^4 - 64*a*c
^7*d^2*e*x^2 + 128*a*c^7*d^3 + 16*(15*b*c^7*e^3*x^6 + 3*b*c^7*d*e^2*x^4 - 4
*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3)*arccsc(c*x) + (40*b*c^5*e^3*x^4 - 41*b*c^5*
d^2*e + 38*b*c^3*d*e^2 + 75*b*c*e^3 + 2*(11*b*c^5*d*e^2 + 25*b*c^3*e^3)*x^2
)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^7*e^3), -1/6720*(256*b*c^7*d^(7/2)
*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt
(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) - (105*b*c^6*d^3 - 35*b*c^4*
d^2*e + 63*b*c^2*d*e^2 + 75*b*e^3)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*
c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^
2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) - 4*(240*a*c^7*e^3*x^6 + 48*a*c^7
*d*e^2*x^4 - 64*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 + 16*(15*b*c^7*e^3*x^6 + 3*
b*c^7*d*e^2*x^4 - 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3)*arccsc(c*x) + (40*b*c^5*
e^3*x^4 - 41*b*c^5*d^2*e + 38*b*c^3*d*e^2 + 75*b*c*e^3 + 2*(11*b*c^5*d*e^2
+ 25*b*c^3*e^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^7*e^3), 1/3360*
(64*b*c^7*sqrt(-d)*d^3*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 -
d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt
(-d) + 8*d^2)/x^4) - (105*b*c^6*d^3 - 35*b*c^4*d^2*e + 63*b*c^2*d*e^2 + 75*
b*e^3)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt
(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(24
0*a*c^7*e^3*x^6 + 48*a*c^7*d*e^2*x^4 - 64*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 +
 16*(15*b*c^7*e^3*x^6 + 3*b*c^7*d*e^2*x^4 - 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3
)*arccsc(c*x) + (40*b*c^5*e^3*x^4 - 41*b*c^5*d^2*e + 38*b*c^3*d*e^2 + 75*b*
c*e^3 + 2*(11*b*c^5*d*e^2 + 25*b*c^3*e^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^
2 + d))/(c^7*e^3), -1/3360*(128*b*c^7*d^(7/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)
*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 -
d*e)*x^2 - d^2)) + (105*b*c^6*d^3 - 35*b*c^4*d^2*e + 63*b*c^2*d*e^2 + 75*b*
e^3)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e
*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) - 2*(240*
a*c^7*e^3*x^6 + 48*a*c^7*d*e^2*x^4 - 64*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 + 1
```

```
6*(15*b*c^7*e^3*x^6 + 3*b*c^7*d*e^2*x^4 - 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3)*
arccsc(c*x) + (40*b*c^5*e^3*x^4 - 41*b*c^5*d^2*e + 38*b*c^3*d*e^2 + 75*b*c*
e^3 + 2*(11*b*c^5*d*e^2 + 25*b*c^3*e^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2
+ d))/(c^7*e^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex^2+d}\,(b\,\mathrm{arccsc}\,(cx)+a)x^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)*x^5, x)
```

maple [F] time = 7.37, size = 0, normalized size = 0.00

$$\int x^5\,(a+b\,\mathrm{arccsc}\,(cx))\,\sqrt{e\,x^2+d}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^5*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{105}\left(\frac{15\left(ex^2+d\right)^{\frac{3}{2}}x^4}{e}-\frac{12\left(ex^2+d\right)^{\frac{3}{2}}dx^2}{e^2}+\frac{8\left(ex^2+d\right)^{\frac{3}{2}}d^2}{e^3}\right)a+\frac{\left(e^3\int\frac{\left(15\,c^2e^3x^7+3\,c^2de^2x^5-4\,c^2d^2ex^3+8\,c^2d^3x\right)e^{\left(\frac{1}{2}\log\left(ex^2+\right.\right.}}{c^2e^3x^2+\left(c^2e^3x^2-e^3\right)(cx+1)(cx-1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/105*(15*(e*x^2 + d)^(3/2)*x^4/e - 12*(e*x^2 + d)^(3/2)*d*x^2/e^2 + 8*(e*x
^2 + d)^(3/2)*d^2/e^3)*a + 1/105*(105*e^3*integrate(1/105*(15*c^2*e^3*x^7 +
 3*c^2*d*e^2*x^5 - 4*c^2*d^2*e*x^3 + 8*c^2*d^3*x)*e^(1/2*log(e*x^2 + d) + 1
/2*log(c*x + 1) + 1/2*log(c*x - 1))/(c^2*e^3*x^2 - e^3 + (c^2*e^3*x^2 - e^3
)*e^(log(c*x + 1) + log(c*x - 1))), x) + (15*e^3*x^6*arctan2(1, sqrt(c*x +
1)*sqrt(c*x - 1)) + 3*d*e^2*x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) - 4
*d^2*e*x^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 8*d^3*arctan2(1, sqrt(
c*x + 1)*sqrt(c*x - 1)))*sqrt(e*x^2 + d))*b/e^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^5\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))),x)
```

```
[Out] int(x^5*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acsc(c*x))*(e*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

3.119 $\int x^3\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)dx$

Optimal. Leaf size=294

$$-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}+\frac{2bcd^{5/2}x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{15e^2\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2x^2-1}\left(d+\right.}{20ce\sqrt{c^2x^2}}$$

```
[Out] -1/3*d*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/e^2+1/5*(e*x^2+d)^(5/2)*(a+b*arccs
c(c*x))/e^2+2/15*b*c*d^(5/2)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(
1/2))/e^2/(c^2*x^2)^(1/2)-1/120*b*(15*c^4*d^2-10*c^2*d*e-9*e^2)*x*arctanh(e
^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^4/e^(3/2)/(c^2*x^2)^(1/2)+1/2
0*b*x*(e*x^2+d)^(3/2)*(c^2*x^2-1)^(1/2)/c/e/(c^2*x^2)^(1/2)+1/120*b*(c^2*d+
9*e)*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c^3/e/(c^2*x^2)^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {266, 43, 5239, 12, 573, 154, 157, 63, 217, 206, 93, 204}

$$-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}+\frac{2bcd^{5/2}x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{15e^2\sqrt{c^2x^2}}-\frac{bx\left(15c^4d^2-10c^2\right.}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*(c^2*d + 9*e)*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(120*c^3*e*Sqrt[c^2*
x^2]) + (b*x*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3/2))/(20*c*e*Sqrt[c^2*x^2]) -
 (d*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^2) + ((d + e*x^2)^(5/2)*(a
+ b*ArcCsc[c*x]))/(5*e^2) + (2*b*c*d^(5/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d
]*Sqrt[-1 + c^2*x^2])])/(15*e^2*Sqrt[c^2*x^2]) - (b*(15*c^4*d^2 - 10*c^2*d*
e - 9*e^2)*x*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(12
0*c^4*e^(3/2)*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
```

```
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 573

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
```

```
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int x^3\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)dx = -\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}+\frac{(bcx)\int}{}$$

$$= -\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}+\frac{(bcx)\int}{}$$

$$= -\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}+\frac{(bcx)\,\mathrm{S}}{}$$

$$= \frac{bx\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}{20ce\sqrt{c^2x^2}}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{\left(d+ex^2\right)^{5/}}{}$$

$$= \frac{b\left(c^2d+9e\right)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{c^2x^2}}+\frac{bx\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}{20ce\sqrt{c^2x^2}}-\frac{d\left(d+\right.}{}$$

$$= \frac{b\left(c^2d+9e\right)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{c^2x^2}}+\frac{bx\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}{20ce\sqrt{c^2x^2}}-\frac{d\left(d+\right.}{}$$

$$= \frac{b\left(c^2d+9e\right)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{c^2x^2}}+\frac{bx\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}{20ce\sqrt{c^2x^2}}-\frac{d\left(d+\right.}{}$$

$$= \frac{b\left(c^2d+9e\right)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{c^2x^2}}+\frac{bx\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}{20ce\sqrt{c^2x^2}}-\frac{d\left(d+\right.}{}$$

$$= \frac{b\left(c^2d+9e\right)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{c^2x^2}}+\frac{bx\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}{20ce\sqrt{c^2x^2}}-\frac{d\left(d+\right.}{}$$

Mathematica [A] time = 0.75, size = 326, normalized size = 1.11

$$\frac{\sqrt{d+ex^2}\left(8ac^3\left(-2d^2+dex^2+3e^2x^4\right)+8bc^3\csc^{-1}(cx)\left(-2d^2+dex^2+3e^2x^4\right)+bex\sqrt{1-\frac{1}{c^2x^2}}\left(c^2\left(7d+6ex^2\right)+\right.\right.}{120c^3e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (Sqrt[d + e*x^2]*(8*a*c^3*(-2*d^2 + d*e*x^2 + 3*e^2*x^4) + b*e*Sqrt[1 - 1/(
c^2*x^2)]*x*(9*e + c^2*(7*d + 6*e*x^2)) + 8*b*c^3*(-2*d^2 + d*e*x^2 + 3*e^2
*x^4)*ArcCsc[c*x]))/(120*c^3*e^2) - (b*Sqrt[1 - 1/(c^2*x^2)]*x*(Sqrt[c^2]*S
qrt[e]*Sqrt[c^2*d + e]*(15*c^4*d^2 - 10*c^2*d*e - 9*e^2)*Sqrt[(c^2*(d + e*x
^2))/(c^2*d + e)]*ArcSinh[(c*Sqrt[e]*Sqrt[-1 + c^2*x^2])/(Sqrt[c^2]*Sqrt[c^
2*d + e])] + 16*c^7*d^(5/2)*Sqrt[d + e*x^2]*ArcTan[(Sqrt[d]*Sqrt[-1 + c^2*x
^2])/Sqrt[d + e*x^2]]))/(120*c^6*e^2*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

fricas [A] time = 2.22, size = 1379, normalized size = 4.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/480*(16*b*c^5*sqrt(-d)*d^2*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2
*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 +
d)*sqrt(-d) + 8*d^2)/x^4) - (15*b*c^4*d^2 - 10*b*c^2*d*e - 9*b*e^2)*sqrt(e)
*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2
*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2)
+ 4*(24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 16*a*c^5*d^2 + 8*(3*b*c^5*e^2*x^4
 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)*arccsc(c*x) + (6*b*c^3*e^2*x^2 + 7*b*c^3*d*
e + 9*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e^2), 1/480*(32*b*c
^5*d^(5/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2
 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) - (15*b*c^4*d^2 -
10*b*c^2*d*e - 9*b*e^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8
*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*
sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 16
*a*c^5*d^2 + 8*(3*b*c^5*e^2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)*arccsc(c*x)
+ (6*b*c^3*e^2*x^2 + 7*b*c^3*d*e + 9*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2
 + d))/(c^5*e^2), 1/240*(8*b*c^5*sqrt(-d)*d^2*log(((c^4*d^2 - 6*c^2*d*e + e
^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*
d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) + (15*b*c^4*d^2 - 10*b*c^2*d*e -
9*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sq
rt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(
24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 16*a*c^5*d^2 + 8*(3*b*c^5*e^2*x^4 + b*
c^5*d*e*x^2 - 2*b*c^5*d^2)*arccsc(c*x) + (6*b*c^3*e^2*x^2 + 7*b*c^3*d*e + 9
*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e^2), 1/240*(16*b*c^5*d^
(5/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)
*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (15*b*c^4*d^2 - 10*b*
c^2*d*e - 9*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x
^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x
^2)) + 2*(24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 16*a*c^5*d^2 + 8*(3*b*c^5*e^
2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)*arccsc(c*x) + (6*b*c^3*e^2*x^2 + 7*b*c
^3*d*e + 9*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex^2 + d}\,(b \operatorname{arccsc}(cx) + a)x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)*x^3, x)
```

maple [F] time = 6.64, size = 0, normalized size = 0.00

$$\int x^3\,(a + b \operatorname{arccsc}(cx))\sqrt{e\,x^2 + d}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^3*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{15}\left(\frac{3\left(ex^2+d\right)^{\frac{3}{2}}x^2}{e}-\frac{2\left(ex^2+d\right)^{\frac{3}{2}}d}{e^2}\right)a+\frac{\left(e^2\int\frac{\left(3\,c^2e^2x^5+c^2dex^3-2\,c^2d^2x\right)e^{\left(\frac{1}{2}\log\left(ex^2+d\right)+\frac{1}{2}\log(cx+1)+\frac{1}{2}\log(cx-1)\right)}}{c^2e^2x^2+\left(c^2e^2x^2-e^2\right)(cx+1)(cx-1)-e^2}\,dx+\left(3\,e^2x^4\,\mathrm{arcta}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*(e*x^2 + d)^(3/2)*x^2/e - 2*(e*x^2 + d)^(3/2)*d/e^2)*a + 1/15*(15*e
^2*integrate(1/15*(3*c^2*e^2*x^5 + c^2*d*e*x^3 - 2*c^2*d^2*x)*e^(1/2*log(e*
x^2 + d) + 1/2*log(c*x + 1) + 1/2*log(c*x - 1))/(c^2*e^2*x^2 - e^2 + (c^2*e
^2*x^2 - e^2)*e^(log(c*x + 1) + log(c*x - 1))), x) + (3*e^2*x^4*arctan2(1,
sqrt(c*x + 1)*sqrt(c*x - 1)) + d*e*x^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x -
1)) - 2*d^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*sqrt(e*x^2 + d))*b/e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))),x)
```

```
[Out] int(x^3*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(a+b\operatorname{acsc}(cx)\right)\sqrt{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsc(c*x))*(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*acsc(c*x))*sqrt(d + e*x**2), x)
```

3.120 $\int x\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)dx$

Optimal. Leaf size=195

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}-\frac{bcd^{3/2}x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3e\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}}+\frac{bx\left(3c^2d+e\right)\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{}}{c\sqrt{}}\right.}{6c^2\sqrt{e}\sqrt{c^2x^2}}$$

```
[Out] 1/3*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/e-1/3*b*c*d^(3/2)*x*arctan((e*x^2+d)^
(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e/(c^2*x^2)^(1/2)+1/6*b*(3*c^2*d+e)*x*arct
anh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^2/e^(1/2)/(c^2*x^2)^(1/2
)+1/6*b*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c/(c^2*x^2)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5237, 446, 102, 157, 63, 217, 206, 93, 204}

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}-\frac{bcd^{3/2}x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3e\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}}+\frac{bx\left(3c^2d+e\right)\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{}}{c\sqrt{}}\right.}{6c^2\sqrt{e}\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(6*c*Sqrt[c^2*x^2]) + ((d + e*x^2)
^(3/2)*(a + b*ArcCsc[c*x]))/(3*e) - (b*c*d^(3/2)*x*ArcTan[Sqrt[d + e*x^2]/(
Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(3*e*Sqrt[c^2*x^2]) + (b*(3*c^2*d + e)*x*ArcT
anh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(6*c^2*Sqrt[e]*Sqrt[
c^2*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5237

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCsc[c*x]))/(2*e*(p + 1)), x
] + Dist[(b*c*x)/(2*e*(p + 1)*Sqrt[c^2*x^2]), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)dx=\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}+\frac{(bcx)\int\frac{\left(d+ex^2\right)^{3/2}}{x\sqrt{-1+c^2x^2}}dx}{3e\sqrt{c^2x^2}}$$

$$=\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}+\frac{(bcx)\operatorname{Subst}\left(\int\frac{(d+ex)^{3/2}}{x\sqrt{-1+c^2x}}dx,x,x^2\right)}{6e\sqrt{c^2x^2}}$$

$$=\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}+\frac{(bx)\operatorname{Subst}\left(\right.}{}$$

$$=\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}+\frac{\left(bcd^2x\right)\operatorname{Subs}}{}$$

$$=\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}+\frac{\left(bcd^2x\right)\operatorname{Subs}}{}$$

$$=\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}-\frac{bcd^{3/2}x\tan^{-1}}{3e\sqrt{}}$$

$$=\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}-\frac{bcd^{3/2}x\tan^{-1}}{3e\sqrt{}}$$

Mathematica [A] time = 0.48, size = 265, normalized size = 1.36

$$\frac{\sqrt{d+ex^2}\left(2ac\left(d+ex^2\right)+bex\sqrt{1-\frac{1}{c^2x^2}}+2bc\csc^{-1}(cx)\left(d+ex^2\right)\right)}{6ce}+\frac{bx\sqrt{1-\frac{1}{c^2x^2}}\left(\sqrt{c^2}\sqrt{e}\sqrt{c^2d+e}\left(3c^2d+e\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (Sqrt[d + e*x^2]*(b*e*Sqrt[1 - 1/(c^2*x^2)]*x + 2*a*c*(d + e*x^2) + 2*b*c*(
d + e*x^2)*ArcCsc[c*x]))/(6*c*e) + (b*Sqrt[1 - 1/(c^2*x^2)]*x*(Sqrt[c^2]*Sq
rt[e]*Sqrt[c^2*d + e]*(3*c^2*d + e)*Sqrt[(c^2*(d + e*x^2))/(c^2*d + e)]*Arc
Sinh[(c*Sqrt[e]*Sqrt[-1 + c^2*x^2])/(Sqrt[c^2]*Sqrt[c^2*d + e])] + 2*c^5*d^
(3/2)*Sqrt[d + e*x^2]*ArcTan[(Sqrt[d]*Sqrt[-1 + c^2*x^2])/Sqrt[d + e*x^2]])
)/(6*c^4*e*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

fricas [A] time = 2.84, size = 1098, normalized size = 5.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(2*b*c^3*sqrt(-d)*d*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2
 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*s
qrt(-d) + 8*d^2)/x^4) + (3*b*c^2*d + b*e)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d
^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*
sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(2*a*c^3*e*x^2 + 2*a*c
```

```
^3*d + sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 + b*c^3*d)*arccsc(c*x))*sqr
t(e*x^2 + d))/(c^3*e), -1/24*(4*b*c^3*d^(3/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)
*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 -
d*e)*x^2 - d^2)) - (3*b*c^2*d + b*e)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 -
6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(
c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) - 4*(2*a*c^3*e*x^2 + 2*a*c^3*d
+ sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 + b*c^3*d)*arccsc(c*x))*sqrt(e*x
^2 + d))/(c^3*e), 1/12*(b*c^3*sqrt(-d)*d*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x
^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sq
rt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) - (3*b*c^2*d + b*e)*sqrt(-e)*arctan(1/
2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3
*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(2*a*c^3*e*x^2 + 2*a*c^3*d +
 sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 + b*c^3*d)*arccsc(c*x))*sqrt(e*x^
2 + d))/(c^3*e), -1/12*(2*b*c^3*d^(3/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2
*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x
^2 - d^2)) + (3*b*c^2*d + b*e)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e
)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*
e - c*e^2)*x^2)) - 2*(2*a*c^3*e*x^2 + 2*a*c^3*d + sqrt(c^2*x^2 - 1)*b*c*e +
 2*(b*c^3*e*x^2 + b*c^3*d)*arccsc(c*x))*sqrt(e*x^2 + d))/(c^3*e)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex^2+d}\,(b\operatorname{arccsc}(cx)+a)x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)*x, x)
```

maple [F] time = 5.06, size = 0, normalized size = 0.00

$$\int x\,(a+b\operatorname{arccsc}(cx))\sqrt{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
[Out] int(x*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(ex^2+d\right)^{\frac{3}{2}}a}{3e}+\frac{\left(e\int\frac{\left(c^2ex^3+c^2dx\right)e^{\left(\frac{1}{2}\log\left(ex^2+d\right)+\frac{1}{2}\log(cx+1)+\frac{1}{2}\log(cx-1)\right)}}{c^2ex^2+\left(c^2ex^2-e\right)(cx+1)(cx-1)-e}\,dx+\left(ex^2\arctan\left(1,\sqrt{cx+1}\sqrt{cx-1}\right)+d\arctan\left(1,\sqrt{\right.\right.\right.}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] 1/3*(e*x^2 + d)^(3/2)*a/e + 1/3*(3*e*integrate(1/3*(c^2*e*x^3 + c^2*d*x)*e^
(1/2*log(e*x^2 + d) + 1/2*log(c*x + 1) + 1/2*log(c*x - 1))/(c^2*e*x^2 + (c^
2*e*x^2 - e)*e^(log(c*x + 1) + log(c*x - 1)) - e), x) + (e*x^2*arctan2(1, s
qrt(c*x + 1)*sqrt(c*x - 1)) + d*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*sq
rt(e*x^2 + d))*b/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))),x)
[Out] int(x*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\operatorname{acsc}(cx)\right)\sqrt{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsc(c*x))*(e*x**2+d)**(1/2),x)
[Out] Integral(x*(a + b*acsc(c*x))*sqrt(d + e*x**2), x)
```

3.121 $\int \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{x},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x,x)
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x,x]
[Out] Defer[Int][(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x, x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{x}\,dx=\int \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{x}\,dx$$

Mathematica [A] time = 5.22, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x,x]
[Out] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x, x]
```

fricas [A] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2+d}\,(b\,\text{arccsc}\,(cx)+a)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2+d}\,(b\,\text{arccsc}\,(cx)+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/x, x)
```

maple [A] time = 4.39, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{arccsc}\,(cx))\,\sqrt{e\,x^2 + d}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x,x)
```

```
[Out] int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\left(\sqrt{d}\,\mathrm{arsinh}\left(\frac{d}{\sqrt{de}\,|x|}\right) - \sqrt{ex^2 + d}\right)a + b\int \frac{\sqrt{ex^2 + d}\,\arctan\left(1, \sqrt{cx + 1}\,\sqrt{cx - 1}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -(sqrt(d)*arcsinh(d/(sqrt(d*e)*abs(x))) - sqrt(e*x^2 + d))*a + b*integrate(
sqrt(e*x^2 + d)*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{e\,x^2 + d}\,\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x,x)
```

```
[Out] int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{acsc}\,(cx))\,\sqrt{d + ex^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))*(e*x**2+d)**(1/2)/x,x)
```

```
[Out] Integral((a + b*acsc(c*x))*sqrt(d + e*x**2)/x, x)
```

3.122 $\int \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{x^3},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^3,x)
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^3,x]
[Out] Defer[Int][(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^3, x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{x^3}\,dx=\int \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{x^3}\,dx$$

Mathematica [A] time = 4.82, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^3,x]
[Out] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^3, x]
```

fricas [A] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2+d}\,(b\,\text{arccsc}(cx)+a)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/x^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2+d}\,(b\,\text{arccsc}(cx)+a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="giac")
```

[Out] integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/x^3, x)

maple [A] time = 1.30, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccsc}\,(cx))\,\sqrt{e\,x^2+d}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^3,x)

[Out] int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^3,x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(\frac{e\,\mathrm{arsinh}\left(\frac{d}{\sqrt{de}\,|x|}\right)}{\sqrt{d}}-\frac{\sqrt{ex^2+d}\,e}{d}+\frac{\left(ex^2+d\right)^{\frac{3}{2}}}{dx^2}\right)a+b\int\frac{\sqrt{ex^2+d}\,\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="maxima")

[Out] -1/2*(e*arcsinh(d/(sqrt(d*e)*abs(x)))/sqrt(d) - sqrt(e*x^2 + d)*e/d + (e*x^2 + d)^(3/2)/(d*x^2))*a + b*integrate(sqrt(e*x^2 + d)*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/x^3, x)

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x^3,x)

[Out] int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x^3, x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{acsc}\,(cx))\,\sqrt{d+ex^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*acsc(c*x))*(e*x**2+d)**(1/2)/x**3,x)

[Out] Integral((a + b*acsc(c*x))*sqrt(d + e*x**2)/x**3, x)

3.123 $\int x^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)dx$

Optimal. Leaf size=26

$$\mathrm{Int}\left(x^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable(x^2*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
[Out] Defer[Int][x^2*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]), x]
```

Rubi steps

$$\int x^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)dx=\int x^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)dx$$

Mathematica [A] time = 9.57, size = 0, normalized size = 0.00

$$\int x^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
[Out] Integrate[x^2*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]), x]
```

fricas [A] time = 0.85, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(bx^2\operatorname{arccsc}(cx)+ax^2\right)\sqrt{ex^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b*x^2*arccsc(c*x) + a*x^2)*sqrt(e*x^2 + d), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{ex^2+d}\,(b\operatorname{arccsc}(cx)+a)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)*x^2, x)
```

maple [A] time = 6.37, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\operatorname{arccsc}(cx)\right)\sqrt{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^2*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}\left(\frac{2\left(ex^2+d\right)^{\frac{3}{2}}x}{e}-\frac{\sqrt{ex^2+d}\,dx}{e}-\frac{d^2\operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{e^{\frac{3}{2}}}\right)a+b\int\sqrt{ex^2+d}\,x^2\arctan\left(1,\sqrt{cx+1}\sqrt{cx-1}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*(2*(e*x^2 + d)^(3/2)*x/e - sqrt(e*x^2 + d)*d*x/e - d^2*arcsinh(e*x/sqrt
(d*e))/e^(3/2))*a + b*integrate(sqrt(e*x^2 + d)*x^2*arctan2(1, sqrt(c*x + 1
)*sqrt(c*x - 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int x^2\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))),x)
```

```
[Out] int(x^2*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\operatorname{acsc}\left(cx\right)\right)\sqrt{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsc(c*x))*(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*acsc(c*x))*sqrt(d + e*x**2), x)
```

3.124 $\int \sqrt{d+ex^2} \left(a+b\csc^{-1}(cx)\right) dx$

Optimal. Leaf size=23

$$\text{Int}\left(\sqrt{d+ex^2} \left(a+b\csc^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{d+ex^2} \left(a+b\csc^{-1}(cx)\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
[Out] Defer[Int][Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]), x]
```

Rubi steps

$$\int \sqrt{d+ex^2} \left(a+b\csc^{-1}(cx)\right) dx = \int \sqrt{d+ex^2} \left(a+b\csc^{-1}(cx)\right) dx$$

Mathematica [A] time = 6.67, size = 0, normalized size = 0.00

$$\int \sqrt{d+ex^2} \left(a+b\csc^{-1}(cx)\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
[Out] Integrate[Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]), x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{ex^2+d}\,(b\,\text{arccsc}\,(cx)+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex^2+d}\,(b\,\text{arccsc}\,(cx)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a), x)
```

maple [A] time = 4.38, size = 0, normalized size = 0.00

$$\int (a+b\,\text{arccsc}\,(cx))\,\sqrt{e\,x^2+d}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(\sqrt{ex^2+d}\,x+\frac{d\,\operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{e}}\right)a+b\int\sqrt{ex^2+d}\,\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(e*x^2 + d)*x + d*arcsinh(e*x/sqrt(d*e))/sqrt(e))*a + b*integrate(
sqrt(e*x^2 + d)*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int\sqrt{e\,x^2+d}\,\left(a+b\,\operatorname{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))),x)
```

```
[Out] int((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\,\operatorname{acsc}(cx))\,\sqrt{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))*(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acsc(c*x))*sqrt(d + e*x**2), x)
```

3.125 $\int \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^2,x)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^2,x]
[Out] Defer[Int][(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^2, x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{x^2}\,dx = \int \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{x^2}\,dx$$

Mathematica [A] time = 1.74, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^2,x]
[Out] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^2, x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2+d}\,(b\,\text{arccsc}\,(cx)+a)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2+d}\,(b\,\text{arccsc}\,(cx)+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/x^2, x)
```

maple [A] time = 3.38, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arccsc}(cx)) \sqrt{e x^2 + d}}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^2,x)
```

```
[Out] int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\left(\sqrt{e} \operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right) - \frac{\sqrt{ex^2 + d}}{x}\right)a + b \int \frac{\sqrt{ex^2 + d} \arctan\left(1, \sqrt{cx + 1}\sqrt{cx - 1}\right)}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] (sqrt(e)*arcsinh(e*x/sqrt(d*e)) - sqrt(e*x^2 + d)/x)*a + b*integrate(sqrt(e
*x^2 + d)*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{e x^2 + d} \left(a + b \operatorname{asin}\left(\frac{1}{c x}\right)\right)}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x^2,x)
```

```
[Out] int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acsc}(cx)) \sqrt{d + ex^2}}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))*(e*x**2+d)**(1/2)/x**2,x)
```

```
[Out] Integral((a + b*acsc(c*x))*sqrt(d + e*x**2)/x**2, x)
```

3.126 $\int \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=328

$$-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3dx^3}-\frac{2bc\sqrt{c^2x^2-1}\left(c^2d+2e\right)\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}}-\frac{bc\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}}-\frac{bx\sqrt{1-c^2x^2}\left(c^2d+\right.}{9d}$$

```
[Out] -1/3*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/d/x^3-2/9*b*c*(c^2*d+2*e)*(c^2*x^2-1
)^(1/2)*(e*x^2+d)^(1/2)/d/(c^2*x^2)^(1/2)-1/9*b*c*(c^2*x^2-1)^(1/2)*(e*x^2+
d)^(1/2)/x^2/(c^2*x^2)^(1/2)+2/9*b*c^2*(c^2*d+2*e)*x*EllipticE(c*x,(-e/c^2/
d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^
(1/2)/(1+e*x^2/d)^(1/2)-1/9*b*(c^2*d+e)*(2*c^2*d+3*e)*x*EllipticF(c*x,(-e/c
^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^
2-1)^(1/2)/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {264, 5239, 12, 474, 583, 524, 427, 426, 424, 421, 419}

$$-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3dx^3}-\frac{2bc\sqrt{c^2x^2-1}\left(c^2d+2e\right)\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}}-\frac{bc\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}}-\frac{bx\sqrt{1-c^2x^2}\left(c^2d+\right.}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^4,x]
```

```
[Out] (-2*b*c*(c^2*d + 2*e)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(9*d*Sqrt[c^2*x^2
]) - (b*c*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(9*x^2*Sqrt[c^2*x^2]) - ((d +
 e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/(3*d*x^3) + (2*b*c^2*(c^2*d + 2*e)*x*Sqr
t[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(9*d*S
qrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) - (b*(c^2*d + e)*(2*c^
2*d + 3*e)*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -
(e/(c^2*d))])/(9*d*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
```

```
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{x^4}\,dx = -\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3dx^3} + \frac{(bcx)\int -\frac{\left(d+ex^2\right)^{3/2}}{3dx^4\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}$$

$$= -\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3dx^3} - \frac{(bcx)\int \frac{\left(d+ex^2\right)^{3/2}}{x^4\sqrt{-1+c^2x^2}}\,dx}{3d\sqrt{c^2x^2}}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} - \frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3dx^3} + \frac{(bcx)\int \frac{-2d\left(c^2d+2\right.}{x^2\sqrt{-1+}}}{9d\sqrt{c}}$$

$$= -\frac{2bc\left(c^2d+2e\right)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} - \frac{\left(d+ex^2\right)}{}$$

$$= -\frac{2bc\left(c^2d+2e\right)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} - \frac{\left(d+ex^2\right)}{}$$

$$= -\frac{2bc\left(c^2d+2e\right)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} - \frac{\left(d+ex^2\right)}{}$$

$$= -\frac{2bc\left(c^2d+2e\right)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} - \frac{\left(d+ex^2\right)}{}$$

$$= -\frac{2bc\left(c^2d+2e\right)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} - \frac{\left(d+ex^2\right)}{}$$

Mathematica [C] time = 0.65, size = 247, normalized size = 0.75

$$-\frac{\sqrt{d+ex^2}\left(3a\left(d+ex^2\right)+bcx\sqrt{1-\frac{1}{c^2x^2}}\left(2c^2dx^2+d+4ex^2\right)+3b\csc^{-1}(cx)\left(d+ex^2\right)\right)}{9dx^3} + ibcx\sqrt{1-\frac{1}{c^2x^2}}\sqrt{\frac{ex^2}{d}+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^4,x]
```

```
[Out] -1/9*(Sqrt[d + e*x^2]*(3*a*(d + e*x^2) + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(d + 2
*c^2*d*x^2 + 4*e*x^2) + 3*b*(d + e*x^2)*ArcCsc[c*x]))/(d*x^3) + ((I/9)*b*c*
Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(2*c^2*d*(c^2*d + 2*e)*Elliptic
E[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))] - (2*c^4*d^2 + 5*c^2*d*e + 3*e^2)*
EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-c^2]*d*Sqrt[1 - c
^2*x^2]*Sqrt[d + e*x^2])
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2+d}\,(b\,\text{arccsc}\,(cx)+a)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2 + d}\,(b\,\mathrm{arccsc}\,(cx) + a)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/x^4, x)
```

maple [F] time = 6.42, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{arccsc}\,(cx))\,\sqrt{e\,x^2 + d}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^4,x)
[Out] int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(ex^2 + d\right)^{\frac{3}{2}} a}{3\,dx^3} - \frac{\left(dx^3 \int \frac{\left(c^2ex^2+c^2d\right)e^{\left(\frac{1}{2}\log\left(ex^2+d\right)+\frac{1}{2}\log(cx+1)+\frac{1}{2}\log(cx-1)\right)}}{c^2dx^4+\left(c^2dx^4-dx^2\right)(cx+1)(cx-1)-dx^2}\,dx + \left(ex^2 \arctan\left(1, \sqrt{cx+1}\,\sqrt{cx-1}\right) + d\,\mathrm{arcta}\right.}{3\,dx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="maxima")
[Out] -1/3*(e*x^2 + d)^(3/2)*a/(d*x^3) - 1/3*(3*d*x^3*integrate(1/3*(c^2*e*x^2 +
c^2*d)*e^(1/2*log(e*x^2 + d) + 1/2*log(c*x + 1) + 1/2*log(c*x - 1))/(c^2*d*
x^4 - d*x^2 + (c^2*d*x^4 - d*x^2)*e^(log(c*x + 1) + log(c*x - 1))), x) + (e
*x^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + d*arctan2(1, sqrt(c*x + 1)*s
qrt(c*x - 1)))*sqrt(e*x^2 + d))*b/(d*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,x^2 + d}\,\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x^4,x)
[Out] int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{acsc}\,(cx))\,\sqrt{d + ex^2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))*(e*x**2+d)**(1/2)/x**4,x)
[Out] Integral((a + b*acsc(c*x))*sqrt(d + e*x**2)/x**4, x)
```

3.127 $\int \frac{\sqrt{d+ex^2} \left(a+b \csc^{-1}(cx)\right)}{x^6} \, dx$

Optimal. Leaf size=453

$$\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{5dx^5}-\frac{bc\sqrt{c^2x^2-1}\left(12c^2d-e\right)\sqrt{d+ex^2}}{225dx^2\sqrt{c^2x^2}}-\frac{bc\sqrt{c^2x^2-1}}{25dx^4}$$

```
[Out] -1/5*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/d/x^5+2/15*e*(e*x^2+d)^(3/2)*(a+b*ar
ccsc(c*x))/d^2/x^3+2/15*b*c*e^2*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*
x^2)^(1/2)-1/45*b*c*e*(2*c^2*d+e)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^
2*x^2)^(1/2)-1/75*b*c*(8*c^4*d^2+3*c^2*d*e-2*e^2)*(c^2*x^2-1)^(1/2)*(e*x^2+
d)^(1/2)/d^2/(c^2*x^2)^(1/2)-1/25*b*c*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/x^4
/(c^2*x^2)^(1/2)-1/45*b*c*e*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/x^2/(c^2*x^
2)^(1/2)-1/75*b*c*(4*c^2*d+e)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/x^2/(c^2*
x^2)^(1/2)-2/15*b*c^2*e^2*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1
/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)
+1/45*b*c^2*e*(2*c^2*d+e)*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1
/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)
+1/75*b*c^2*(8*c^4*d^2+3*c^2*d*e-2*e^2)*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(
-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+
e*x^2/d)^(1/2)-1/75*b*c^2*(8*c^2*d-e)*(c^2*d+e)*x*EllipticF(c*x,(-e/c^2/d)^
(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(
1/2)/(e*x^2+d)^(1/2)-2/45*b*c^2*e*(c^2*d+e)*x*EllipticF(c*x,(-e/c^2/d)^(1/2
))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)
/(e*x^2+d)^(1/2)+2/15*b*e^2*(c^2*d+e)*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(-c
^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*
x^2+d)^(1/2)
```

Rubi [A] time = 0.62, antiderivative size = 453, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {271, 264, 5239, 12, 580, 583, 524, 427, 426, 424, 421, 419}

$$\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{5dx^5}-\frac{bc\sqrt{c^2x^2-1}\left(24c^4d^2+19c^2de-31e^2\right)\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^6,x]
```

```
[Out] -(b*c*(24*c^4*d^2 + 19*c^2*d*e - 31*e^2)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2]
)/(225*d^2*Sqrt[c^2*x^2]) - (b*c*(12*c^2*d - e)*Sqrt[-1 + c^2*x^2]*Sqrt[d +
 e*x^2])/(225*d*x^2*Sqrt[c^2*x^2]) - (b*c*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3
/2))/(25*d*x^4*Sqrt[c^2*x^2]) - ((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/(5*
d*x^5) + (2*e*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/(15*d^2*x^3) + (b*c^2*
(24*c^4*d^2 + 19*c^2*d*e - 31*e^2)*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*Elli
pticE[ArcSin[c*x], -(e/(c^2*d))])/(225*d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]
*Sqrt[1 + (e*x^2)/d]) - (b*(c^2*d + e)*(24*c^4*d^2 + 7*c^2*d*e - 30*e^2)*x*
Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])
/(225*d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
```

```
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}\,\left(a+b\csc^{-1}(cx)\right)}{x^6}\,dx = -\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{5dx^5} + \frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{15d^2x^3} + \frac{(bcx)}{}$$

$$= -\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{5dx^5} + \frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{15d^2x^3} + \frac{(bcx)}{}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\,\left(d+ex^2\right)^{3/2}}{25dx^4\sqrt{c^2x^2}} - \frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{5dx^5} + \frac{2e\left(d+ex^2\right.}{}$$

$$= -\frac{bc\left(12c^2d-e\right)\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{225dx^2\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}\,\left(d+ex^2\right)^{3/2}}{25dx^4\sqrt{c^2x^2}} - \frac{\left(d\right.}{}$$

$$= -\frac{bc\left(24c^4d^2+19c^2de-31e^2\right)\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}} - \frac{bc\left(12c^2d-e\right)\sqrt{-1}}{225dx^2}$$

$$= -\frac{bc\left(24c^4d^2+19c^2de-31e^2\right)\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}} - \frac{bc\left(12c^2d-e\right)\sqrt{-1}}{225dx^2}$$

$$= -\frac{bc\left(24c^4d^2+19c^2de-31e^2\right)\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}} - \frac{bc\left(12c^2d-e\right)\sqrt{-1}}{225dx^2}$$

$$= -\frac{bc\left(24c^4d^2+19c^2de-31e^2\right)\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}} - \frac{bc\left(12c^2d-e\right)\sqrt{-1}}{225dx^2}$$

$$= -\frac{bc\left(24c^4d^2+19c^2de-31e^2\right)\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}} - \frac{bc\left(12c^2d-e\right)\sqrt{-1}}{225dx^2}$$

Mathematica [C] time = 0.72, size = 325, normalized size = 0.72

$$-\frac{\sqrt{d+ex^2}\,\left(15a\left(3d^2+dex^2-2e^2x^4\right)+bcx\sqrt{1-\frac{1}{c^2x^2}}\,\left(dex^2\left(19c^2x^2+8\right)+3d^2\left(8c^4x^4+4c^2x^2+3\right)-31e^2x^4\right)\right.}{225d^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^6,x]
```

```
[Out] -1/225*(Sqrt[d + e*x^2]*(15*a*(3*d^2 + d*e*x^2 - 2*e^2*x^4) + b*c*Sqrt[1 -
1/(c^2*x^2)]*x*(-31*e^2*x^4 + d*e*x^2*(8 + 19*c^2*x^2) + 3*d^2*(3 + 4*c^2*x
^2 + 8*c^4*x^4)) + 15*b*(3*d^2 + d*e*x^2 - 2*e^2*x^4)*ArcCsc[c*x]))/(d^2*x^
5) + ((I/225)*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(24*c^
4*d^2 + 19*c^2*d*e - 31*e^2)*EllipticE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d)
)] + (-24*c^6*d^3 - 31*c^4*d^2*e + 23*c^2*d*e^2 + 30*e^3)*EllipticF[I*ArcSi
nh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-c^2]*d^2*Sqrt[1 - c^2*x^2]*Sqrt[d
+ e*x^2])
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2+d}\,(b\,\text{arccsc}\,(cx)+a)}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2+d}\,(b\,\mathrm{arccsc}\,(cx)+a)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/x^6, x)
```

maple [F] time = 8.95, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccsc}\,(cx))\,\sqrt{e\,x^2+d}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^6,x)
```

```
[Out] int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^6,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{15}\,a\left(\frac{2\left(ex^2+d\right)^{\frac{3}{2}}e}{d^2x^3}-\frac{3\left(ex^2+d\right)^{\frac{3}{2}}}{dx^5}\right)+\frac{\left(d^2x^5\int\frac{\left(2\,c^2e^2x^4-c^2dex^2-3\,c^2d^2\right)e^{\left(\frac{1}{2}\,\log\left(ex^2+d\right)+\frac{1}{2}\,\log(cx+1)+\frac{1}{2}\,\log(cx-1)\right)}}{c^2d^2x^6-d^2x^4+\left(c^2d^2x^6-d^2x^4\right)(cx+1)(cx-1)}\,dx+\left(2\,e^2x^4\,\mathrm{arctan}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] 1/15*a*(2*(e*x^2 + d)^(3/2)*e/(d^2*x^3) - 3*(e*x^2 + d)^(3/2)/(d*x^5)) + 1/
15*(15*d^2*x^5*integrate(1/15*(2*c^2*e^2*x^4 - c^2*d*e*x^2 - 3*c^2*d^2)*e^(
1/2*log(e*x^2 + d) + 1/2*log(c*x + 1) + 1/2*log(c*x - 1))/(c^2*d^2*x^6 - d^
2*x^4 + (c^2*d^2*x^6 - d^2*x^4)*e^(log(c*x + 1) + log(c*x - 1))), x) + (2*e
^2*x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) - d*e*x^2*arctan2(1, sqrt(c*
x + 1)*sqrt(c*x - 1)) - 3*d^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*sqrt
(e*x^2 + d))*b/(d^2*x^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x^6,x)
```

```
[Out] int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{acsc}\,(cx))\,\sqrt{d+ex^2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))*(e*x**2+d)**(1/2)/x**6,x)
```

```
[Out] Integral((a + b*acsc(c*x))*sqrt(d + e*x**2)/x**6, x)
```

3.128 $\int x^3 \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right) dx$

Optimal. Leaf size=374

$$-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^2}+\frac{2bcd^{7/2}x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{35e^2\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2x^2-1}\left(d+\right.}{42ce\sqrt{c^2x^2}}$$

```
[Out] -1/5*d*(e*x^2+d)^(5/2)*(a+b*arccsc(c*x))/e^2+1/7*(e*x^2+d)^(7/2)*(a+b*arccs
c(c*x))/e^2+2/35*b*c*d^(7/2)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(
1/2))/e^2/(c^2*x^2)^(1/2)-1/560*b*(35*c^6*d^3-35*c^4*d^2*e-63*c^2*d*e^2-25*
e^3)*x*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^6/e^(3/2)/(c^
2*x^2)^(1/2)+1/840*b*(13*c^2*d+25*e)*x*(e*x^2+d)^(3/2)*(c^2*x^2-1)^(1/2)/c^
3/e/(c^2*x^2)^(1/2)+1/42*b*x*(e*x^2+d)^(5/2)*(c^2*x^2-1)^(1/2)/c/e/(c^2*x^2
)^(1/2)-1/560*b*(3*c^4*d^2-38*c^2*d*e-25*e^2)*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)
^(1/2)/c^5/e/(c^2*x^2)^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {266, 43, 5239, 12, 573, 154, 157, 63, 217, 206, 93, 204}

$$-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^2}-\frac{bx\sqrt{c^2x^2-1}\left(3c^4d^2-38c^2de-25e^2\right)\sqrt{d+ex^2}}{560c^5e\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] -(b*(3*c^4*d^2 - 38*c^2*d*e - 25*e^2)*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
/(560*c^5*e*Sqrt[c^2*x^2]) + (b*(13*c^2*d + 25*e)*x*Sqrt[-1 + c^2*x^2]*(d +
 e*x^2)^(3/2))/(840*c^3*e*Sqrt[c^2*x^2]) + (b*x*Sqrt[-1 + c^2*x^2]*(d + e*x
^2)^(5/2))/(42*c*e*Sqrt[c^2*x^2]) - (d*(d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]
))/(5*e^2) + ((d + e*x^2)^(7/2)*(a + b*ArcCsc[c*x]))/(7*e^2) + (2*b*c*d^(7/
2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(35*e^2*Sqrt[c^2
*x^2]) - (b*(35*c^6*d^3 - 35*c^4*d^2*e - 63*c^2*d*e^2 - 25*e^3)*x*ArcTanh[(
Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(560*c^6*e^(3/2)*Sqrt[c^2
*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 573

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int x^3 \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right) dx = -\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2} + \frac{\left(d+ex^2\right)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^2} + \frac{(bcx}{}$$

$$= -\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2} + \frac{\left(d+ex^2\right)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^2} + \frac{(bcx}{}$$

$$= -\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2} + \frac{\left(d+ex^2\right)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^2} + \frac{(bcx}{}$$

$$= \frac{bx\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{5/2}}{42ce\sqrt{c^2x^2}} - \frac{d\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2} + \frac{\left(d+ex^2\right.}{}$$

$$= \frac{b\left(13c^2d+25e\right)x\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}{840c^3e\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{5/2}}{42ce\sqrt{c^2x^2}}$$

$$= -\frac{b\left(3c^4d^2-38c^2de-25e^2\right)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{c^2x^2}} + \frac{b\left(13c^2d+25e\right)x\sqrt{}}{840c}$$

$$= -\frac{b\left(3c^4d^2-38c^2de-25e^2\right)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{c^2x^2}} + \frac{b\left(13c^2d+25e\right)x\sqrt{}}{840c}$$

$$= -\frac{b\left(3c^4d^2-38c^2de-25e^2\right)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{c^2x^2}} + \frac{b\left(13c^2d+25e\right)x\sqrt{}}{840c}$$

$$= -\frac{b\left(3c^4d^2-38c^2de-25e^2\right)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{c^2x^2}} + \frac{b\left(13c^2d+25e\right)x\sqrt{}}{840c}$$

$$= -\frac{b\left(3c^4d^2-38c^2de-25e^2\right)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{c^2x^2}} + \frac{b\left(13c^2d+25e\right)x\sqrt{}}{840c}$$

Mathematica [C] time = 0.62, size = 339, normalized size = 0.91

$$\frac{\sqrt{d+ex^2}\left(-48ac^5\left(2d-5ex^2\right)\left(d+ex^2\right)^2-48bc^5\csc^{-1}(cx)\left(2d-5ex^2\right)\left(d+ex^2\right)^2+bex\sqrt{1-\frac{1}{c^2x^2}}\left(c^4\left(57d^2+10\right.\right.\right.}{1680c^5e^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*(32*c^4*d^4*Sqrt[1 + d/(e*x^2)]*(-1 + c^2*x^2)*AppellF1[1, 1/2, 1/2, 2,
1/(c^2*x^2), -(d/(e*x^2))] + e*(-35*c^6*d^3 + 35*c^4*d^2*e + 63*c^2*d*e^2 +
 25*e^3)*Sqrt[1 - 1/(c^2*x^2)]*x^4*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*Ap
pellF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/d)]))/(1120*c^5*e^2*x*(-1 + c^2*x
^2)*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(-48*a*c^5*(2*d - 5*e*x^2)*(d + e*x
^2)^2 + b*e*Sqrt[1 - 1/(c^2*x^2)]*x*(75*e^2 + 2*c^2*e*(82*d + 25*e*x^2) + c
^4*(57*d^2 + 106*d*e*x^2 + 40*e^2*x^4)) - 48*b*c^5*(2*d - 5*e*x^2)*(d + e*x
^2)^2*ArcCsc[c*x]))/(1680*c^5*e^2)
```

fricas [A] time = 9.25, size = 1697, normalized size = 4.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] [1/6720*(96*b*c^7*sqrt(-d)*d^3*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^
2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 +
 d)*sqrt(-d) + 8*d^2)/x^4) - 3*(35*b*c^6*d^3 - 35*b*c^4*d^2*e - 63*b*c^2*d*
e^2 - 25*b*e^3)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*
e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x
^2 + d)*sqrt(e) + e^2) + 4*(240*a*c^7*e^3*x^6 + 384*a*c^7*d*e^2*x^4 + 48*a*
c^7*d^2*e*x^2 - 96*a*c^7*d^3 + 48*(5*b*c^7*e^3*x^6 + 8*b*c^7*d*e^2*x^4 + b*
c^7*d^2*e*x^2 - 2*b*c^7*d^3)*arccsc(c*x) + (40*b*c^5*e^3*x^4 + 57*b*c^5*d^2
*e + 164*b*c^3*d*e^2 + 75*b*c*e^3 + 2*(53*b*c^5*d*e^2 + 25*b*c^3*e^3)*x^2)*
sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^7*e^2), 1/6720*(192*b*c^7*d^(7/2)*ar
ctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)
/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) - 3*(35*b*c^6*d^3 - 35*b*c^4*d^
2*e - 63*b*c^2*d*e^2 - 25*b*e^3)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^
2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*
x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(240*a*c^7*e^3*x^6 + 384*a*c^7*
d*e^2*x^4 + 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 + 48*(5*b*c^7*e^3*x^6 + 8*b*c
^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*arccsc(c*x) + (40*b*c^5*e^3*x
^4 + 57*b*c^5*d^2*e + 164*b*c^3*d*e^2 + 75*b*c*e^3 + 2*(53*b*c^5*d*e^2 + 25
*b*c^3*e^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^7*e^2), 1/3360*(48*
b*c^7*sqrt(-d)*d^3*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)
*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d)
 + 8*d^2)/x^4) + 3*(35*b*c^6*d^3 - 35*b*c^4*d^2*e - 63*b*c^2*d*e^2 - 25*b*e
^3)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*
x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(240*a
*c^7*e^3*x^6 + 384*a*c^7*d*e^2*x^4 + 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 + 48
*(5*b*c^7*e^3*x^6 + 8*b*c^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*arcc
sc(c*x) + (40*b*c^5*e^3*x^4 + 57*b*c^5*d^2*e + 164*b*c^3*d*e^2 + 75*b*c*e^3
 + 2*(53*b*c^5*d*e^2 + 25*b*c^3*e^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d
))/(c^7*e^2), 1/3360*(96*b*c^7*d^(7/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*
d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^
2 - d^2)) + 3*(35*b*c^6*d^3 - 35*b*c^4*d^2*e - 63*b*c^2*d*e^2 - 25*b*e^3)*s
qrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 +
 d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(240*a*c^7*
e^3*x^6 + 384*a*c^7*d*e^2*x^4 + 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 + 48*(5*b
*c^7*e^3*x^6 + 8*b*c^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*arccsc(c*
x) + (40*b*c^5*e^3*x^4 + 57*b*c^5*d^2*e + 164*b*c^3*d*e^2 + 75*b*c*e^3 + 2*
(53*b*c^5*d*e^2 + 25*b*c^3*e^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c
^7*e^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2 + d\right)^{\frac{3}{2}} (b \operatorname{arccsc}(cx) + a) x^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)*x^3, x)
```

maple [F] time = 6.43, size = 0, normalized size = 0.00

$$\int x^{3} \left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\,\mathrm{arccsc}\left(c x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

```
[Out] int(x^3*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{35}\left(\frac{5\left(ex^{2}+d\right)^{\frac{5}{2}}x^{2}}{e}-\frac{2\left(ex^{2}+d\right)^{\frac{5}{2}}d}{e^{2}}\right)a+\frac{\left(e^{2}\int\frac{\left(5\,c^{2}e^{3}x^{7}+8\,c^{2}de^{2}x^{5}+c^{2}d^{2}ex^{3}-2\,c^{2}d^{3}x\right)e^{\left(\frac{1}{2}\log\left(ex^{2}+d\right)+\frac{1}{2}\log(cx+1)+\frac{1}{2}\log(cx-1)\right)}}{c^{2}e^{2}x^{2}+\left(c^{2}e^{2}x^{2}-e^{2}\right)(cx+1)(cx-1)-e^{2}}\,dx+\left(5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/35*(5*(e*x^2 + d)^(5/2)*x^2/e - 2*(e*x^2 + d)^(5/2)*d/e^2)*a + 1/35*(35*e
^2*integrate(1/35*(5*c^2*e^3*x^7 + 8*c^2*d*e^2*x^5 + c^2*d^2*e*x^3 - 2*c^2*
d^3*x)*e^(1/2*log(e*x^2 + d) + 1/2*log(c*x + 1) + 1/2*log(c*x - 1))/(c^2*e^
2*x^2 - e^2 + (c^2*e^2*x^2 - e^2)*e^(log(c*x + 1) + log(c*x - 1))), x) + (5
*e^3*x^6*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 8*d*e^2*x^4*arctan2(1, s
qrt(c*x + 1)*sqrt(c*x - 1)) + d^2*e*x^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x -
 1)) - 2*d^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*sqrt(e*x^2 + d))*b/e^
2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,{\left(e\,x^2+d\right)}^{3/2}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))),x)
```

```
[Out] int(x^3*(d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**(3/2)*(a+b*acsc(c*x)),x)
```

```
[Out] Timed out
```

3.129 $\int x\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx$

Optimal. Leaf size=262

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}-\frac{bcd^{5/2}x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{5e\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{20c\sqrt{c^2x^2}}+\frac{bx\left(15c^4d^2+10c^2de+3\right.}{40c^4\sqrt{e}}$$

```
[Out] 1/5*(e*x^2+d)^(5/2)*(a+b*arccsc(c*x))/e-1/5*b*c*d^(5/2)*x*arctan((e*x^2+d)^
(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e/(c^2*x^2)^(1/2)+1/40*b*(15*c^4*d^2+10*c^
2*d*e+3*e^2)*x*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^4/e^(
1/2)/(c^2*x^2)^(1/2)+1/20*b*x*(e*x^2+d)^(3/2)*(c^2*x^2-1)^(1/2)/c/(c^2*x^2)
^(1/2)+1/40*b*(7*c^2*d+3*e)*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c^3/(c^2*x^
2)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {5237, 446, 102, 154, 157, 63, 217, 206, 93, 204}

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}+\frac{bx\left(15c^4d^2+10c^2de+3e^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{40c^4\sqrt{e}\sqrt{c^2x^2}}-\frac{bcd^{5/2}x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{5e\sqrt{c^2x^2}}+\frac{b}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*(7*c^2*d + 3*e)*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(40*c^3*Sqrt[c^2*x
^2]) + (b*x*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3/2))/(20*c*Sqrt[c^2*x^2]) + ((
d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]))/(5*e) - (b*c*d^(5/2)*x*ArcTan[Sqrt[d
+ e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(5*e*Sqrt[c^2*x^2]) + (b*(15*c^4*d^
2 + 10*c^2*d*e + 3*e^2)*x*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d +
e*x^2])])/(40*c^4*Sqrt[e]*Sqrt[c^2*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5237

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCsc[c*x]))/(2*e*(p + 1)), x
] + Dist[(b*c*x)/(2*e*(p + 1)*Sqrt[c^2*x^2]), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)dx = \frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e} + \frac{(bcx)\int\frac{\left(d+ex^2\right)^{5/2}}{x\sqrt{-1+c^2x^2}}dx}{5e\sqrt{c^2x^2}}$$

$$= \frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e} + \frac{(bcx)\operatorname{Subst}\left(\int\frac{(d+ex)^{5/2}}{x\sqrt{-1+c^2x}}dx, x, x^2\right)}{10e\sqrt{c^2x^2}}$$

$$= \frac{bx\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}{20c\sqrt{c^2x^2}} + \frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e} + \frac{(bx)\operatorname{Sul}}{}$$

$$= \frac{b\left(7c^2d+3e\right)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}{20c\sqrt{c^2x^2}} +$$

$$= \frac{b\left(7c^2d+3e\right)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}{20c\sqrt{c^2x^2}} +$$

$$= \frac{b\left(7c^2d+3e\right)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}{20c\sqrt{c^2x^2}} +$$

$$= \frac{b\left(7c^2d+3e\right)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}{20c\sqrt{c^2x^2}} +$$

$$= \frac{b\left(7c^2d+3e\right)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}{20c\sqrt{c^2x^2}} +$$

Mathematica [A] time = 0.76, size = 304, normalized size = 1.16

$$\frac{\sqrt{d+ex^2}\left(8ac^3\left(d+ex^2\right)^2+8bc^3\csc^{-1}(cx)\left(d+ex^2\right)^2+bex\sqrt{1-\frac{1}{c^2x^2}}\left(c^2\left(9d+2ex^2\right)+3e\right)\right)}{40c^3e} + \frac{bx\sqrt{1-\frac{1}{c^2x^2}}\left(8\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (Sqrt[d + e*x^2]*(8*a*c^3*(d + e*x^2)^2 + b*e*Sqrt[1 - 1/(c^2*x^2)]*x*(3*e
+ c^2*(9*d + 2*e*x^2)) + 8*b*c^3*(d + e*x^2)^2*ArcCsc[c*x]))/(40*c^3*e) + (
b*Sqrt[1 - 1/(c^2*x^2)]*x*(Sqrt[c^2]*Sqrt[e]*Sqrt[c^2*d + e]*(15*c^4*d^2 +
10*c^2*d*e + 3*e^2)*Sqrt[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcSinh[(c*Sqrt[e]*
Sqrt[-1 + c^2*x^2])/(Sqrt[c^2]*Sqrt[c^2*d + e])] + 8*c^7*d^(5/2)*Sqrt[d + e
*x^2]*ArcTan[(Sqrt[d]*Sqrt[-1 + c^2*x^2])/Sqrt[d + e*x^2]]))/(40*c^6*e*Sqrt
[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

fricas [A] time = 4.50, size = 1375, normalized size = 5.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] [1/160*(8*b*c^5*sqrt(-d)*d^2*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*
d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d
)*sqrt(-d) + 8*d^2)/x^4) + (15*b*c^4*d^2 + 10*b*c^2*d*e + 3*b*e^2)*sqrt(e)*
log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*
c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) +
 4*(8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 + 8*(b*c^5*e^2*x^4 + 2
*b*c^5*d*e*x^2 + b*c^5*d^2)*arccsc(c*x) + (2*b*c^3*e^2*x^2 + 9*b*c^3*d*e +
3*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e), -1/160*(16*b*c^5*d^
(5/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)
*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) - (15*b*c^4*d^2 + 10*b*
c^2*d*e + 3*b*e^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4
*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(
e*x^2 + d)*sqrt(e) + e^2) - 4*(8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5
*d^2 + 8*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2 + b*c^5*d^2)*arccsc(c*x) + (2*b*c
^3*e^2*x^2 + 9*b*c^3*d*e + 3*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(
c^5*e), 1/80*(4*b*c^5*sqrt(-d)*d^2*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8
*(c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x
^2 + d)*sqrt(-d) + 8*d^2)/x^4) - (15*b*c^4*d^2 + 10*b*c^2*d*e + 3*b*e^2)*sq
rt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 +
d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(8*a*c^5*e^2
*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 + 8*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2
+ b*c^5*d^2)*arccsc(c*x) + (2*b*c^3*e^2*x^2 + 9*b*c^3*d*e + 3*b*c*e^2)*sqrt
(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e), -1/80*(8*b*c^5*d^(5/2)*arctan(-1/2
*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e
*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (15*b*c^4*d^2 + 10*b*c^2*d*e + 3*b*e^2
)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^
2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) - 2*(8*a*c^5
*e^2*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 + 8*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*
x^2 + b*c^5*d^2)*arccsc(c*x) + (2*b*c^3*e^2*x^2 + 9*b*c^3*d*e + 3*b*c*e^2)*
sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2+d\right)^{\frac{3}{2}}(b\operatorname{arccsc}(cx)+a)x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)*x, x)
```

maple [F] time = 4.74, size = 0, normalized size = 0.00

$$\int x\left(e\,x^2+d\right)^{\frac{3}{2}}\left(a+b\operatorname{arccsc}(cx)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

```
[Out] int(x*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(ex^2+d\right)^{\frac{5}{2}}a}{5\,e}+\frac{\left(e\int\frac{\left(c^2e^2x^5+2\,c^2dex^3+c^2d^2x\right)e^{\left(\frac{1}{2}\log\left(ex^2+d\right)+\frac{1}{2}\log(cx+1)+\frac{1}{2}\log(cx-1)\right)}}{c^2ex^2+\left(c^2ex^2-e\right)(cx+1)(cx-1)-e}\,dx+\left(e^2x^4\arctan\left(1,\sqrt{cx+1}\sqrt{cx-1}\right)+2\,de\right.\right.}{5\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*(e*x^2 + d)^(5/2)*a/e + 1/5*(5*e*integrate(1/5*(c^2*e^2*x^5 + 2*c^2*d*e
*x^3 + c^2*d^2*x)*e^(1/2*log(e*x^2 + d) + 1/2*log(c*x + 1) + 1/2*log(c*x -
1))/(c^2*e*x^2 + (c^2*e*x^2 - e)*e^(log(c*x + 1) + log(c*x - 1)) - e), x) +
 (e^2*x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 2*d*e*x^2*arctan2(1, sq
rt(c*x + 1)*sqrt(c*x - 1)) + d^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*s
qrt(e*x^2 + d))*b/e
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(e\,x^2+d\right)^{3/2}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))),x)
```

```
[Out] int(x*(d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**(3/2)*(a+b*acsc(c*x)),x)
```

```
[Out] Timed out
```

3.130 $\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=26

$$\mathrm{Int}\left(\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x},x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x,x)
```

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x,x]
[Out] Defer[Int][((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x, x]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x}\,dx=\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x}\,dx$$

Mathematica [A] time = 6.09, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x,x]
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x, x]
```

fricas [A] time = 0.88, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(aex^2+ad+\left(bex^2+bd\right)\operatorname{arccsc}(cx)\right)\sqrt{ex^2+d}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x,x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsc(c*x))*sqrt(e*x^2 + d)/x, x
)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}\left(b\operatorname{arccsc}(cx)+a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)/x, x)
```

maple [A] time = 3.98, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\,\mathrm{arccsc}\left(c x\right)\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x,x)
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}\left(3\,d^{\frac{3}{2}}\,\mathrm{arsinh}\left(\frac{d}{\sqrt{de}\,|x|}\right)-\left(ex^{2}+d\right)^{\frac{3}{2}}-3\,\sqrt{ex^{2}+d}\,d\right)a+\left(e\int\sqrt{ex^{2}+d}\,x\,\mathrm{arctan}\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right)\,dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x,x, algorithm="maxima")
[Out] -1/3*(3*d^(3/2)*arcsinh(d/(sqrt(d*e)*abs(x))) - (e*x^2 + d)^(3/2) - 3*sqrt(
e*x^2 + d)*d)*a + (e*integrate(sqrt(e*x^2 + d)*x*arctan2(1, sqrt(c*x + 1)*s
qrt(c*x - 1)), x) + d*integrate(sqrt(e*x^2 + d)*arctan2(1, sqrt(c*x + 1)*sq
rt(c*x - 1))/x, x))*b
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(e\,x^2+d\right)^{3/2}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x,x)
[Out] int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{acsc}{\left(c x \right)}\right) \left(d + e x^{2}\right)^{\frac{3}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsc(c*x))/x,x)
[Out] Integral((a + b*acsc(c*x))*(d + e*x**2)**(3/2)/x, x)
```

3.131 $\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x^3},x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^3,x)
```

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^3,x]
[Out] Defer[Int][((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^3, x]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x^3}\,dx=\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x^3}\,dx$$

Mathematica [A] time = 6.06, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^3,x]
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^3, x]
```

fricas [A] time = 2.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(aex^2+ad+\left(bex^2+bd\right)\text{arccsc}\left(cx\right)\right)\sqrt{ex^2+d}}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^3,x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsc(c*x))*sqrt(e*x^2 + d)/x^3,
 x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}\left(b\,\text{arccsc}\left(cx\right)+a\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)/x^3, x)
```

maple [A] time = 1.07, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\,\mathrm{arccsc}\left(c x\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^3,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(3\,\sqrt{d}\,e\,\mathrm{arsinh}\left(\frac{d}{\sqrt{de}\,|x|}\right)-3\,\sqrt{ex^{2}+d}\,e-\frac{\left(ex^{2}+d\right)^{\frac{3}{2}}e}{d}+\frac{\left(ex^{2}+d\right)^{\frac{5}{2}}}{dx^{2}}\right)a+\left(e\int\frac{\sqrt{ex^{2}+d}\;\arctan\left(1,\sqrt{cx+1}\right.}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(3*sqrt(d)*e*arcsinh(d/(sqrt(d*e)*abs(x))) - 3*sqrt(e*x^2 + d)*e - (e*
x^2 + d)^(3/2)*e/d + (e*x^2 + d)^(5/2)/(d*x^2))*a + (e*integrate(sqrt(e*x^2
 + d)*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/x, x) + d*integrate(sqrt(e*x^
2 + d)*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/x^3, x))*b
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{{\left(e\,x^2+d\right)}^{3/2}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^3,x)
```

```
[Out] int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^3, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{acsc}\left(cx\right)\right)\left(d+ex^{2}\right)^{\frac{3}{2}}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsc(c*x))/x**3,x)
```

```
[Out] Integral((a + b*acsc(c*x))*(d + e*x**2)**(3/2)/x**3, x)
```

3.132 $\int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx$

Optimal. Leaf size=26

$$\text{Int}\left(x^2 \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable(x^2*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
[Out] Defer[Int][x^2*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]), x]
```

Rubi steps

$$\int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx=\int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx$$

Mathematica [A] time = 9.91, size = 0, normalized size = 0.00

$$\int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
[Out] Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]), x]
```

fricas [A] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(aex^4+adx^2+\left(bex^4+bdx^2\right)\operatorname{arccsc}\left(cx\right)\right)\sqrt{ex^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="fricas")
[Out] integral((a*e*x^4 + a*d*x^2 + (b*e*x^4 + b*d*x^2)*arccsc(c*x))*sqrt(e*x^2 +
 d), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2+d\right)^{\frac{3}{2}}(b\operatorname{arccsc}\left(cx\right)+a)x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)*x^2, x)
```

maple [A] time = 5.86, size = 0, normalized size = 0.00

$$\int x^{2} \left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\,\mathrm{arccsc}\left(cx\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

```
[Out] int(x^2*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{48}\left(\frac{8\left(ex^{2}+d\right)^{\frac{5}{2}}x}{e}-\frac{2\left(ex^{2}+d\right)^{\frac{3}{2}}dx}{e}-\frac{3\sqrt{ex^{2}+d}\,d^{2}x}{e}-\frac{3\,d^{3}\,\mathrm{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{e^{\frac{3}{2}}}\right)a+b\int\left(ex^{4}\arctan\left(1,\sqrt{cx+1}\,\sqrt{c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/48*(8*(e*x^2 + d)^(5/2)*x/e - 2*(e*x^2 + d)^(3/2)*d*x/e - 3*sqrt(e*x^2 +
d)*d^2*x/e - 3*d^3*arcsinh(e*x/sqrt(d*e))/e^(3/2))*a + b*integrate((e*x^4*a
rctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + d*x^2*arctan2(1, sqrt(c*x + 1)*sqr
t(c*x - 1)))*sqrt(e*x^2 + d), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int x^{2} \left(e\,x^{2}+d\right)^{3/2} \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))),x)
```

```
[Out] int(x^2*(d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**(3/2)*(a+b*acsc(c*x)),x)
```

```
[Out] Timed out
```

3.133 $\int \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx$

Optimal. Leaf size=23

$$\text{Int}\left(\left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
[Out] Defer[Int][(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]), x]
```

Rubi steps

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx = \int \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx$$

Mathematica [A] time = 8.47, size = 0, normalized size = 0.00

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
[Out] Integrate[(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]), x]
```

fricas [A] time = 1.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(aex^2+ad+\left(bex^2+bd\right)\text{arccsc}\,(cx)\right)\sqrt{ex^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsc(c*x))*sqrt(e*x^2 + d), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2+d\right)^{\frac{3}{2}}(b\,\text{arccsc}\,(cx)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a), x)
```

maple [A] time = 4.24, size = 0, normalized size = 0.00

$$\int \left(e\,x^2+d\right)^{\frac{3}{2}}(a+b\,\text{arccsc}\,(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}\left(2\left(ex^2+d\right)^{\frac{3}{2}}x+3\sqrt{ex^2+d}\,dx+\frac{3\,d^2\,\mathrm{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{e}}\right)a+b\int\left(ex^2\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right)+d\arctan\left(1,\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*(2*(e*x^2 + d)^(3/2)*x + 3*sqrt(e*x^2 + d)*d*x + 3*d^2*arcsinh(e*x/sqrt
(d*e))/sqrt(e))*a + b*integrate((e*x^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x -
1)) + d*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*sqrt(e*x^2 + d), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \left(e\,x^2+d\right)^{3/2}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))),x)
```

```
[Out] int((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{acsc}\,(cx)\right)\left(d+ex^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsc(c*x)),x)
```

```
[Out] Integral((a + b*acsc(c*x))*(d + e*x**2)**(3/2), x)
```

3.134 $\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x^2},x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^2,x)
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^2,x]
```

```
[Out] Defer[Int][((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^2, x]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x^2}\,dx=\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x^2}\,dx$$

Mathematica [A] time = 13.65, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^2,x]
```

```
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^2, x]
```

fricas [A] time = 1.24, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(aex^2+ad+\left(bex^2+bd\right)\text{arccsc}\,(cx)\right)\sqrt{ex^2+d}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsc(c*x))*sqrt(e*x^2 + d)/x^2,
 x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}\left(b\,\text{arccsc}\,(cx)+a\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^2,x, algorithm="giac")

[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)/x^2, x)

maple [A] time = 3.71, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\,\mathrm{arccsc}\left(c x\right)\right)}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^2,x)

[Out] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^2,x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(3\sqrt{ex^{2}+d}\,ex+3\,d\sqrt{e}\,\operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)-\frac{2\left(ex^{2}+d\right)^{\frac{3}{2}}}{x}\right)a+\left(e\int\sqrt{ex^{2}+d}\,\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right)dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^2,x, algorithm="maxima")

[Out] 1/2*(3*sqrt(e*x^2 + d)*e*x + 3*d*sqrt(e)*arcsinh(e*x/sqrt(d*e)) - 2*(e*x^2 + d)^(3/2)/x)*a + (e*integrate(sqrt(e*x^2 + d)*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)), x) + d*integrate(sqrt(e*x^2 + d)*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/x^2, x))*b

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(e\,x^2+d\right)^{3/2}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^2,x)

[Out] int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^2, x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{acsc}{\left(c x \right)}\right) \left(d + e x^{2}\right)^{\frac{3}{2}}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**(3/2)*(a+b*acsc(c*x))/x**2,x)

[Out] Integral((a + b*acsc(c*x))*(d + e*x**2)**(3/2)/x**2, x)

3.135 $\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^4}\,dx$

Optimal. Leaf size=26

$$\mathrm{Int}\left(\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^4},x\right)$$

[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^4,x)

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^4}\,dx$$

Verification is Not applicable to the result.

[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^4,x]

[Out] Defer[Int][((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^4, x]

Rubi steps

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^4}\,dx = \int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^4}\,dx$$

Mathematica [A] time = 6.49, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^4}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^4,x]

[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^4, x]

fricas [A] time = 1.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(aex^2+ad+\left(bex^2+bd\right)\mathrm{arccsc}\left(cx\right)\right)\sqrt{ex^2+d}}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^4,x, algorithm="fricas")

[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsc(c*x))*sqrt(e*x^2 + d)/x^4, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arccsc}\left(cx\right)+a)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^4,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)/x^4, x)
```

maple [A] time = 6.64, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a +b \,\mathrm{arccsc}\left(c x \right)\right)}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^4,x)
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^4,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\left(\frac{3\,\sqrt{ex^2+d}\,e^2x}{d}+3\,e^{\frac{3}{2}}\,\mathrm{arsinh}\left(\frac{ex}{\sqrt{de}}\right)-\frac{2\left(ex^2+d\right)^{\frac{3}{2}}e}{dx}-\frac{\left(ex^2+d\right)^{\frac{5}{2}}}{dx^3}\right)a+\left(e\int\frac{\sqrt{ex^2+d}\,\arctan\left(1,\sqrt{cx+1}\,\sqrt{\right.}\right.}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^4,x, algorithm="maxima")
[Out] 1/3*(3*sqrt(e*x^2 + d)*e^2*x/d + 3*e^(3/2)*arcsinh(e*x/sqrt(d*e)) - 2*(e*x^
2 + d)^(3/2)*e/(d*x) - (e*x^2 + d)^(5/2)/(d*x^3))*a + (e*integrate(sqrt(e*x
^2 + d)*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/x^2, x) + d*integrate(sqrt(
e*x^2 + d)*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/x^4, x))*b
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{{\left(e\,x^2+d\right)}^{3/2}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^4,x)
[Out] int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^4, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{acsc}{\left(c x \right)}\right) \left(d + e x^{2}\right)^{\frac{3}{2}}}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsc(c*x))/x**4,x)
[Out] Integral((a + b*acsc(c*x))*(d + e*x**2)**(3/2)/x**4, x)
```

3.136 $\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x^6}\,dx$

Optimal. Leaf size=416

$$-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5dx^5}-\frac{4bc\sqrt{c^2x^2-1}\left(c^2d+2e\right)\sqrt{d+ex^2}}{75x^2\sqrt{c^2x^2}}-\frac{bc\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{25x^4\sqrt{c^2x^2}}-\frac{bc\sqrt{c^2x^2-1}\left(8c^4\right.}{}$$

```
[Out] -1/5*(e*x^2+d)^(5/2)*(a+b*arccsc(c*x))/d/x^5-1/25*b*c*(e*x^2+d)^(3/2)*(c^2*
x^2-1)^(1/2)/x^4/(c^2*x^2)^(1/2)-1/75*b*c*(8*c^4*d^2+23*c^2*d*e+23*e^2)*(c^
2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/(c^2*x^2)^(1/2)-4/75*b*c*(c^2*d+2*e)*(c^2*
x^2-1)^(1/2)*(e*x^2+d)^(1/2)/x^2/(c^2*x^2)^(1/2)+1/75*b*c^2*(8*c^4*d^2+23*c
^2*d*e+23*e^2)*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+
d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)-1/75*b*(c^2*
d+e)*(8*c^4*d^2+19*c^2*d*e+15*e^2)*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(-c^2*
x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*x^2+d
)^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {264, 5239, 12, 474, 580, 583, 524, 427, 426, 424, 421, 419}

$$-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5dx^5}-\frac{bc\sqrt{c^2x^2-1}\left(8c^4d^2+23c^2de+23e^2\right)\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}}-\frac{bx\sqrt{1-c^2x^2}\left(c^2d+e\right)\left(8c^4d^2+\right.}{75d\sqrt{c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^6,x]
```

```
[Out] -(b*c*(8*c^4*d^2 + 23*c^2*d*e + 23*e^2)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
/(75*d*Sqrt[c^2*x^2]) - (4*b*c*(c^2*d + 2*e)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*
x^2])/(75*x^2*Sqrt[c^2*x^2]) - (b*c*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3/2))/(
25*x^4*Sqrt[c^2*x^2]) - ((d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]))/(5*d*x^5) +
 (b*c^2*(8*c^4*d^2 + 23*c^2*d*e + 23*e^2)*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^
2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(75*d*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*
x^2]*Sqrt[1 + (e*x^2)/d]) - (b*(c^2*d + e)*(8*c^4*d^2 + 19*c^2*d*e + 15*e^2
)*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d
))])/(75*d*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
```

```
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x^6}\,dx = -\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5dx^5} + \frac{(bcx)\int -\frac{\left(d+ex^2\right)^{5/2}}{5dx^6\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}$$

$$= -\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5dx^5} - \frac{(bcx)\int \frac{\left(d+ex^2\right)^{5/2}}{x^6\sqrt{-1+c^2x^2}}\,dx}{5d\sqrt{c^2x^2}}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}{25x^4\sqrt{c^2x^2}} - \frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5dx^5} + \frac{(bcx)\int \frac{\sqrt{d+}}{}}{}$$

$$= -\frac{4bc\left(c^2d+2e\right)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}{25x^4\sqrt{c^2x^2}} - \left(d+\right.$$

$$= -\frac{bc\left(8c^4d^2+23c^2de+23e^2\right)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}} - \frac{4bc\left(c^2d+2e\right)\sqrt{-1+}}{75x^2\sqrt{c^2}}$$

$$= -\frac{bc\left(8c^4d^2+23c^2de+23e^2\right)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}} - \frac{4bc\left(c^2d+2e\right)\sqrt{-1+}}{75x^2\sqrt{c^2}}$$

$$= -\frac{bc\left(8c^4d^2+23c^2de+23e^2\right)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}} - \frac{4bc\left(c^2d+2e\right)\sqrt{-1+}}{75x^2\sqrt{c^2}}$$

$$= -\frac{bc\left(8c^4d^2+23c^2de+23e^2\right)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}} - \frac{4bc\left(c^2d+2e\right)\sqrt{-1+}}{75x^2\sqrt{c^2}}$$

$$= -\frac{bc\left(8c^4d^2+23c^2de+23e^2\right)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}} - \frac{4bc\left(c^2d+2e\right)\sqrt{-1+}}{75x^2\sqrt{c^2}}$$

Mathematica [C] time = 0.69, size = 303, normalized size = 0.73

$$-\frac{\sqrt{d+ex^2}\left(15a\left(d+ex^2\right)^2+bcx\sqrt{1-\frac{1}{c^2x^2}}\left(dex^2\left(23c^2x^2+11\right)+d^2\left(8c^4x^4+4c^2x^2+3\right)+23e^2x^4\right)+15b\csc^{-1}(c\right.}{75dx^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^6,x]
```

```
[Out] -1/75*(Sqrt[d + e*x^2]*(15*a*(d + e*x^2)^2 + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(2
3*e^2*x^4 + d*e*x^2*(11 + 23*c^2*x^2) + d^2*(3 + 4*c^2*x^2 + 8*c^4*x^4)) +
15*b*(d + e*x^2)^2*ArcCsc[c*x]))/(d*x^5) + ((I/75)*b*c*Sqrt[1 - 1/(c^2*x^2)
]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(8*c^4*d^2 + 23*c^2*d*e + 23*e^2)*EllipticE[
I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))] - (8*c^6*d^3 + 27*c^4*d^2*e + 34*c^2
*d*e^2 + 15*e^3)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-
c^2]*d*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

fricas [F] time = 2.03, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(aex^2 + ad + \left(bex^2 + bd\right)\text{arccsc}\left(cx\right)\right)\sqrt{ex^2 + d}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsc(c*x))*sqrt(e*x^2 + d)/x^6,
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2 + d\right)^{\frac{3}{2}}\left(b\,\text{arccsc}\left(cx\right) + a\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^6,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)/x^6, x)
```

maple [F] time = 9.02, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^2 + d\right)^{\frac{3}{2}}\left(a + b\,\text{arccsc}\left(cx\right)\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^6,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^6,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(ex^2 + d\right)^{\frac{5}{2}}a}{5\,dx^5} - \frac{\left(dx^5 \int \frac{\left(c^2e^2x^4 + 2\,c^2dex^2 + c^2d^2\right)e^{\left(\frac{1}{2}\,\log\left(ex^2+d\right) + \frac{1}{2}\,\log\left(cx+1\right) + \frac{1}{2}\,\log\left(cx-1\right)\right)}}{c^2dx^6 - dx^4 + \left(c^2dx^6 - dx^4\right)\left(cx+1\right)\left(cx-1\right)}\,dx + \left(e^2x^4\,\arctan\left(1, \sqrt{cx+1}\,\sqrt{cx-1}\right)\right.}{5\,dx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] -1/5*(e*x^2 + d)^(5/2)*a/(d*x^5) - 1/5*(5*d*x^5*integrate(1/5*(c^2*e^2*x^4
+ 2*c^2*d*e*x^2 + c^2*d^2)*e^(1/2*log(e*x^2 + d) + 1/2*log(c*x + 1) + 1/2*l
og(c*x - 1))/(c^2*d*x^6 - d*x^4 + (c^2*d*x^6 - d*x^4)*e^(log(c*x + 1) + log
(c*x - 1))), x) + (e^2*x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 2*d*e*
x^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + d^2*arctan2(1, sqrt(c*x + 1)*
sqrt(c*x - 1)))*sqrt(e*x^2 + d))*b/(d*x^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)^{3/2}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^6,x)
```

```
[Out] int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^6, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsc(c*x))/x**6,x)
```

```
[Out] Timed out
```

3.137 $\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^8}\,dx$

Optimal. Leaf size=554

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{7dx^7}-\frac{bc\sqrt{c^2x^2-1}\left(d+ex^2\right)^{5/2}}{49dx^6\sqrt{c^2x^2}}-\frac{bc\sqrt{c^2x^2-1}\left(30c^2\right.}{1225d}$$

```
[Out] -1/7*(e*x^2+d)^(5/2)*(a+b*arccsc(c*x))/d/x^7+2/35*e*(e*x^2+d)^(5/2)*(a+b*ar
ccsc(c*x))/d^2/x^5-1/1225*b*c*(30*c^2*d+11*e)*(e*x^2+d)^(3/2)*(c^2*x^2-1)^(
1/2)/d/x^4/(c^2*x^2)^(1/2)-1/49*b*c*(e*x^2+d)^(5/2)*(c^2*x^2-1)^(1/2)/d/x^6
/(c^2*x^2)^(1/2)-1/3675*b*c*(240*c^6*d^3+528*c^4*d^2*e+193*c^2*d*e^2-247*e^
3)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)-1/3675*b*c*(120*c^
4*d^2+159*c^2*d*e-37*e^2)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/x^2/(c^2*x^2)
^(1/2)+1/3675*b*c^2*(240*c^6*d^3+528*c^4*d^2*e+193*c^2*d*e^2-247*e^3)*x*Ell
ipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^
2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)-2/3675*b*(c^2*d+e)*(120*c^6*d^
3+204*c^4*d^2*e+17*c^2*d*e^2-105*e^3)*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(-c
^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*
x^2+d)^(1/2)
```

Rubi [A] time = 0.89, antiderivative size = 554, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {271, 264, 5239, 12, 580, 583, 524, 427, 426, 424, 421, 419}

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{7dx^7}-\frac{bc\sqrt{c^2x^2-1}\left(528c^4d^2e+240c^6d^3+193c^2de^2\right.}{3675d^2\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^8,x]
```

```
[Out] -(b*c*(240*c^6*d^3 + 528*c^4*d^2*e + 193*c^2*d*e^2 - 247*e^3)*Sqrt[-1 + c^2
*x^2]*Sqrt[d + e*x^2])/(3675*d^2*Sqrt[c^2*x^2]) - (b*c*(120*c^4*d^2 + 159*c
^2*d*e - 37*e^2)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(3675*d*x^2*Sqrt[c^2*x
^2]) - (b*c*(30*c^2*d + 11*e)*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3/2))/(1225*d
*x^4*Sqrt[c^2*x^2]) - (b*c*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(5/2))/(49*d*x^6*
Sqrt[c^2*x^2]) - ((d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]))/(7*d*x^7) + (2*e*(
d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]))/(35*d^2*x^5) + (b*c^2*(240*c^6*d^3 +
528*c^4*d^2*e + 193*c^2*d*e^2 - 247*e^3)*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2
]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(3675*d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c
^2*x^2]*Sqrt[1 + (e*x^2)/d]) - (2*b*(c^2*d + e)*(120*c^6*d^3 + 204*c^4*d^2*
e + 17*c^2*d*e^2 - 105*e^3)*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*Ellipti
cF[ArcSin[c*x], -(e/(c^2*d))])/(3675*d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*S
qrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x^8}\,dx = -\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{7dx^7}+\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{35d^2x^5}+\frac{(bc}{}$$

$$= -\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{7dx^7}+\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{35d^2x^5}+\frac{(bc}{}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{5/2}}{49dx^6\sqrt{c^2x^2}}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{7dx^7}+\frac{2e\left(d+e\right.}{}$$

$$= -\frac{bc\left(30c^2d+11e\right)\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}{1225dx^4\sqrt{c^2x^2}}-\frac{bc\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{5/2}}{49dx^6\sqrt{c^2x^2}}$$

$$= -\frac{bc\left(120c^4d^2+159c^2de-37e^2\right)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675dx^2\sqrt{c^2x^2}}-\frac{bc\left(30c^2d+11\right.}{12}$$

$$= -\frac{bc\left(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3\right)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675d^2\sqrt{c^2x^2}}-$$

$$= -\frac{bc\left(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3\right)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675d^2\sqrt{c^2x^2}}-$$

$$= -\frac{bc\left(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3\right)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675d^2\sqrt{c^2x^2}}-$$

$$= -\frac{bc\left(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3\right)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675d^2\sqrt{c^2x^2}}-$$

$$= -\frac{bc\left(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3\right)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675d^2\sqrt{c^2x^2}}-$$

Mathematica [C] time = 0.87, size = 383, normalized size = 0.69

$$-\frac{\sqrt{d+ex^2}\left(105a\left(5d-2ex^2\right)\left(d+ex^2\right)^2+bcx\sqrt{1-\frac{1}{c^2x^2}}\left(de^2x^4\left(193c^2x^2+71\right)+3d^2ex^2\left(176c^4x^4+83c^2x^2+61\right)\right.\right.}{3675d^2x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^8,x]
```

```
[Out] -1/3675*(Sqrt[d + e*x^2]*(105*a*(5*d - 2*e*x^2)*(d + e*x^2)^2 + b*c*Sqrt[1
- 1/(c^2*x^2)]*x*(-247*e^3*x^6 + d*e^2*x^4*(71 + 193*c^2*x^2) + 3*d^2*e*x^2
*(61 + 83*c^2*x^2 + 176*c^4*x^4) + 15*d^3*(5 + 6*c^2*x^2 + 8*c^4*x^4 + 16*c
^6*x^6)) + 105*b*(5*d - 2*e*x^2)*(d + e*x^2)^2*ArcCsc[c*x]))/(d^2*x^7) + ((
I/3675)*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(240*c^6*d^3
 + 528*c^4*d^2*e + 193*c^2*d*e^2 - 247*e^3)*EllipticE[I*ArcSinh[Sqrt[-c^2]*
x], -(e/(c^2*d))] - 2*(120*c^8*d^4 + 324*c^6*d^3*e + 221*c^4*d^2*e^2 - 88*c
^2*d*e^3 - 105*e^4)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqr
t[-c^2]*d^2*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(aex^2+ad+\left(bex^2+bd\right)\text{arccsc}\left(cx\right)\right)\sqrt{ex^2+d}}{x^8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^8,x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsc(c*x))*sqrt(e*x^2 + d)/x^8,
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(ex^2+d\right)^{\frac{3}{2}}\left(b\,\text{arccsc}\left(cx\right)+a\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^8,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)/x^8, x)
```

maple [F] time = 7.49, size = 0, normalized size = 0.00

$$\int\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}\left(a+b\,\text{arccsc}\left(cx\right)\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^8,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^8,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{35}a\left(\frac{2\left(ex^2+d\right)^{\frac{5}{2}}e}{d^2x^5}-\frac{5\left(ex^2+d\right)^{\frac{5}{2}}}{dx^7}\right)+\frac{\left(d^2x^7\int\frac{\left(2c^2e^3x^6-c^2de^2x^4-8c^2d^2ex^2-5c^2d^3\right)e^{\left(\frac{1}{2}\log\left(ex^2+d\right)+\frac{1}{2}\log\left(cx+1\right)+\frac{1}{2}\log\left(cx-1\right)\right)}}{c^2d^2x^8-d^2x^6+\left(c^2d^2x^8-d^2x^6\right)\left(cx+1\right)\left(cx-1\right)}\,dx+\left(2e\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^8,x, algorithm="maxima")
```

```
[Out] 1/35*a*(2*(e*x^2 + d)^(5/2)*e/(d^2*x^5) - 5*(e*x^2 + d)^(5/2)/(d*x^7)) + 1/
35*(35*d^2*x^7*integrate(1/35*(2*c^2*e^3*x^6 - c^2*d*e^2*x^4 - 8*c^2*d^2*e*
x^2 - 5*c^2*d^3)*e^(1/2*log(e*x^2 + d) + 1/2*log(c*x + 1) + 1/2*log(c*x - 1
))/(c^2*d^2*x^8 - d^2*x^6 + (c^2*d^2*x^8 - d^2*x^6)*e^(log(c*x + 1) + log(c
*x - 1))), x) + (2*e^3*x^6*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) - d*e^2*
x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) - 8*d^2*e*x^2*arctan2(1, sqrt(c
*x + 1)*sqrt(c*x - 1)) - 5*d^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*sqr
t(e*x^2 + d))*b/(d^2*x^7)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)^{3/2}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^8,x)
```

```
[Out] int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^8, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsc(c*x))/x**8,x)
```

```
[Out] Timed out
```

3.138 $\int \frac{x^5\left(a+b \csc^{-1}(cx)\right)}{\sqrt{d+ex^2}} dx$

Optimal. Leaf size=321

$$\frac{d^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}-\frac{8bcd^{5/2}x\tan^{-1}\left(\right.}{15e^3\sqrt{c^2}}$$

```
[Out] -2/3*d*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/e^3+1/5*(e*x^2+d)^(5/2)*(a+b*arccs
c(c*x))/e^3-8/15*b*c*d^(5/2)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(
1/2))/e^3/(c^2*x^2)^(1/2)+1/120*b*(45*c^4*d^2-10*c^2*d*e+9*e^2)*x*arctanh(e
^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^4/e^(5/2)/(c^2*x^2)^(1/2)+1/2
0*b*x*(e*x^2+d)^(3/2)*(c^2*x^2-1)^(1/2)/c/e^2/(c^2*x^2)^(1/2)+d^2*(a+b*arcc
sc(c*x))*(e*x^2+d)^(1/2)/e^3-1/120*b*(19*c^2*d-9*e)*x*(c^2*x^2-1)^(1/2)*(e*
x^2+d)^(1/2)/c^3/e^2/(c^2*x^2)^(1/2)
```

Rubi [A] time = 1.05, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {266, 43, 5239, 12, 1615, 154, 157, 63, 217, 206, 93, 204}

$$\frac{d^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}-\frac{8bcd^{5/2}x\tan^{-1}\left(\right.}{15e^3\sqrt{c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] -(b*(19*c^2*d - 9*e)*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(120*c^3*e^2*Sqr
t[c^2*x^2]) + (b*x*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3/2))/(20*c*e^2*Sqrt[c^2
*x^2]) + (d^2*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/e^3 - (2*d*(d + e*x^2)^(
3/2)*(a + b*ArcCsc[c*x]))/(3*e^3) + ((d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]))
/(5*e^3) - (8*b*c*d^(5/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x
^2])])/(15*e^3*Sqrt[c^2*x^2]) + (b*(45*c^4*d^2 - 10*c^2*d*e + 9*e^2)*x*ArcT
anh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(120*c^4*e^(5/2)*Sqr
t[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
```

```
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{x^5\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \frac{d^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3} - \frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} + \frac{\left(d+ex^2\right)^{5/2}\left(a+\right.}{5e^3}$$

$$= \frac{d^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3} - \frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} + \frac{\left(d+ex^2\right)^{5/2}\left(a+\right.}{5e^3}$$

$$= \frac{d^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3} - \frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} + \frac{\left(d+ex^2\right)^{5/2}\left(a+\right.}{5e^3}$$

$$= \frac{bx\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}{20ce^2\sqrt{c^2x^2}} + \frac{d^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3} - \frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\right.}{3e^3}$$

$$= -\frac{b\left(19c^2d-9e\right)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e^2\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}{20ce^2\sqrt{c^2x^2}} + \frac{d^2\sqrt{d+ex^2}}{}$$

$$= -\frac{b\left(19c^2d-9e\right)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e^2\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}{20ce^2\sqrt{c^2x^2}} + \frac{d^2\sqrt{d+ex^2}}{}$$

$$= -\frac{b\left(19c^2d-9e\right)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e^2\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}{20ce^2\sqrt{c^2x^2}} + \frac{d^2\sqrt{d+ex^2}}{}$$

$$= -\frac{b\left(19c^2d-9e\right)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e^2\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}{20ce^2\sqrt{c^2x^2}} + \frac{d^2\sqrt{d+ex^2}}{}$$

$$= -\frac{b\left(19c^2d-9e\right)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e^2\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}{20ce^2\sqrt{c^2x^2}} + \frac{d^2\sqrt{d+ex^2}}{}$$

Mathematica [A] time = 0.75, size = 328, normalized size = 1.02

$$\frac{\sqrt{d+ex^2}\left(8ac^3\left(8d^2-4dex^2+3e^2x^4\right)+8bc^3\csc^{-1}(cx)\left(8d^2-4dex^2+3e^2x^4\right)+bex\sqrt{1-\frac{1}{c^2x^2}}\left(c^2\left(6ex^2-13d\right)+\right.\right.}{120c^3e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (Sqrt[d + e*x^2]*(8*a*c^3*(8*d^2 - 4*d*e*x^2 + 3*e^2*x^4) + b*e*Sqrt[1 - 1/
(c^2*x^2)]*x*(9*e + c^2*(-13*d + 6*e*x^2)) + 8*b*c^3*(8*d^2 - 4*d*e*x^2 + 3
*e^2*x^4)*ArcCsc[c*x]))/(120*c^3*e^3) + (b*Sqrt[1 - 1/(c^2*x^2)]*x*(Sqrt[c^
2]*Sqrt[e]*Sqrt[c^2*d + e]*(45*c^4*d^2 - 10*c^2*d*e + 9*e^2)*Sqrt[(c^2*(d +
 e*x^2))/(c^2*d + e)]*ArcSinh[(c*Sqrt[e]*Sqrt[-1 + c^2*x^2])/(Sqrt[c^2]*Sqr
t[c^2*d + e])] + 64*c^7*d^(5/2)*Sqrt[d + e*x^2]*ArcTan[(Sqrt[d]*Sqrt[-1 + c
^2*x^2])/Sqrt[d + e*x^2]]))/(120*c^6*e^3*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2]
)
```

fricas [A] time = 4.80, size = 1383, normalized size = 4.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/480*(64*b*c^5*sqrt(-d)*d^2*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2
*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 +
d)*sqrt(-d) + 8*d^2)/x^4) + (45*b*c^4*d^2 - 10*b*c^2*d*e + 9*b*e^2)*sqrt(e)
*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2
*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2)
+ 4*(24*a*c^5*e^2*x^4 - 32*a*c^5*d*e*x^2 + 64*a*c^5*d^2 + 8*(3*b*c^5*e^2*x^
4 - 4*b*c^5*d*e*x^2 + 8*b*c^5*d^2)*arccsc(c*x) + (6*b*c^3*e^2*x^2 - 13*b*c^
3*d*e + 9*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e^3), -1/480*(1
28*b*c^5*d^(5/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt
(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) - (45*b*c^4*
d^2 - 10*b*c^2*d*e + 9*b*e^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d
*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2
 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) - 4*(24*a*c^5*e^2*x^4 - 32*a*c^5*d*e*x
^2 + 64*a*c^5*d^2 + 8*(3*b*c^5*e^2*x^4 - 4*b*c^5*d*e*x^2 + 8*b*c^5*d^2)*arc
csc(c*x) + (6*b*c^3*e^2*x^2 - 13*b*c^3*d*e + 9*b*c*e^2)*sqrt(c^2*x^2 - 1))*
sqrt(e*x^2 + d))/(c^5*e^3), 1/240*(32*b*c^5*sqrt(-d)*d^2*log(((c^4*d^2 - 6*
c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d -
e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) - (45*b*c^4*d^2 - 10*b
*c^2*d*e + 9*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*
x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*
x^2)) + 2*(24*a*c^5*e^2*x^4 - 32*a*c^5*d*e*x^2 + 64*a*c^5*d^2 + 8*(3*b*c^5*
e^2*x^4 - 4*b*c^5*d*e*x^2 + 8*b*c^5*d^2)*arccsc(c*x) + (6*b*c^3*e^2*x^2 - 1
3*b*c^3*d*e + 9*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e^3), -1/
240*(64*b*c^5*d^(5/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)
*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (45*b
*c^4*d^2 - 10*b*c^2*d*e + 9*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d
 - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^
3*d*e - c*e^2)*x^2)) - 2*(24*a*c^5*e^2*x^4 - 32*a*c^5*d*e*x^2 + 64*a*c^5*d^
2 + 8*(3*b*c^5*e^2*x^4 - 4*b*c^5*d*e*x^2 + 8*b*c^5*d^2)*arccsc(c*x) + (6*b*
c^3*e^2*x^2 - 13*b*c^3*d*e + 9*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))
/(c^5*e^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x^5}{\sqrt{ex^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^5/sqrt(e*x^2 + d), x)
```

maple [F] time = 7.87, size = 0, normalized size = 0.00

$$\int \frac{x^5 (a + b \operatorname{arccsc}(cx))}{\sqrt{e x^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{15}\left(\frac{3\sqrt{ex^2+d}\,x^4}{e} - \frac{4\sqrt{ex^2+d}\,dx^2}{e^2} + \frac{8\sqrt{ex^2+d}\,d^2}{e^3}\right)a + \frac{\left(e^3 \int \frac{\left(3c^2e^3x^7 - c^2de^2x^5 + 4c^2d^2ex^3 + 8c^2d^3x\right)e^{\left(-\frac{1}{2}\log\left(ex^2+d\right) + \frac{1}{2}\log(cx+1) + \cdots\right.}}{c^2e^3x^2 + \left(c^2e^3x^2 - e^3\right)(cx+1)(cx-1) - e^3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*sqrt(e*x^2 + d)*x^4/e - 4*sqrt(e*x^2 + d)*d*x^2/e^2 + 8*sqrt(e*x^2
+ d)*d^2/e^3)*a + 1/15*(15*e^3*integrate(1/15*(3*c^2*e^3*x^7 - c^2*d*e^2*x^
5 + 4*c^2*d^2*e*x^3 + 8*c^2*d^3*x)*e^(-1/2*log(e*x^2 + d) + 1/2*log(c*x + 1
) + 1/2*log(c*x - 1))/(c^2*e^3*x^2 - e^3 + (c^2*e^3*x^2 - e^3)*e^(log(c*x +
 1) + log(c*x - 1))), x) + (3*e^2*x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1
)) - 4*d*e*x^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 8*d^2*arctan2(1, s
qrt(c*x + 1)*sqrt(c*x - 1)))*sqrt(e*x^2 + d))*b/e^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5 \left(a + b \operatorname{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

```
[Out] int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 (a + b \operatorname{acsc}(cx))}{\sqrt{d + ex^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acsc(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**5*(a + b*acsc(c*x))/sqrt(d + e*x**2), x)
```

3.139 $\int \frac{x^3\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=225

$$-\frac{d\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{2bcd^{3/2}x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3e^2\sqrt{c^2x^2}}-\frac{bx\left(3c^2d-e\right)\tanh}{6c^2e^{3/2}}$$

```
[Out] 1/3*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/e^2+2/3*b*c*d^(3/2)*x*arctan((e*x^2+d
)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e^2/(c^2*x^2)^(1/2)-1/6*b*(3*c^2*d-e)*x*
arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^2/e^(3/2)/(c^2*x^2)^
(1/2)-d*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/e^2+1/6*b*x*(c^2*x^2-1)^(1/2)*(e*
x^2+d)^(1/2)/c/e/(c^2*x^2)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {266, 43, 5239, 12, 573, 154, 157, 63, 217, 206, 93, 204}

$$-\frac{d\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{2bcd^{3/2}x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3e^2\sqrt{c^2x^2}}-\frac{bx\left(3c^2d-e\right)\tanh}{6c^2e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (b*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(6*c*e*Sqrt[c^2*x^2]) - (d*Sqrt[d
+ e*x^2]*(a + b*ArcCsc[c*x]))/e^2 + ((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))
/(3*e^2) + (2*b*c*d^(3/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x
^2])])/(3*e^2*Sqrt[c^2*x^2]) - (b*(3*c^2*d - e)*x*ArcTanh[(Sqrt[e]*Sqrt[-1
+ c^2*x^2])/(c*Sqrt[d + e*x^2])])/(6*c^2*e^(3/2)*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 573

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
```

```
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{x^3\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=-\frac{d\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{(bcx)\int\frac{\left(-2d+ex^2\right)}{3e^2x\sqrt{-}}}{\sqrt{c^2x^2}}$$

$$=-\frac{d\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{(bcx)\int\frac{\left(-2d+ex^2\right)}{x\sqrt{-1+}}}{3e^2\sqrt{c^2x}}$$

$$=-\frac{d\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{(bcx)\,\mathrm{Subst}\left(\int\right.}{6}$$

$$=\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce\sqrt{c^2x^2}}-\frac{d\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc\right.}{3e^2}$$

$$=\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce\sqrt{c^2x^2}}-\frac{d\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc\right.}{3e^2}$$

$$=\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce\sqrt{c^2x^2}}-\frac{d\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc\right.}{3e^2}$$

$$=\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce\sqrt{c^2x^2}}-\frac{d\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc\right.}{3e^2}$$

$$=\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce\sqrt{c^2x^2}}-\frac{d\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc\right.}{3e^2}$$

Mathematica [A] time = 0.53, size = 271, normalized size = 1.20

$$\frac{\sqrt{d+ex^2}\left(-4acd+2acex^2+bex\sqrt{1-\frac{1}{c^2x^2}}+2bc\csc^{-1}(cx)\left(ex^2-2d\right)\right)}{6ce^2}-\frac{bx\sqrt{1-\frac{1}{c^2x^2}}\left(\sqrt{c^2}\sqrt{e}\left(3c^2d-e\right)\sqrt{c^2d}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (Sqrt[d + e*x^2]*(-4*a*c*d + b*e*Sqrt[1 - 1/(c^2*x^2)]*x + 2*a*c*e*x^2 + 2*
b*c*(-2*d + e*x^2)*ArcCsc[c*x]))/(6*c*e^2) - (b*Sqrt[1 - 1/(c^2*x^2)]*x*(Sq
rt[c^2]*(3*c^2*d - e)*Sqrt[e]*Sqrt[c^2*d + e]*Sqrt[(c^2*(d + e*x^2))/(c^2*d
 + e)]*ArcSinh[(c*Sqrt[e]*Sqrt[-1 + c^2*x^2])/(Sqrt[c^2]*Sqrt[c^2*d + e])]
+ 4*c^5*d^(3/2)*Sqrt[d + e*x^2]*ArcTan[(Sqrt[d]*Sqrt[-1 + c^2*x^2])/Sqrt[d
+ e*x^2]]))/(6*c^4*e^2*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

fricas [A] time = 2.40, size = 1107, normalized size = 4.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(4*b*c^3*sqrt(-d)*d*log((((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2
 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*s
qrt(-d) + 8*d^2)/x^4) - (3*b*c^2*d - b*e)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d
^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*
sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(2*a*c^3*e*x^2 - 4*a*c
^3*d + sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 - 2*b*c^3*d)*arccsc(c*x))*s
qrt(e*x^2 + d))/(c^3*e^2), 1/24*(8*b*c^3*d^(3/2)*arctan(-1/2*sqrt(c^2*x^2 -
 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2
 - d*e)*x^2 - d^2)) - (3*b*c^2*d - b*e)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2
 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sq
rt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(2*a*c^3*e*x^2 - 4*a*c^3
*d + sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 - 2*b*c^3*d)*arccsc(c*x))*sqr
t(e*x^2 + d))/(c^3*e^2), 1/12*(2*b*c^3*sqrt(-d)*d*log((((c^4*d^2 - 6*c^2*d*e
 + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2
- 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) + (3*b*c^2*d - b*e)*sqrt(-e)*
arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt
(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(2*a*c^3*e*x^2 - 4*
a*c^3*d + sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 - 2*b*c^3*d)*arccsc(c*x)
)*sqrt(e*x^2 + d))/(c^3*e^2), 1/12*(4*b*c^3*d^(3/2)*arctan(-1/2*sqrt(c^2*x^
2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*
d^2 - d*e)*x^2 - d^2)) + (3*b*c^2*d - b*e)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2
 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d
*e + (c^3*d*e - c*e^2)*x^2)) + 2*(2*a*c^3*e*x^2 - 4*a*c^3*d + sqrt(c^2*x^2
- 1)*b*c*e + 2*(b*c^3*e*x^2 - 2*b*c^3*d)*arccsc(c*x))*sqrt(e*x^2 + d))/(c^3
*e^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x^3}{\sqrt{ex^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^3/sqrt(e*x^2 + d), x)
```

maple [F] time = 7.30, size = 0, normalized size = 0.00

$$\int \frac{x^3 (a + b \operatorname{arccsc}(cx))}{\sqrt{e\,x^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\left(\frac{\sqrt{ex^2 + d}\,x^2}{e} - \frac{2\sqrt{ex^2 + d}\,d}{e^2}\right)a + \frac{\left(e^2x^4 \arctan\left(1, \sqrt{cx+1}\,\sqrt{cx-1}\right) - dex^2 \arctan\left(1, \sqrt{cx+1}\,\sqrt{cx-1}\right) + \sqrt{ex^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(sqrt(e*x^2 + d)*x^2/e - 2*sqrt(e*x^2 + d)*d/e^2)*a + 1/3*(e^2*x^4*arct
an2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) - d*e*x^2*arctan2(1, sqrt(c*x + 1)*sqrt
```

```
(c*x - 1)) + 3*sqrt(e*x^2 + d)*e^2*integrate(1/3*(c^2*e^2*x^5 - c^2*d*e*x^3
 - 2*c^2*d^2*x)*e^(-1/2*log(e*x^2 + d) + 1/2*log(c*x + 1) + 1/2*log(c*x - 1
))/(c^2*e^2*x^2 - e^2 + (c^2*e^2*x^2 - e^2)*e^(log(c*x + 1) + log(c*x - 1))
), x) - 2*d^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*b/(sqrt(e*x^2 + d)*e
^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2),x)
[Out] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\,(a+b\,\mathrm{acsc}\,(cx))}{\sqrt{d+ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsc(c*x))/(e*x**2+d)**(1/2),x)
[Out] Integral(x**3*(a + b*acsc(c*x))/sqrt(d + e*x**2), x)
```

3.140 $\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=132

$$\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e}-\frac{bc\sqrt{d}\,x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{e\sqrt{c^2x^2}}+\frac{bx\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{c^2x^2}}$$

```
[Out] -b*c*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))*d^(1/2)/e/(c^2*x^2
)^(1/2)+b*x*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/e^(1/2)/(c
^2*x^2)^(1/2)+(a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/e
```

Rubi [A] time = 0.15, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {5237, 446, 105, 63, 217, 206, 93, 204}

$$\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e}-\frac{bc\sqrt{d}\,x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{e\sqrt{c^2x^2}}+\frac{bx\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/e - (b*c*Sqrt[d]*x*ArcTan[Sqrt[d + e*
x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(e*Sqrt[c^2*x^2]) + (b*x*ArcTanh[(Sqrt[
e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(Sqrt[e]*Sqrt[c^2*x^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5237

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCsc[c*x]))/(2*e*(p + 1)), x
] + Dist[(b*c*x)/(2*e*(p + 1)*Sqrt[c^2*x^2]), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e} + \frac{(bcx)\int \frac{\sqrt{d+ex^2}}{x\sqrt{-1+c^2x^2}}\,dx}{e\sqrt{c^2x^2}}$$

$$= \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e} + \frac{(bcx)\,\text{Subst}\left(\int \frac{\sqrt{d+ex}}{x\sqrt{-1+c^2x}}\,dx,x,x^2\right)}{2e\sqrt{c^2x^2}}$$

$$= \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e} + \frac{(bcx)\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+c^2x}\,\sqrt{d+ex}}\,dx,x,x^2\right)}{2\sqrt{c^2x^2}} + \frac{(bcdx)\,\text{Sub}}{}$$

$$= \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e} + \frac{(bx)\,\text{Subst}\left(\int \frac{1}{\sqrt{d+\frac{e}{c^2}+\frac{ex^2}{c^2}}}\,dx,x,\sqrt{-1+c^2x^2}\right)}{c\sqrt{c^2x^2}} + \frac{(bcd}{}$$

$$= \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e} - \frac{bc\sqrt{d}\,x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\,\sqrt{-1+c^2x^2}}\right)}{e\sqrt{c^2x^2}} + \frac{(bx)\,\text{Subst}\left(\int \frac{1}{1-\frac{ex^2}{c^2}}\,d\right.}{c\sqrt{c^2x^2}}$$

$$= \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e} - \frac{bc\sqrt{d}\,x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\,\sqrt{-1+c^2x^2}}\right)}{e\sqrt{c^2x^2}} + \frac{bx\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{-1+c^2x}}{c\sqrt{d+ex^2}}\right.}{\sqrt{e}\,\sqrt{c^2x^2}}$$

Mathematica [A] time = 0.22, size = 210, normalized size = 1.59

$$\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e} + \frac{bx\sqrt{1-\frac{1}{c^2x^2}}\left(\sqrt{c^2}\,\sqrt{e}\,\sqrt{c^2d+e}\,\sqrt{\frac{c^2(d+ex^2)}{c^2d+e}}\,\sinh^{-1}\left(\frac{c\sqrt{e}\,\sqrt{c^2x^2-1}}{\sqrt{c^2}\,\sqrt{c^2d+e}}\right)+c^3\sqrt{d}\,\sqrt{d+ex^2}\right.}{c^2e\sqrt{c^2x^2-1}\,\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/e + (b*Sqrt[1 - 1/(c^2*x^2)]*x*(Sqrt[
c^2]*Sqrt[e]*Sqrt[c^2*d + e]*Sqrt[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcSinh[(c
*Sqrt[e]*Sqrt[-1 + c^2*x^2])/(Sqrt[c^2]*Sqrt[c^2*d + e])] + c^3*Sqrt[d]*Sqr
t[d + e*x^2]*ArcTan[(Sqrt[d]*Sqrt[-1 + c^2*x^2])/Sqrt[d + e*x^2]]))/(c^2*e*
Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

fricas [A] time = 1.65, size = 870, normalized size = 6.59

$$\left[\frac{bc\sqrt{-d}\log\left(\frac{\left(c^4d^2-6c^2de+e^2\right)x^4-8\left(c^2d^2-de\right)x^2+4\sqrt{c^2x^2-1}\left(\left(c^2d-e\right)x^2-2d\right)\sqrt{ex^2+d}\sqrt{-d}+8d^2}{x^4}\right)+b\sqrt{e}\log\left(8c^4e^2x^4+c^4d^2-6c^2d\right.}{4ce}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(b*c*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)
*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d)
 + 8*d^2)/x^4) + b*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4
*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(
e*x^2 + d)*sqrt(e) + e^2) + 4*sqrt(e*x^2 + d)*(b*c*arccsc(c*x) + a*c))/(c*e
), -1/4*(2*b*c*sqrt(d)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d
)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) - b*sq
rt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 +
 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) +
e^2) - 4*sqrt(e*x^2 + d)*(b*c*arccsc(c*x) + a*c))/(c*e), 1/4*(b*c*sqrt(-d)*
log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x
^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) - 2*
b*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^
2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 4*sqrt(e*x
^2 + d)*(b*c*arccsc(c*x) + a*c))/(c*e), -1/2*(b*c*sqrt(d)*arctan(-1/2*sqrt(
c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 +
 (c^2*d^2 - d*e)*x^2 - d^2)) + b*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d -
 e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*
d*e - c*e^2)*x^2)) - 2*sqrt(e*x^2 + d)*(b*c*arccsc(c*x) + a*c))/(c*e)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x}{\sqrt{ex^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x/sqrt(e*x^2 + d), x)
```

maple [F] time = 5.44, size = 0, normalized size = 0.00

$$\int \frac{x\,(a+b\,\mathrm{arccsc}\,(cx))}{\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(\sqrt{ex^2+d}\arctan\left(1,\sqrt{cx+1}\sqrt{cx-1}\right)+\int\frac{\sqrt{ex^2+d}}{\sqrt{cx+1}\sqrt{cx-1}\,x}\,dx\right)b}{e}+\frac{\sqrt{ex^2+d}\,a}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] (e*integrate((c^2*e*x^3 + c^2*d*x)*e^(-1/2*log(e*x^2 + d) + 1/2*log(c*x + 1
) + 1/2*log(c*x - 1))/(c^2*e*x^2 + (c^2*e*x^2 - e)*e^(log(c*x + 1) + log(c*
x - 1)) - e), x) + sqrt(e*x^2 + d)*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))
*b/e + sqrt(e*x^2 + d)*a/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

```
[Out] int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x\left(a+b\,\mathrm{acsc}\,(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsc(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*(a + b*acsc(c*x))/sqrt(d + e*x**2), x)
```

3.141 $\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=26

$$\mathrm{Int}\left(\frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x/(e*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCsc[c*x])/(x*Sqrt[d + e*x^2]),x]
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x*Sqrt[d + e*x^2]), x]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx = \int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$$

Mathematica [A] time = 1.71, size = 0, normalized size = 0.00

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x*Sqrt[d + e*x^2]),x]
[Out] Integrate[(a + b*ArcCsc[c*x])/(x*Sqrt[d + e*x^2]), x]
```

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{ex^2+d}\,(b\,\mathrm{arccsc}\,(cx)+a)}{ex^3+dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/(e*x^3 + d*x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\mathrm{arccsc}\,(cx)+a}{\sqrt{ex^2+d}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)/(sqrt(e*x^2 + d)*x), x)
```

maple [A] time = 4.18, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{arccsc}\,(cx)}{x\sqrt{e\,x^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/x/(e*x^2+d)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$b \int \frac{\arctan\left(1, \sqrt{cx + 1}\,\sqrt{cx - 1}\right)}{\sqrt{ex^2 + d}\,x}\, dx - \frac{a\,\mathrm{arsinh}\left(\frac{d}{\sqrt{de}\,|x|}\right)}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] b*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(sqrt(e*x^2 + d)*x), x)
 - a*arcsinh(d/(sqrt(d*e)*abs(x)))/sqrt(d)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x\,\sqrt{e\,x^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^(1/2)),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{acsc}\,(cx)}{x\sqrt{d + ex^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acsc(c*x))/(x*sqrt(d + e*x**2)), x)
```

3.142 $\int \frac{a+b\csc^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{a+b\csc^{-1}(cx)}{x^3\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCsc[c*x])/(x^3*Sqrt[d + e*x^2]),x]
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x^3*Sqrt[d + e*x^2]), x]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx = \int \frac{a+b\csc^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

Mathematica [A] time = 12.33, size = 0, normalized size = 0.00

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x^3*Sqrt[d + e*x^2]),x]
[Out] Integrate[(a + b*ArcCsc[c*x])/(x^3*Sqrt[d + e*x^2]), x]
```

fricas [A] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2+d}\,(b\,\text{arccsc}\,(cx)+a)}{ex^5+dx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/(e*x^5 + d*x^3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arccsc}\,(cx)+a}{\sqrt{ex^2+d}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)/(sqrt(e*x^2 + d)*x^3), x)
```

maple [A] time = 1.44, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{arccsc}\,(cx)}{x^3\sqrt{e\,x^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,a\left(\frac{e\,\mathrm{arsinh}\left(\frac{d}{\sqrt{de}\,|x|}\right)}{d^{\frac{3}{2}}} - \frac{\sqrt{ex^2 + d}}{dx^2}\right) + b\int \frac{\arctan\left(1, \sqrt{cx + 1}\,\sqrt{cx - 1}\right)}{\sqrt{ex^2 + d}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*a*(e*arcsinh(d/(sqrt(d*e)*abs(x)))/d^(3/2) - sqrt(e*x^2 + d)/(d*x^2)) +
 b*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(sqrt(e*x^2 + d)*x^3),
 x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^3\,\sqrt{e\,x^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x^3*(d + e*x^2)^(1/2)),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(x^3*(d + e*x^2)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{acsc}\,(cx)}{x^3\sqrt{d + ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x**3/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acsc(c*x))/(x**3*sqrt(d + e*x**2)), x)
```

3.143 $\int \frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=26

$$\mathrm{Int}\left(\frac{x^2\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^2*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
[Out] Defer[Int][(x^2*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

Mathematica [A] time = 16.60, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^2*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
[Out] Integrate[(x^2*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2], x]
```

fricas [A] time = 0.59, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{bx^2\,\mathrm{arccsc}\,(cx)+ax^2}{\sqrt{ex^2+d}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b*x^2*arccsc(c*x) + a*x^2)/sqrt(e*x^2 + d), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^2}{\sqrt{ex^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)*x^2/sqrt(e*x^2 + d), x)
```

maple [A] time = 7.45, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + b \,\mathrm{arccsc}\,(cx))}{\sqrt{e\,x^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,a\left(\frac{\sqrt{ex^2+d}\,x}{e} - \frac{d\,\mathrm{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{e^{\frac{3}{2}}}\right) + b\int \frac{x^2\arctan\left(1, \sqrt{cx+1}\,\sqrt{cx-1}\right)}{\sqrt{ex^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*a*(sqrt(e*x^2 + d)*x/e - d*arcsinh(e*x/sqrt(d*e))/e^(3/2)) + b*integrat
e(x^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/sqrt(e*x^2 + d), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

```
[Out] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + b \,\mathrm{acsc}\,(cx))}{\sqrt{d + ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsc(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*acsc(c*x))/sqrt(d + e*x**2), x)
```

3.144 $\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCsc[c*x])/Sqrt[d + e*x^2],x]
[Out] Defer[Int][(a + b*ArcCsc[c*x])/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex^2}}\,dx = \int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Mathematica [A] time = 1.02, size = 0, normalized size = 0.00

$$\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCsc[c*x])/Sqrt[d + e*x^2],x]
[Out] Integrate[(a + b*ArcCsc[c*x])/Sqrt[d + e*x^2], x]
```

fricas [A] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arccsc}\,(cx)+a}{\sqrt{ex^2+d}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b*arccsc(c*x) + a)/sqrt(e*x^2 + d), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arccsc}\,(cx)+a}{\sqrt{ex^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)/sqrt(e*x^2 + d), x)
```

maple [A] time = 4.69, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arccsc}\left(cx\right)}{\sqrt{e\,x^{2}+d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$b\int \frac{\arctan\left(1, \sqrt{cx+1}\,\sqrt{cx-1}\right)}{\sqrt{ex^{2}+d}}\, dx + \frac{a\,\mathrm{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] b*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/sqrt(e*x^2 + d), x) + a
*arcsinh(e*x/sqrt(d*e))/sqrt(e)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{\sqrt{e\,x^{2}+d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x^2)^(1/2),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(d + e*x^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

3.145 $\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=247

$$-\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{dx}-\frac{bc\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{d\sqrt{c^2x^2}}-\frac{bx\sqrt{1-c^2x^2}\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\,F\left(\sin^{-1}(cx)|-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}+\frac{bc^2x}{}$$

```
[Out] -(a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/d/x-b*c*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2
)/d/(c^2*x^2)^(1/2)+b*c^2*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1
/2)*(e*x^2+d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)-b
*(c^2*d+e)*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)
^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {264, 5239, 12, 475, 21, 423, 427, 426, 424, 421, 419}

$$-\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{dx}-\frac{bc\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{d\sqrt{c^2x^2}}-\frac{bx\sqrt{1-c^2x^2}\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\,F\left(\sin^{-1}(cx)|-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}+\frac{bc^2x}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(x^2*Sqrt[d + e*x^2]),x]
```

```
[Out] -((b*c*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(d*Sqrt[c^2*x^2])) - (Sqrt[d + e
*x^2]*(a + b*ArcCsc[c*x]))/(d*x) + (b*c^2*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^
2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(d*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2
]*Sqrt[1 + (e*x^2)/d]) - (b*(c^2*d + e)*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2
)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(d*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x
^2]*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx = -\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{dx} - \frac{(bcx)\int \frac{\sqrt{d+ex^2}}{dx^2\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}$$

$$= -\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{dx} - \frac{(bcx)\int \frac{\sqrt{d+ex^2}}{x^2\sqrt{-1+c^2x^2}}\,dx}{d\sqrt{c^2x^2}}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{dx} + \frac{(bcx)\int \frac{-e+c^2ex^2}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{d\sqrt{c^2x^2}}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{dx} + \frac{(bcex)\int \frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\,dx}{d\sqrt{c^2x^2}}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{dx} + \frac{\left(bc^3x\right)\int \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\,dx}{d\sqrt{c^2x^2}} - \frac{\left(bc\left(c^{2}\right.\right.}{}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{dx} + \frac{\left(bc^3x\sqrt{1-c^2x^2}\right)\int \frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\,dx}{d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{dx} + \frac{\left(bc^3x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\right)\int}{d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+}}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{dx} + \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\,E\left(\sin\right.}{d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1}}$$

Mathematica [A] time = 0.20, size = 140, normalized size = 0.57

$$\frac{bcex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{\frac{ex^2}{d}+1}\,E\left(\sin^{-1}\left(\sqrt{-\frac{e}{d}}x\right)\middle|-\frac{c^2d}{e}\right)}{d\sqrt{1-c^2x^2}\sqrt{-\frac{e}{d}}\sqrt{d+ex^2}} - \frac{\sqrt{d+ex^2}\left(a+bcx\sqrt{1-\frac{1}{c^2x^2}}+b\csc^{-1}(cx)\right)}{dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x^2*Sqrt[d + e*x^2]),x]
```

```
[Out] -((Sqrt[d + e*x^2]*(a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x + b*ArcCsc[c*x]))/(d*x)
) + (b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*EllipticE[ArcSin[Sqr
t[-(e/d)]*x], -((c^2*d)/e)])/(d*Sqrt[-(e/d)]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x
^2])
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{ex^2+d}\,(b\,\mathrm{arccsc}\,(cx)+a)}{ex^4+dx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/(e*x^4 + d*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arccsc}(cx) + a}{\sqrt{ex^2 + d}\, x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)/(sqrt(e*x^2 + d)*x^2), x)
```

maple [F] time = 4.44, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arccsc}(cx)}{x^2 \sqrt{e\, x^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(1/2),x)
[Out] int((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(x \int \frac{\sqrt{ex^2+d}}{\sqrt{cx+1}\,\sqrt{cx-1}\,x^2}\, dx + \sqrt{ex^2 + d} \arctan\left(1, \sqrt{cx+1}\,\sqrt{cx-1}\right)\right)b}{dx} - \frac{\sqrt{ex^2 + d}\, a}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] -(d*x*integrate((c^2*e*x^2 + c^2*d)*e^(-1/2*log(e*x^2 + d) + 1/2*log(c*x +
1) + 1/2*log(c*x - 1))/(c^2*d*x^2 + (c^2*d*x^2 - d)*e^(log(c*x + 1) + log(c
*x - 1)) - d), x) + sqrt(e*x^2 + d)*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))
)*b/(d*x) - sqrt(e*x^2 + d)*a/(d*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{asin}\left(\frac{1}{c\,x}\right)}{x^2\, \sqrt{e\,x^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x^2)^(1/2)),x)
[Out] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsc}(cx)}{x^2 \sqrt{d + ex^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x**2/(e*x**2+d)**(1/2),x)
[Out] Integral((a + b*acsc(c*x))/(x**2*sqrt(d + e*x**2)), x)
```

3.146 $\int \frac{a+b\csc^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=362

$$\frac{2e\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{3dx^3}-\frac{bc\sqrt{c^2x^2-1}\left(2c^2d-5e\right)\sqrt{d+ex^2}}{9d^2\sqrt{c^2x^2}}-2bx\sqrt{1-c^2x^2}\ (c$$

```
[Out] -1/3*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/d/x^3+2/3*e*(a+b*arccsc(c*x))*(e*x^2
+d)^(1/2)/d^2/x-1/9*b*c*(2*c^2*d-5*e)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2
/(c^2*x^2)^(1/2)-1/9*b*c*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/x^2/(c^2*x^2)^
(1/2)+1/9*b*c^2*(2*c^2*d-5*e)*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1
)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(
1/2)-2/9*b*(c^2*d-3*e)*(c^2*d+e)*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^
2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*x^2+d
)^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {271, 264, 5239, 12, 580, 583, 524, 427, 426, 424, 421, 419}

$$\frac{2e\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{3dx^3}-\frac{bc\sqrt{c^2x^2-1}\left(2c^2d-5e\right)\sqrt{d+ex^2}}{9d^2\sqrt{c^2x^2}}-2bx\sqrt{1-c^2x^2}\ (c$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(x^4*Sqrt[d + e*x^2]),x]
```

```
[Out] -(b*c*(2*c^2*d - 5*e)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(9*d^2*Sqrt[c^2*x
^2]) - (b*c*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(9*d*x^2*Sqrt[c^2*x^2]) - (
Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/(3*d*x^3) + (2*e*Sqrt[d + e*x^2]*(a +
b*ArcCsc[c*x]))/(3*d^2*x) + (b*c^2*(2*c^2*d - 5*e)*x*Sqrt[1 - c^2*x^2]*Sqrt
[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(9*d^2*Sqrt[c^2*x^2]*Sqrt
[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) - (2*b*(c^2*d - 3*e)*(c^2*d + e)*x*Sqrt
[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(9*
d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx = -\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{3dx^3}+\frac{2e\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{3d^2x}+\frac{(bcx)\int\frac{\sqrt{d+ex^2}\left(-d+2ex^2\right)}{3d^2x^4\sqrt{-1+c^2x^2}}\,d}{\sqrt{c^2x^2}}$$

$$= -\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{3dx^3}+\frac{2e\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{3d^2x}+\frac{(bcx)\int\frac{\sqrt{d+ex^2}\left(-d+2ex^2\right)}{x^4\sqrt{-1+c^2x^2}}\,d}{3d^2\sqrt{c^2x^2}}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9dx^2\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{3dx^3}+\frac{2e\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{3d^2x}$$

$$= -\frac{bc\left(2c^2d-5e\right)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^2\sqrt{c^2x^2}}-\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9dx^2\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\csc^{-}\right.}{3dx^3}$$

$$= -\frac{bc\left(2c^2d-5e\right)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^2\sqrt{c^2x^2}}-\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9dx^2\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\csc^{-}\right.}{3dx^3}$$

$$= -\frac{bc\left(2c^2d-5e\right)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^2\sqrt{c^2x^2}}-\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9dx^2\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\csc^{-}\right.}{3dx^3}$$

$$= -\frac{bc\left(2c^2d-5e\right)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^2\sqrt{c^2x^2}}-\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9dx^2\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\csc^{-}\right.}{3dx^3}$$

$$= -\frac{bc\left(2c^2d-5e\right)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^2\sqrt{c^2x^2}}-\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9dx^2\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\csc^{-}\right.}{3dx^3}$$

Mathematica [C] time = 0.66, size = 249, normalized size = 0.69

$$-\frac{\sqrt{d+ex^2}\left(3a\left(d-2ex^2\right)+bcx\sqrt{1-\frac{1}{c^2x^2}}\left(2c^2dx^2+d-5ex^2\right)+3b\csc^{-1}(cx)\left(d-2ex^2\right)\right)}{9d^2x^3}+\frac{ibcx\sqrt{1-\frac{1}{c^2x^2}}\sqrt{\frac{ex^2}{d}+}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x^4*Sqrt[d + e*x^2]),x]
```

```
[Out] -1/9*(Sqrt[d + e*x^2]*(b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(d + 2*c^2*d*x^2 - 5*e*x
^2) + 3*a*(d - 2*e*x^2) + 3*b*(d - 2*e*x^2)*ArcCsc[c*x]))/(d^2*x^3) + ((I/9
)*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(2*c^2*d - 5*e)*El
lipticE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))] + 2*(-(c^4*d^2) + 2*c^2*d*e
```

```
+ 3*e^2)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-c^2]*d^2
*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2+d}\,(b\,\text{arccsc}\,(cx)+a)}{ex^6+dx^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/(e*x^6 + d*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arccsc}\,(cx)+a}{\sqrt{ex^2+d}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/(sqrt(e*x^2 + d)*x^4), x)
```

maple [F] time = 8.42, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\text{arccsc}\,(cx)}{x^4\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}a\left(\frac{2\sqrt{ex^2+d}\,e}{d^2x}-\frac{\sqrt{ex^2+d}}{dx^3}\right)+\frac{\left(2\,e^2x^4\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right)+\sqrt{ex^2+d}\,d^2x^3\int\frac{\left(2\,c^2e^2x^4+c^2dex^2-c^2d^2\right)e^{\left(-\frac{1}{2}\right.}}{c^2d^2x^4-d^2x^2+\left(c^2d\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*(2*sqrt(e*x^2 + d)*e/(d^2*x) - sqrt(e*x^2 + d)/(d*x^3)) + 1/3*(2*e^2*
x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 3*sqrt(e*x^2 + d)*d^2*x^3*int
egrate(1/3*(2*c^2*e^2*x^4 + c^2*d*e*x^2 - c^2*d^2)*e^(-1/2*log(e*x^2 + d) +
 1/2*log(c*x + 1) + 1/2*log(c*x - 1))/(c^2*d^2*x^4 - d^2*x^2 + (c^2*d^2*x^4
 - d^2*x^2)*e^(log(c*x + 1) + log(c*x - 1))), x) + d*e*x^2*arctan2(1, sqrt(
c*x + 1)*sqrt(c*x - 1)) - d^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*b/(s
qrt(e*x^2 + d)*d^2*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^4\,\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x^4*(d + e*x^2)^(1/2)),x)
[Out] int((a + b*asin(1/(c*x)))/(x^4*(d + e*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsc}(cx)}{x^4 \sqrt{d + ex^2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x**4/(e*x**2+d)**(1/2),x)
[Out] Integral((a + b*acsc(c*x))/(x**4*sqrt(d + e*x**2)), x)
```

3.147 $\int \frac{x^5\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=252

$$-\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{8bcd^{3/2}x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2}}\right)}{3e^3\sqrt{c^2x^2}}$$

```
[Out] 1/3*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/e^3+8/3*b*c*d^(3/2)*x*arctan((e*x^2+d
)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e^3/(c^2*x^2)^(1/2)-1/6*b*(9*c^2*d-e)*x*
arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^2/e^(5/2)/(c^2*x^2)^
(1/2)-d^2*(a+b*arccsc(c*x))/e^3/(e*x^2+d)^(1/2)-2*d*(a+b*arccsc(c*x))*(e*x^
2+d)^(1/2)/e^3+1/6*b*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c/e^2/(c^2*x^2)^(1
/2)
```

Rubi [A] time = 1.07, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {266, 43, 5239, 12, 1615, 157, 63, 217, 206, 93, 204}

$$-\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{8bcd^{3/2}x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2}}\right)}{3e^3\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] (b*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(6*c*e^2*Sqrt[c^2*x^2]) - (d^2*(a
+ b*ArcCsc[c*x]))/(e^3*Sqrt[d + e*x^2]) - (2*d*Sqrt[d + e*x^2]*(a + b*ArcCs
c[c*x]))/e^3 + ((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^3) + (8*b*c*d^(
3/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(3*e^3*Sqrt[c^
2*x^2]) - (b*(9*c^2*d - e)*x*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d
 + e*x^2])])/(6*c^2*e^(5/2)*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{x^5\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = -\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}\right.}{3e^3}$$

$$= -\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}\right.}{3e^3}$$

$$= -\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}\right.}{3e^3}$$

$$= \frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce^2\sqrt{c^2x^2}} - \frac{d^2\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3} +$$

$$= \frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce^2\sqrt{c^2x^2}} - \frac{d^2\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3} +$$

$$= \frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce^2\sqrt{c^2x^2}} - \frac{d^2\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3} +$$

$$= \frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce^2\sqrt{c^2x^2}} - \frac{d^2\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3} +$$

$$= \frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce^2\sqrt{c^2x^2}} - \frac{d^2\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3} +$$

Mathematica [A] time = 0.64, size = 302, normalized size = 1.20

$$\frac{-2ac\left(8d^2+4dex^2-e^2x^4\right)+bex\sqrt{1-\frac{1}{c^2x^2}}\left(d+ex^2\right)-2bc\csc^{-1}(cx)\left(8d^2+4dex^2-e^2x^4\right)}{6ce^3\sqrt{d+ex^2}} - \frac{bx\sqrt{1-\frac{1}{c^2x^2}}\left(\sqrt{c^2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] (b*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2) - 2*a*c*(8*d^2 + 4*d*e*x^2 - e^2*x
^4) - 2*b*c*(8*d^2 + 4*d*e*x^2 - e^2*x^4)*ArcCsc[c*x])/(6*c*e^3*Sqrt[d + e*
x^2]) - (b*Sqrt[1 - 1/(c^2*x^2)]*x*(Sqrt[c^2]*(9*c^2*d - e)*Sqrt[e]*Sqrt[c^
2*d + e]*Sqrt[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcSinh[(c*Sqrt[e]*Sqrt[-1 + c
^2*x^2])/(Sqrt[c^2]*Sqrt[c^2*d + e])] + 16*c^5*d^(3/2)*Sqrt[d + e*x^2]*ArcT
an[(Sqrt[d]*Sqrt[-1 + c^2*x^2])/Sqrt[d + e*x^2]]))/(6*c^4*e^3*Sqrt[-1 + c^2
*x^2]*Sqrt[d + e*x^2])
```

fricas [A] time = 1.68, size = 1480, normalized size = 5.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/24*((9*b*c^2*d^2 - b*d*e + (9*b*c^2*d*e - b*e^2)*x^2)*sqrt(e)*log(8*c^4
*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2
```

```
 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) - 16*(b*c^
3*d*e*x^2 + b*c^3*d^2)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c
^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2
+ d)*sqrt(-d) + 8*d^2)/x^4) - 4*(2*a*c^3*e^2*x^4 - 8*a*c^3*d*e*x^2 - 16*a*c
^3*d^2 + 2*(b*c^3*e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b*c^3*d^2)*arccsc(c*x) + (b
*c*e^2*x^2 + b*c*d*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^3*e^4*x^2 + c^
3*d*e^3), 1/24*(32*(b*c^3*d*e*x^2 + b*c^3*d^2)*sqrt(d)*arctan(-1/2*sqrt(c^2
*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c
^2*d^2 - d*e)*x^2 - d^2)) - (9*b*c^2*d^2 - b*d*e + (9*b*c^2*d*e - b*e^2)*x^
2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*
x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(
e) + e^2) + 4*(2*a*c^3*e^2*x^4 - 8*a*c^3*d*e*x^2 - 16*a*c^3*d^2 + 2*(b*c^3*
e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b*c^3*d^2)*arccsc(c*x) + (b*c*e^2*x^2 + b*c*d
*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^3*e^4*x^2 + c^3*d*e^3), 1/12*((9
*b*c^2*d^2 - b*d*e + (9*b*c^2*d*e - b*e^2)*x^2)*sqrt(-e)*arctan(1/2*(2*c^2*
e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4
- c*d*e + (c^3*d*e - c*e^2)*x^2)) + 8*(b*c^3*d*e*x^2 + b*c^3*d^2)*sqrt(-d)*
log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x
^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) + 2*
(2*a*c^3*e^2*x^4 - 8*a*c^3*d*e*x^2 - 16*a*c^3*d^2 + 2*(b*c^3*e^2*x^4 - 4*b*
c^3*d*e*x^2 - 8*b*c^3*d^2)*arccsc(c*x) + (b*c*e^2*x^2 + b*c*d*e)*sqrt(c^2*x
^2 - 1))*sqrt(e*x^2 + d))/(c^3*e^4*x^2 + c^3*d*e^3), 1/12*(16*(b*c^3*d*e*x^
2 + b*c^3*d^2)*sqrt(d)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d
)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (9*b
*c^2*d^2 - b*d*e + (9*b*c^2*d*e - b*e^2)*x^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*
x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 -
c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(2*a*c^3*e^2*x^4 - 8*a*c^3*d*e*x^2 - 16
*a*c^3*d^2 + 2*(b*c^3*e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b*c^3*d^2)*arccsc(c*x)
+ (b*c*e^2*x^2 + b*c*d*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^3*e^4*x^2
+ c^3*d*e^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x^5}{\left(ex^2 + d\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^5/(e*x^2 + d)^(3/2), x)
```

maple [F] time = 7.40, size = 0, normalized size = 0.00

$$\int \frac{x^5 (a + b \operatorname{arccsc}(cx))}{\left(e\,x^2 + d\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\left(\frac{x^4}{\sqrt{ex^2+d}\,e} - \frac{4\,dx^2}{\sqrt{ex^2+d}\,e^2} - \frac{8\,d^2}{\sqrt{ex^2+d}\,e^3}\right)a + \frac{\left(e^2x^4 \arctan\left(1, \sqrt{cx+1}\,\sqrt{cx-1}\right) - 4\,dex^2 \arctan\left(1, \sqrt{cx+1}\,\sqrt{c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*(x^4/(sqrt(e*x^2 + d)*e) - 4*d*x^2/(sqrt(e*x^2 + d)*e^2) - 8*d^2/(sqrt(
e*x^2 + d)*e^3))*a + 1/3*(e^2*x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) -
 4*d*e*x^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 3*sqrt(e*x^2 + d)*e^3*
integrate(1/3*(c^2*e^2*x^5 - 4*c^2*d*e*x^3 - 8*c^2*d^2*x)*e^(-1/2*log(e*x^2
 + d) + 1/2*log(c*x + 1) + 1/2*log(c*x - 1))/(c^2*e^3*x^2 - e^3 + (c^2*e^3*
x^2 - e^3)*e^(log(c*x + 1) + log(c*x - 1))), x) - 8*d^2*arctan2(1, sqrt(c*x
 + 1)*sqrt(c*x - 1)))*b/(sqrt(e*x^2 + d)*e^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

```
[Out] int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acsc(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

3.148 $\int \frac{x^3\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=156

$$\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{d\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}+\frac{bx\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{e^{3/2}\sqrt{c^2x^2}}-\frac{2bc\sqrt{d}\,x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{e^2\sqrt{c^2x^2}}$$

```
[Out] b*x*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/e^(3/2)/(c^2*x^2)^
(1/2)-2*b*c*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))*d^(1/2)/e^2
/(c^2*x^2)^(1/2)+d*(a+b*arccsc(c*x))/e^2/(e*x^2+d)^(1/2)+(a+b*arccsc(c*x))*
(e*x^2+d)^(1/2)/e^2
```

Rubi [A] time = 0.32, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {266, 43, 5239, 12, 573, 157, 63, 217, 206, 93, 204}

$$\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{d\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}-\frac{2bc\sqrt{d}\,x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{e^2\sqrt{c^2x^2}}+\frac{bx\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{e^{3/2}\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] (d*(a + b*ArcCsc[c*x]))/(e^2*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(a + b*Arc
Csc[c*x]))/e^2 - (2*b*c*Sqrt[d]*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 +
 c^2*x^2])])/(e^2*Sqrt[c^2*x^2]) + (b*x*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2]
)/(c*Sqrt[d + e*x^2])])/(e^(3/2)*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 573

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{x^3\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}dx = \frac{d\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^2} + \frac{(bcx)\int \frac{2d+ex^2}{e^2x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}dx}{\sqrt{c^2x^2}}$$

$$= \frac{d\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^2} + \frac{(bcx)\int \frac{2d+ex^2}{x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}dx}{e^2\sqrt{c^2x^2}}$$

$$= \frac{d\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^2} + \frac{(bcx)\,\mathrm{Subst}\left(\int \frac{2d+ex}{x\sqrt{-1+c^2x}\sqrt{d+ex}}d\right.}{2e^2\sqrt{c^2x^2}}$$

$$= \frac{d\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^2} + \frac{(bcdx)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\right.}{e^2\sqrt{c^2x^2}}$$

$$= \frac{d\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^2} + \frac{(2bcdx)\,\mathrm{Subst}\left(\int \frac{1}{-d-x^2}dx, x, \right.}{e^2\sqrt{c^2x^2}}$$

$$= \frac{d\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^2} - \frac{2bc\sqrt{d}\,x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{e^2\sqrt{c^2x^2}} +$$

$$= \frac{d\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^2} - \frac{2bc\sqrt{d}\,x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{e^2\sqrt{c^2x^2}} +$$

Mathematica [A] time = 0.36, size = 220, normalized size = 1.41

$$\frac{\left(2d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}} + \frac{bx\sqrt{1-\frac{1}{c^2x^2}}\left(\sqrt{c^2}\sqrt{e}\sqrt{c^2d+e}\sqrt{\frac{c^2(d+ex^2)}{c^2d+e}}\sinh^{-1}\left(\frac{c\sqrt{e}\sqrt{c^2x^2-1}}{\sqrt{c^2}\sqrt{c^2d+e}}\right) + 2c^3\sqrt{d}\sqrt{d+ex^2}\right.}{c^2e^2\sqrt{c^2x^2-1}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] ((2*d + e*x^2)*(a + b*ArcCsc[c*x]))/(e^2*Sqrt[d + e*x^2]) + (b*Sqrt[1 - 1/(
c^2*x^2)]*x*(Sqrt[c^2]*Sqrt[e]*Sqrt[c^2*d + e]*Sqrt[(c^2*(d + e*x^2))/(c^2*
d + e)]*ArcSinh[(c*Sqrt[e]*Sqrt[-1 + c^2*x^2])/(Sqrt[c^2]*Sqrt[c^2*d + e])]
 + 2*c^3*Sqrt[d]*Sqrt[d + e*x^2]*ArcTan[(Sqrt[d]*Sqrt[-1 + c^2*x^2])/Sqrt[d
 + e*x^2]]))/(c^2*e^2*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

fricas [A] time = 0.96, size = 1072, normalized size = 6.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b*e*x^2 + b*d)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(
c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sq
rt(e*x^2 + d)*sqrt(e) + e^2) + 2*(b*c*e*x^2 + b*c*d)*sqrt(-d)*log(((c^4*d^2
 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2
*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) + 4*(a*c*e*x^2 +
2*a*c*d + (b*c*e*x^2 + 2*b*c*d)*arccsc(c*x))*sqrt(e*x^2 + d))/(c*e^3*x^2 +
```

```
c*d*e^2), -1/4*(4*(b*c*e*x^2 + b*c*d)*sqrt(d)*arctan(-1/2*sqrt(c^2*x^2 - 1)
*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 -
d*e)*x^2 - d^2)) - (b*e*x^2 + b*d)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*
c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^
2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) - 4*(a*c*e*x^2 + 2*a*c*d + (b*c*e
*x^2 + 2*b*c*d)*arccsc(c*x))*sqrt(e*x^2 + d))/(c*e^3*x^2 + c*d*e^2), -1/2*(
(b*e*x^2 + b*d)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2
- 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)
) - (b*c*e*x^2 + b*c*d)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(
c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2
 + d)*sqrt(-d) + 8*d^2)/x^4) - 2*(a*c*e*x^2 + 2*a*c*d + (b*c*e*x^2 + 2*b*c*
d)*arccsc(c*x))*sqrt(e*x^2 + d))/(c*e^3*x^2 + c*d*e^2), -1/2*(2*(b*c*e*x^2
+ b*c*d)*sqrt(d)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt
(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (b*e*x^2 +
 b*d)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(
e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) - 2*(a*c
*e*x^2 + 2*a*c*d + (b*c*e*x^2 + 2*b*c*d)*arccsc(c*x))*sqrt(e*x^2 + d))/(c*e
^3*x^2 + c*d*e^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x^3}{\left(ex^2 + d\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)*x^3/(e*x^2 + d)^(3/2), x)
```

maple [F] time = 6.68, size = 0, normalized size = 0.00

$$\int \frac{x^3 \left(a + b \operatorname{arccsc}(cx)\right)}{\left(e\, x^2 + d\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
[Out] int(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \left(a + b \operatorname{asin}\left(\frac{1}{c\, x}\right)\right)}{\left(e\, x^2 + d\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

```
[Out] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 (a + b \operatorname{acsc}(cx))}{\left(d + ex^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsc(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**3*(a + b*acsc(c*x))/(d + e*x**2)**(3/2), x)
```

3.149 $\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=79

$$\frac{bcx\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{\sqrt{d}\,e\sqrt{c^2x^2}}-\frac{a+b\csc^{-1}(cx)}{e\sqrt{d+ex^2}}$$

```
[Out] b*c*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e/d^(1/2)/(c^2*x^2)
^(1/2)+(-a-b*arccsc(c*x))/e/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5237, 446, 93, 204}

$$\frac{bcx\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{\sqrt{d}\,e\sqrt{c^2x^2}}-\frac{a+b\csc^{-1}(cx)}{e\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] -((a + b*ArcCsc[c*x])/(e*Sqrt[d + e*x^2])) + (b*c*x*ArcTan[Sqrt[d + e*x^2]/
(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(Sqrt[d]*e*Sqrt[c^2*x^2])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5237

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCsc[c*x]))/(2*e*(p + 1)), x
] + Dist[(b*c*x)/(2*e*(p + 1)*Sqrt[c^2*x^2]), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = -\frac{a+b\csc^{-1}(cx)}{e\sqrt{d+ex^2}} - \frac{(bcx)\int \frac{1}{x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{e\sqrt{c^2x^2}}$$

$$= -\frac{a+b\csc^{-1}(cx)}{e\sqrt{d+ex^2}} - \frac{(bcx)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{2e\sqrt{c^2x^2}}$$

$$= -\frac{a+b\csc^{-1}(cx)}{e\sqrt{d+ex^2}} - \frac{(bcx)\,\mathrm{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\right)}{e\sqrt{c^2x^2}}$$

$$= -\frac{a+b\csc^{-1}(cx)}{e\sqrt{d+ex^2}} + \frac{bcx\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{\sqrt{d}\,e\sqrt{c^2x^2}}$$

Mathematica [A] time = 0.16, size = 96, normalized size = 1.22

$$-\frac{a+b\csc^{-1}(cx)}{e\sqrt{d+ex^2}} - \frac{bcx\sqrt{1-\frac{1}{c^2x^2}}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{c^2x^2-1}}{\sqrt{d+ex^2}}\right)}{\sqrt{d}\,e\sqrt{c^2x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] -((a + b*ArcCsc[c*x])/(e*Sqrt[d + e*x^2])) - (b*c*Sqrt[1 - 1/(c^2*x^2)]*x*A
rcTan[(Sqrt[d]*Sqrt[-1 + c^2*x^2])/Sqrt[d + e*x^2]])/(Sqrt[d]*e*Sqrt[-1 + c
^2*x^2])
```

fricas [A] time = 0.71, size = 283, normalized size = 3.58

$$\left[-\frac{\left(bex^2+bd\right)\sqrt{-d}\log\left(\frac{\left(c^4d^2-6c^2de+e^2\right)x^4-8\left(c^2d^2-de\right)x^2+4\sqrt{c^2x^2-1}\left(\left(c^2d-e\right)x^2-2d\right)\sqrt{ex^2+d}\sqrt{-d}+8d^2}{x^4}\right)+4\sqrt{ex^2+d}\,(bd\,\mathrm{arccsc}}{4\left(de^2x^2+d^2e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b*e*x^2 + b*d)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c
^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2
+ d)*sqrt(-d) + 8*d^2)/x^4) + 4*sqrt(e*x^2 + d)*(b*d*arccsc(c*x) + a*d))/(d
*e^2*x^2 + d^2*e), 1/2*((b*e*x^2 + b*d)*sqrt(d)*arctan(-1/2*sqrt(c^2*x^2 -
1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2
- d*e)*x^2 - d^2)) - 2*sqrt(e*x^2 + d)*(b*d*arccsc(c*x) + a*d))/(d*e^2*x^2
+ d^2*e)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x}{\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x/(e*x^2 + d)^(3/2), x)
```

maple [F] time = 4.79, size = 0, normalized size = 0.00

$$\int \frac{x \left(a+b \,\mathrm{arccsc}\left(c x\right)\right)}{\left(e \,x^{2}+d \right)^{\frac{3}{2}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(\sqrt{ex^2+d}\int \frac{1}{\sqrt{ex^2+d}\,\sqrt{cx+1}\,\sqrt{cx-1}\,x}\,dx+\arctan\left(1, \sqrt{cx+1}\,\sqrt{cx-1}\right)\right)b}{\sqrt{ex^2+d}\,e}-\frac{a}{\sqrt{ex^2+d}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] -(sqrt(e*x^2 + d)*c^2*e*integrate(x*e^(-1/2*log(e*x^2 + d) + 1/2*log(c*x +
1) + 1/2*log(c*x - 1))/(c^2*e*x^2 + (c^2*e*x^2 - e)*e^(log(c*x + 1) + log(c
*x - 1)) - e), x) + arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*b/(sqrt(e*x^2
+ d)*e) - a/(sqrt(e*x^2 + d)*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

```
[Out] int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \left(a + b \operatorname{acsc}{\left(c x \right)}\right)}{\left(d + e x^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsc(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral(x*(a + b*acsc(c*x))/(d + e*x**2)**(3/2), x)
```

3.150 $\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=26

$$\mathrm{Int}\left(\frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x/(e*x^2+d)^(3/2),x)
```

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^(3/2)),x]
```

```
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^(3/2)), x]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [A] time = 19.37, size = 0, normalized size = 0.00

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^(3/2)),x]
```

```
[Out] Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^(3/2)), x]
```

fricas [A] time = 0.68, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{ex^2+d}\,(b\,\mathrm{arccsc}\,(cx)+a)}{e^2x^5+2\,dex^3+d^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x),
 x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\mathrm{arccsc}\,(cx)+a}{\left(ex^2+d\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)/((e*x^2 + d)^(3/2)*x), x)
```

maple [A] time = 3.84, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arccsc}\left(c x\right)}{x \left(e\,x^{2}+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x/(e*x^2+d)^(3/2),x)
[Out] int((a+b*arccsc(c*x))/x/(e*x^2+d)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-a\left(\frac{\mathrm{arsinh}\left(\frac{d}{\sqrt{de}\,|x|}\right)}{d^{\frac{3}{2}}}-\frac{1}{\sqrt{ex^{2}+d}\,d}\right)+b\int \frac{\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right)}{\left(ex^{3}+dx\right)\sqrt{ex^{2}+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="maxima")
[Out] -a*(arcsinh(d/(sqrt(d*e)*abs(x)))/d^(3/2) - 1/(sqrt(e*x^2 + d)*d)) + b*inte
grate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/((e*x^3 + d*x)*sqrt(e*x^2 + d
)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x\,{\left(e\,x^2+d\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^(3/2)),x)
[Out] int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x/(e*x**2+d)**(3/2),x)
[Out] Timed out
```

3.151 $\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=26

$$\operatorname{Int}\left(\frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

Rubi [A] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCsc[c*x])/(x^3*(d + e*x^2)^(3/2)),x]
```

```
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x^3*(d + e*x^2)^(3/2)), x]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [A] time = 25.01, size = 0, normalized size = 0.00

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x^3*(d + e*x^2)^(3/2)),x]
```

```
[Out] Integrate[(a + b*ArcCsc[c*x])/(x^3*(d + e*x^2)^(3/2)), x]
```

fricas [A] time = 0.76, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{ex^2+d}\,(b\operatorname{arccsc}(cx)+a)}{e^2x^7+2\,dex^5+d^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/(e^2*x^7 + 2*d*e*x^5 + d^2*x^3
), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\operatorname{arccsc}(cx)+a}{\left(ex^2+d\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/((e*x^2 + d)^(3/2)*x^3), x)
```

maple [A] time = 1.34, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arccsc}(cx)}{x^3 \left(e\,x^2 + d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a\left(\frac{3\,e\operatorname{arsinh}\left(\frac{d}{\sqrt{de}\,|x|}\right)}{d^{\frac{5}{2}}} - \frac{3\,e}{\sqrt{ex^2+d}\,d^2} - \frac{1}{\sqrt{ex^2+d}\,dx^2}\right) + b\int \frac{\arctan\left(1, \sqrt{cx+1}\,\sqrt{cx-1}\right)}{\left(ex^5+dx^3\right)\sqrt{ex^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*a*(3*e*arcsinh(d/(sqrt(d*e)*abs(x)))/d^(5/2) - 3*e/(sqrt(e*x^2 + d)*d^2
) - 1/(sqrt(e*x^2 + d)*d*x^2)) + b*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(
c*x - 1))/((e*x^5 + d*x^3)*sqrt(e*x^2 + d)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b \operatorname{asin}\left(\frac{1}{c\,x}\right)}{x^3 \left(e\,x^2 + d\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x^3*(d + e*x^2)^(3/2)),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(x^3*(d + e*x^2)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x**3/(e*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

3.152 $\int \frac{x^4\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{x^4\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^4\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] Defer[Int][(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^4\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{x^4\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [A] time = 9.54, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] Integrate[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2), x]
```

fricas [A] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4\,\text{arccsc}\,(cx)+ax^4\right)\sqrt{ex^2+d}}{e^2x^4+2\,dex^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arccsc(c*x) + a*x^4)*sqrt(e*x^2 + d)/(e^2*x^4 + 2*d*e*x^2 +
 d^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arccsc}\,(cx)+a)x^4}{\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)*x^4/(e*x^2 + d)^(3/2), x)
```

maple [A] time = 6.20, size = 0, normalized size = 0.00

$$\int \frac{x^{4} \left(a + b \,\mathrm{arccsc}\left(c x\right)\right)}{\left(e \,x^{2}+d \right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
[Out] int(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} \left(\frac{x^{3}}{\sqrt{ex^{2}+d}\, e} + \frac{3\,dx}{\sqrt{ex^{2}+d}\, e^{2}} - \frac{3\,d\,\mathrm{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{e^{\frac{5}{2}}}\right)a + b\int \frac{x^{4} \arctan\left(1, \sqrt{cx+1}\,\sqrt{cx-1}\right)}{\left(ex^{2}+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
[Out] 1/2*(x^3/(sqrt(e*x^2 + d)*e) + 3*d*x/(sqrt(e*x^2 + d)*e^2) - 3*d*arcsinh(e*
x/sqrt(d*e))/e^(5/2))*a + b*integrate(x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x
 - 1))/(e*x^2 + d)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^4\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2),x)
[Out] int((x^4*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acsc(c*x))/(e*x**2+d)**(3/2),x)
[Out] Timed out
```

3.153 $\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{x^2\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
[Out] Defer[Int][(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [A] time = 4.83, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
[Out] Integrate[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2), x]
```

fricas [A] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2\operatorname{arccsc}(cx)+ax^2\right)\sqrt{ex^2+d}}{e^2x^4+2\,dex^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
[Out] integral((b*x^2*arccsc(c*x) + a*x^2)*sqrt(e*x^2 + d)/(e^2*x^4 + 2*d*e*x^2 +
 d^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arccsc}(cx)+a)x^2}{\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)*x^2/(e*x^2 + d)^(3/2), x)
```

maple [A] time = 6.92, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + b \,\mathrm{arccsc}\,(cx))}{\left(e\,x^2 + d\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
[Out] int(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-a\left(\frac{x}{\sqrt{ex^2+d}\,e} - \frac{\mathrm{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{e^{\frac{3}{2}}}\right) + b\int \frac{x^2 \arctan\left(1, \sqrt{cx+1}\,\sqrt{cx-1}\right)}{\left(ex^2+d\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
[Out] -a*(x/(sqrt(e*x^2 + d)*e) - arcsinh(e*x/sqrt(d*e))/e^(3/2)) + b*integrate(x
^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x^2 + d)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^2 \left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2),x)
[Out] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + b \,\mathrm{acsc}\,(cx))}{\left(d + ex^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsc(c*x))/(e*x**2+d)**(3/2),x)
[Out] Integral(x**2*(a + b*acsc(c*x))/(d + e*x**2)**(3/2), x)
```

3.154 $\int \frac{a+b\csc^{-1}(cx)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=108

$$\frac{x\left(a+b\csc^{-1}(cx)\right)}{d\sqrt{d+ex^2}}+\frac{bx\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\,F\left(\sin^{-1}(cx)|-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}$$

```
[Out] x*(a+b*arccsc(c*x))/d/(e*x^2+d)^(1/2)+b*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(
-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*
x^2+d)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {191, 5229, 12, 421, 419}

$$\frac{x\left(a+b\csc^{-1}(cx)\right)}{d\sqrt{d+ex^2}}+\frac{bx\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\,F\left(\sin^{-1}(cx)|-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(d + e*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcCsc[c*x]))/(d*Sqrt[d + e*x^2]) + (b*x*Sqrt[1 - c^2*x^2]*Sqrt[1
 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(d*Sqrt[c^2*x^2]*Sqrt[-
1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 5229

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCsc[c*x], u, x]
+ Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1])
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1/2,
0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\csc^{-1}(cx)}{\left(d+ex^2\right)^{3/2}}\,dx &= \frac{x\left(a+b\csc^{-1}(cx)\right)}{d\sqrt{d+ex^2}}+\frac{(bcx)\int \frac{1}{d\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{\sqrt{c^2x^2}}\\
&= \frac{x\left(a+b\csc^{-1}(cx)\right)}{d\sqrt{d+ex^2}}+\frac{(bcx)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{d\sqrt{c^2x^2}}\\
&= \frac{x\left(a+b\csc^{-1}(cx)\right)}{d\sqrt{d+ex^2}}+\frac{\left(bcx\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{d\sqrt{c^2x^2}\sqrt{d+ex^2}}\\
&= \frac{x\left(a+b\csc^{-1}(cx)\right)}{d\sqrt{d+ex^2}}+\frac{\left(bcx\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\\
&= \frac{x\left(a+b\csc^{-1}(cx)\right)}{d\sqrt{d+ex^2}}+\frac{bx\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,F\left(\sin^{-1}(cx)\middle|-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}$$

Mathematica [A] time = 0.21, size = 112, normalized size = 1.04

$$\frac{x\left(a+b\csc^{-1}(cx)\right)}{d\sqrt{d+ex^2}}+\frac{bcx\sqrt{1-\frac{1}{c^2x^2}}\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\,F\left(\sin^{-1}(cx)\middle|-\frac{e}{c^2d}\right)}{d\left(c^3x^2-c\right)\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(d + e*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcCsc[c*x]))/(d*Sqrt[d + e*x^2]) + (b*c*Sqrt[1 - 1/(c^2*x^2)]*x*
Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])
/(d*(-c + c^3*x^2)*Sqrt[d + e*x^2])
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{ex^2+d}\,(b\,\mathrm{arccsc}\,(cx)+a)}{e^2x^4+2\,dex^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/(e^2*x^4 + 2*d*e*x^2 + d^2), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\mathrm{arccsc}\,(cx)+a}{\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/(e*x^2 + d)^(3/2), x)
```

maple [F] time = 4.35, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{arccsc}\,(cx)}{\left(e\,x^{2} + d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\int \frac{\arctan\left(1, \sqrt{cx+1}\,\sqrt{cx-1}\right)}{\left(ex^{2} + d\right)^{\frac{3}{2}}}\,dx + \frac{ax}{\sqrt{ex^{2} + d}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] b*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x^2 + d)^(3/2), x) +
 a*x/(sqrt(e*x^2 + d)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{\left(e\,x^{2} + d\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x^2)^(3/2),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(d + e*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{acsc}\,(cx)}{\left(d + ex^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acsc(c*x))/(d + e*x**2)**(3/2), x)
```

3.155 $\int \frac{a+b\csc^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=275

$$-\frac{2ex\left(a+b\csc^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}}-\frac{bc\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{d^2\sqrt{c^2x^2}}-\frac{bx\sqrt{1-c^2x^2}\left(c^2d+2e\right)\sqrt{\frac{ex^2}{d}+1}\,F\left(\sin^{-1}(cx\right.}{d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}$$

```
[Out] (-a-b*arccsc(c*x))/d/x/(e*x^2+d)^(1/2)-2*e*x*(a+b*arccsc(c*x))/d^2/(e*x^2+d
)^(1/2)-b*c*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)+b*c^2*x*E
llipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*
x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)-b*(c^2*d+2*e)*x*EllipticF(c*
x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d^2/(c^2*x^2)^(1/2
)/(c^2*x^2-1)^(1/2)/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {271, 191, 5239, 12, 583, 524, 427, 426, 424, 421, 419}

$$-\frac{2ex\left(a+b\csc^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}}-\frac{bc\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{d^2\sqrt{c^2x^2}}-\frac{bx\sqrt{1-c^2x^2}\left(c^2d+2e\right)\sqrt{\frac{ex^2}{d}+1}\,F\left(\sin^{-1}(cx\right.}{d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(x^2*(d + e*x^2)^(3/2)),x]
```

```
[Out] -((b*c*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(d^2*Sqrt[c^2*x^2])) - (a + b*Ar
cCsc[c*x])/(d*x*Sqrt[d + e*x^2]) - (2*e*x*(a + b*ArcCsc[c*x]))/(d^2*Sqrt[d
+ e*x^2]) + (b*c^2*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x
], -(e/(c^2*d))])/(d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]
) - (b*(c^2*d + 2*e)*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcS
in[c*x], -(e/(c^2*d))])/(d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^
2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx = -\frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex\left(a+b\csc^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} + \frac{(bcx)\int \frac{-d-2ex^2}{d^2x^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{\sqrt{c^2x^2}}$$

$$= -\frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex\left(a+b\csc^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} + \frac{(bcx)\int \frac{-d-2ex^2}{x^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{d^2\sqrt{c^2x^2}}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{c^2x^2}} - \frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex\left(a+b\csc^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} + \frac{(bcx)\int \frac{-2de+}{\sqrt{-1+c^2x}}}{d^3\sqrt{c^2}}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{c^2x^2}} - \frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex\left(a+b\csc^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} + \frac{\left(bc^3x\right)\int \frac{\sqrt{d+ex}}{\sqrt{-1+c^2}}}{d^2\sqrt{c^2x^2}}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{c^2x^2}} - \frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex\left(a+b\csc^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} + \frac{\left(bc^3x\sqrt{1-c^2x^2}\right.}{d^2\sqrt{c^2x^2}\,\sqrt{}}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{c^2x^2}} - \frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex\left(a+b\csc^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} + \frac{\left(bc^3x\sqrt{1-c^2x^2}\right.}{d^2\sqrt{c^2x^2}\,\sqrt{}}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{c^2x^2}} - \frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex\left(a+b\csc^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} + \frac{bc^2x\sqrt{1-c^2x^2}}{d^2\sqrt{c^2x^2}}$$

Mathematica [C] time = 0.49, size = 213, normalized size = 0.77

$$\frac{-a\left(d+2ex^2\right)-bcx\sqrt{1-\frac{1}{c^2x^2}}\left(d+ex^2\right)-b\csc^{-1}(cx)\left(d+2ex^2\right)}{d^2x\sqrt{d+ex^2}} + \frac{ibcx\sqrt{1-\frac{1}{c^2x^2}}\sqrt{\frac{ex^2}{d}+1}\left(c^2dE\left(i\sinh^{-1}\left(\sqrt{-}\right.\right.\right.}{\sqrt{-c^2}\,d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x^2*(d + e*x^2)^(3/2)),x]
```

```
[Out] (-(b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2)) - a*(d + 2*e*x^2) - b*(d + 2*e*
x^2)*ArcCsc[c*x])/(d^2*x*Sqrt[d + e*x^2]) + (I*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*
Sqrt[1 + (e*x^2)/d]*(c^2*d*EllipticE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]
 - (c^2*d + 2*e)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-
c^2]*d^2*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2+d}\,(b\,\text{arccsc}\,(cx)+a)}{e^2x^6+2\,dex^4+d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/(e^2*x^6 + 2*d*e*x^4 + d^2*x^2
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arccsc}\,(cx)+a}{\left(ex^2+d\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/((e*x^2 + d)^(3/2)*x^2), x)
```

maple [F] time = 4.03, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{arccsc}\,(cx)}{x^2 \left(e\,x^2 + d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(3/2),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^2 \left(e\,x^2 + d\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x^2)^(3/2)),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x^2)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x**2/(e*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

3.156 $\int \frac{x^5\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=243

$$-\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{bx\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{e^{5/2}\sqrt{c^2x^2}}-\frac{8bc\sqrt{d}\,x\,\text{ta}}{3e}$$

```
[Out] -1/3*d^2*(a+b*arccsc(c*x))/e^3/(e*x^2+d)^(3/2)+b*x*arctanh(e^(1/2)*(c^2*x^2
-1)^(1/2)/c/(e*x^2+d)^(1/2))/e^(5/2)/(c^2*x^2)^(1/2)-8/3*b*c*x*arctan((e*x^
2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))*d^(1/2)/e^3/(c^2*x^2)^(1/2)+2*d*(a+b*
arccsc(c*x))/e^3/(e*x^2+d)^(1/2)+1/3*b*c*d*x*(c^2*x^2-1)^(1/2)/e^2/(c^2*d+e
)/(c^2*x^2)^(1/2)/(e*x^2+d)^(1/2)+(a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/e^3
```

Rubi [A] time = 1.06, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {266, 43, 5239, 12, 1614, 157, 63, 217, 206, 93, 204}

$$-\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{bcdx\sqrt{c^2x^2-1}}{3e^2\sqrt{c^2x^2}\left(c^2d+e\right)\sqrt{d+ex^2}}-\frac{8bc}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*c*d*x*Sqrt[-1 + c^2*x^2])/(3*e^2*(c^2*d + e)*Sqrt[c^2*x^2]*Sqrt[d + e*x^
2]) - (d^2*(a + b*ArcCsc[c*x]))/(3*e^3*(d + e*x^2)^(3/2)) + (2*d*(a + b*Arc
Csc[c*x]))/(e^3*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/e^
3 - (8*b*c*Sqrt[d]*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/
(3*e^3*Sqrt[c^2*x^2]) + (b*x*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d
 + e*x^2])])/(e^(5/2)*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1614

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[(b*R*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{x^5 \left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = -\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3} + \frac{(bc\cdots}{\cdots}$$

$$= -\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3} + \frac{(bc\cdots}{\cdots}$$

$$= -\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3} + \frac{(bc\cdots}{\cdots}$$

$$= \frac{bcdx\sqrt{-1+c^2x^2}}{3e^2\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d\cdots}}{\cdots}$$

$$= \frac{bcdx\sqrt{-1+c^2x^2}}{3e^2\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d\cdots}}{\cdots}$$

$$= \frac{bcdx\sqrt{-1+c^2x^2}}{3e^2\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d\cdots}}{\cdots}$$

$$= \frac{bcdx\sqrt{-1+c^2x^2}}{3e^2\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d\cdots}}{\cdots}$$

$$= \frac{bcdx\sqrt{-1+c^2x^2}}{3e^2\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d\cdots}}{\cdots}$$

Mathematica [A] time = 0.54, size = 311, normalized size = 1.28

$$\frac{a\left(c^2d+e\right)\left(8d^2+12dex^2+3e^2x^4\right)+b\left(c^2d+e\right)\csc^{-1}(cx)\left(8d^2+12dex^2+3e^2x^4\right)+bcdex\sqrt{1-\frac{1}{c^2x^2}}\left(d+ex^2\right)\cdots}{3e^3\left(c^2d+e\right)\left(d+ex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*c*d*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2) + a*(c^2*d + e)*(8*d^2 + 12*d*
e*x^2 + 3*e^2*x^4) + b*(c^2*d + e)*(8*d^2 + 12*d*e*x^2 + 3*e^2*x^4)*ArcCsc[
c*x])/(3*e^3*(c^2*d + e)*(d + e*x^2)^(3/2)) + (b*Sqrt[1 - 1/(c^2*x^2)]*x*(3
*Sqrt[c^2]*Sqrt[e]*Sqrt[c^2*d + e]*Sqrt[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcS
inh[(c*Sqrt[e]*Sqrt[-1 + c^2*x^2])/(Sqrt[c^2]*Sqrt[c^2*d + e])] + 8*c^3*Sqr
t[d]*Sqrt[d + e*x^2]*ArcTan[(Sqrt[d]*Sqrt[-1 + c^2*x^2])/Sqrt[d + e*x^2]]))
/(3*c^2*e^3*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

fricas [B] time = 1.38, size = 2119, normalized size = 8.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e
+ b*d*e^2)*x^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*
e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x
^2 + d)*sqrt(e) + e^2) + 8*(b*c^3*d^3 + b*c*d^2*e + (b*c^3*d*e^2 + b*c*e^3)
*x^4 + 2*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e
+ e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 -
 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) + 4*(8*a*c^3*d^3 + 8*a*c*d^2*e
 + 3*(a*c^3*d*e^2 + a*c*e^3)*x^4 + 12*(a*c^3*d^2*e + a*c*d*e^2)*x^2 + (8*b*
c^3*d^3 + 8*b*c*d^2*e + 3*(b*c^3*d*e^2 + b*c*e^3)*x^4 + 12*(b*c^3*d^2*e + b
*c*d*e^2)*x^2)*arccsc(c*x) + (b*c*d*e^2*x^2 + b*c*d^2*e)*sqrt(c^2*x^2 - 1))
*sqrt(e*x^2 + d))/(c^3*d^3*e^3 + c*d^2*e^4 + (c^3*d*e^5 + c*e^6)*x^4 + 2*(c
^3*d^2*e^4 + c*d*e^5)*x^2), -1/12*(16*(b*c^3*d^3 + b*c*d^2*e + (b*c^3*d*e^2
 + b*c*e^3)*x^4 + 2*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(d)*arctan(-1/2*sqrt
(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4
+ (c^2*d^2 - d*e)*x^2 - d^2)) - 3*(b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 +
b*d^2*e + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^
2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*s
qrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) - 4*(8*a*c^3*d^3 + 8*a*c*d^
2*e + 3*(a*c^3*d*e^2 + a*c*e^3)*x^4 + 12*(a*c^3*d^2*e + a*c*d*e^2)*x^2 + (8
*b*c^3*d^3 + 8*b*c*d^2*e + 3*(b*c^3*d*e^2 + b*c*e^3)*x^4 + 12*(b*c^3*d^2*e
+ b*c*d*e^2)*x^2)*arccsc(c*x) + (b*c*d*e^2*x^2 + b*c*d^2*e)*sqrt(c^2*x^2 -
1))*sqrt(e*x^2 + d))/(c^3*d^3*e^3 + c*d^2*e^4 + (c^3*d*e^5 + c*e^6)*x^4 + 2
*(c^3*d^2*e^4 + c*d*e^5)*x^2), -1/6*(3*(b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x
^4 + b*d^2*e + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*
x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 -
c*d*e + (c^3*d*e - c*e^2)*x^2)) - 4*(b*c^3*d^3 + b*c*d^2*e + (b*c^3*d*e^2 +
 b*c*e^3)*x^4 + 2*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(-d)*log(((c^4*d^2 - 6
*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d -
 e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) - 2*(8*a*c^3*d^3 + 8*
a*c*d^2*e + 3*(a*c^3*d*e^2 + a*c*e^3)*x^4 + 12*(a*c^3*d^2*e + a*c*d*e^2)*x^
2 + (8*b*c^3*d^3 + 8*b*c*d^2*e + 3*(b*c^3*d*e^2 + b*c*e^3)*x^4 + 12*(b*c^3*
d^2*e + b*c*d*e^2)*x^2)*arccsc(c*x) + (b*c*d*e^2*x^2 + b*c*d^2*e)*sqrt(c^2*
x^2 - 1))*sqrt(e*x^2 + d))/(c^3*d^3*e^3 + c*d^2*e^4 + (c^3*d*e^5 + c*e^6)*x
^4 + 2*(c^3*d^2*e^4 + c*d*e^5)*x^2), -1/6*(8*(b*c^3*d^3 + b*c*d^2*e + (b*c^
3*d*e^2 + b*c*e^3)*x^4 + 2*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(d)*arctan(-1
/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d
*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + 3*(b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)
*x^4 + b*d^2*e + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(-e)*arctan(1/2*(2*c^2*
e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4
- c*d*e + (c^3*d*e - c*e^2)*x^2)) - 2*(8*a*c^3*d^3 + 8*a*c*d^2*e + 3*(a*c^3
*d*e^2 + a*c*e^3)*x^4 + 12*(a*c^3*d^2*e + a*c*d*e^2)*x^2 + (8*b*c^3*d^3 + 8
*b*c*d^2*e + 3*(b*c^3*d*e^2 + b*c*e^3)*x^4 + 12*(b*c^3*d^2*e + b*c*d*e^2)*x
^2)*arccsc(c*x) + (b*c*d*e^2*x^2 + b*c*d^2*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2
 + d))/(c^3*d^3*e^3 + c*d^2*e^4 + (c^3*d*e^5 + c*e^6)*x^4 + 2*(c^3*d^2*e^4
+ c*d*e^5)*x^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x^5}{\left(ex^2 + d\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^5/(e*x^2 + d)^(5/2), x)
```

maple [F] time = 7.34, size = 0, normalized size = 0.00

$$\int \frac{x^{5} \left(a + b \,\mathrm{arccsc}\left(c x\right)\right)}{\left(e \,x^{2}+d \right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\left(\frac{3\,x^4}{\left(ex^2+d\right)^{\frac{3}{2}}e}+\frac{12\,dx^2}{\left(ex^2+d\right)^{\frac{3}{2}}e^2}+\frac{8\,d^2}{\left(ex^2+d\right)^{\frac{3}{2}}e^3}\right)a+\frac{\left(3\,e^2x^4\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right)+12\,dex^2\arctan\left(1,\sqrt{c}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*x^4/((e*x^2 + d)^(3/2)*e) + 12*d*x^2/((e*x^2 + d)^(3/2)*e^2) + 8*d^2
/((e*x^2 + d)^(3/2)*e^3))*a + 1/3*(3*e^2*x^4*arctan2(1, sqrt(c*x + 1)*sqrt(
c*x - 1)) + 12*d*e*x^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 8*d^2*arct
an2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 3*(e^4*x^2 + d*e^3)*sqrt(e*x^2 + d)*i
ntegrate(1/3*(3*c^2*e^2*x^5 + 12*c^2*d*e*x^3 + 8*c^2*d^2*x)*e^(-1/2*log(e*x
^2 + d) + 1/2*log(c*x + 1) + 1/2*log(c*x - 1))/(c^2*e^4*x^4 - d*e^3 + (c^2*
d*e^3 - e^4)*x^2 + (c^2*e^4*x^4 - d*e^3 + (c^2*d*e^3 - e^4)*x^2)*e^(log(c*x
 + 1) + log(c*x - 1))), x))*b/((e^4*x^2 + d*e^3)*sqrt(e*x^2 + d))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

```
[Out] int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acsc(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

3.157 $\int \frac{x^3\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=163

$$-\frac{a+b\csc^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d\left(a+b\csc^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}+\frac{2bcx\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3\sqrt{d}\,e^2\sqrt{c^2x^2}}-\frac{bcx\sqrt{c^2x^2-1}}{3e\sqrt{c^2x^2}\left(c^2d+e\right)\sqrt{d+ex^2}}$$

```
[Out] 1/3*d*(a+b*arccsc(c*x))/e^2/(e*x^2+d)^(3/2)+2/3*b*c*x*arctan((e*x^2+d)^(1/2
)/d^(1/2)/(c^2*x^2-1)^(1/2))/e^2/d^(1/2)/(c^2*x^2)^(1/2)+(-a-b*arccsc(c*x))
/e^2/(e*x^2+d)^(1/2)-1/3*b*c*x*(c^2*x^2-1)^(1/2)/e/(c^2*d+e)/(c^2*x^2)^(1/2
)/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {266, 43, 5239, 12, 573, 152, 93, 204}

$$-\frac{a+b\csc^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d\left(a+b\csc^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}+\frac{2bcx\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3\sqrt{d}\,e^2\sqrt{c^2x^2}}-\frac{bcx\sqrt{c^2x^2-1}}{3e\sqrt{c^2x^2}\left(c^2d+e\right)\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] -(b*c*x*Sqrt[-1 + c^2*x^2])/(3*e*(c^2*d + e)*Sqrt[c^2*x^2]*Sqrt[d + e*x^2])
 + (d*(a + b*ArcCsc[c*x]))/(3*e^2*(d + e*x^2)^(3/2)) - (a + b*ArcCsc[c*x])/
(e^2*Sqrt[d + e*x^2]) + (2*b*c*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 +
c^2*x^2])])/(3*Sqrt[d]*e^2*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
```

```
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 573

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{x^3\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{d\left(a+b\csc^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\csc^{-1}(cx)}{e^2\sqrt{d+ex^2}} + \frac{(bcx)\int \frac{-2d-3ex^2}{3e^2x\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}\,dx}{\sqrt{c^2x^2}}$$

$$= \frac{d\left(a+b\csc^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\csc^{-1}(cx)}{e^2\sqrt{d+ex^2}} + \frac{(bcx)\int \frac{-2d-3ex^2}{x\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}\,dx}{3e^2\sqrt{c^2x^2}}$$

$$= \frac{d\left(a+b\csc^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\csc^{-1}(cx)}{e^2\sqrt{d+ex^2}} + \frac{(bcx)\,\text{Subst}\left(\int \frac{-2d-3ex}{x\sqrt{-1+c^2x}\,(d+ex)^{3/2}}\,dx, x, x^2\right)}{6e^2\sqrt{c^2x^2}}$$

$$= -\frac{bcx\sqrt{-1+c^2x^2}}{3e\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{d\left(a+b\csc^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\csc^{-1}(cx)}{e^2\sqrt{d+ex^2}} - \frac{(bcx)\,\text{Subst}}{3c}$$

$$= -\frac{bcx\sqrt{-1+c^2x^2}}{3e\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{d\left(a+b\csc^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\csc^{-1}(cx)}{e^2\sqrt{d+ex^2}} - \frac{(bcx)\,\text{Subst}}{}$$

$$= -\frac{bcx\sqrt{-1+c^2x^2}}{3e\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{d\left(a+b\csc^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\csc^{-1}(cx)}{e^2\sqrt{d+ex^2}} - \frac{(2bcx)\,\text{Sub}}{}$$

$$= -\frac{bcx\sqrt{-1+c^2x^2}}{3e\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{d\left(a+b\csc^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\csc^{-1}(cx)}{e^2\sqrt{d+ex^2}} + \frac{2bcx\tan^{-1}}{3\sqrt{d}\,}$$

Mathematica [A] time = 0.27, size = 173, normalized size = 1.06

$$\frac{-a\left(c^2d+e\right)\left(2d+3ex^2\right)-bcex\sqrt{1-\frac{1}{c^2x^2}}\left(d+ex^2\right)-b\left(c^2d+e\right)\csc^{-1}(cx)\left(2d+3ex^2\right)}{3e^2\left(c^2d+e\right)\left(d+ex^2\right)^{3/2}} - \frac{2bcx\sqrt{1-\frac{1}{c^2x^2}}\tan^{-1}\left(\sqrt{\right.}{3\sqrt{d}\,e^2\sqrt{c^2x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (-(b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2)) - a*(c^2*d + e)*(2*d + 3*e*x^
2) - b*(c^2*d + e)*(2*d + 3*e*x^2)*ArcCsc[c*x])/(3*e^2*(c^2*d + e)*(d + e*x
^2)^(3/2)) - (2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*ArcTan[(Sqrt[d]*Sqrt[-1 + c^2*x
^2])/Sqrt[d + e*x^2]])/(3*Sqrt[d]*e^2*Sqrt[-1 + c^2*x^2])
```

fricas [B] time = 1.10, size = 663, normalized size = 4.07

$$\left[-\frac{\left(bc^2d^3+\left(bc^2de^2+be^3\right)x^4+bd^2e+2\left(bc^2d^2e+bde^2\right)x^2\right)\sqrt{-d}\log\left(\frac{\left(c^4d^2-6c^2de+e^2\right)x^4-8\left(c^2d^2-de\right)x^2+4\sqrt{c^2x^2-1}\left(\left(c^2d-e\right)x\right.}{x^4}\right.}{6\left(c^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e +
b*d*e^2)*x^2)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 -
```

```
d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt
(-d) + 8*d^2)/x^4) + 2*(2*a*c^2*d^3 + 2*a*d^2*e + 3*(a*c^2*d^2*e + a*d*e^2)
*x^2 + (2*b*c^2*d^3 + 2*b*d^2*e + 3*(b*c^2*d^2*e + b*d*e^2)*x^2)*arccsc(c*x
) + (b*d*e^2*x^2 + b*d^2*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^2*d^4*e^
2 + d^3*e^3 + (c^2*d^2*e^4 + d*e^5)*x^4 + 2*(c^2*d^3*e^3 + d^2*e^4)*x^2), 1
/3*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e + b*d
*e^2)*x^2)*sqrt(d)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sq
rt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) - (2*a*c^2
*d^3 + 2*a*d^2*e + 3*(a*c^2*d^2*e + a*d*e^2)*x^2 + (2*b*c^2*d^3 + 2*b*d^2*e
 + 3*(b*c^2*d^2*e + b*d*e^2)*x^2)*arccsc(c*x) + (b*d*e^2*x^2 + b*d^2*e)*sqr
t(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^2*d^4*e^2 + d^3*e^3 + (c^2*d^2*e^4 + d*
e^5)*x^4 + 2*(c^2*d^3*e^3 + d^2*e^4)*x^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x^3}{(ex^2 + d)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^3/(e*x^2 + d)^(5/2), x)
```

maple [F] time = 6.73, size = 0, normalized size = 0.00

$$\int \frac{x^3 (a + b \operatorname{arccsc}(cx))}{(e x^2 + d)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3} a \left(\frac{3 x^2}{(ex^2 + d)^{\frac{3}{2}} e} + \frac{2 d}{(ex^2 + d)^{\frac{3}{2}} e^2} \right) + b \int \frac{x^3 \arctan\left(1, \sqrt{cx+1}\sqrt{cx-1}\right)}{\left(e^2x^4 + 2\,dex^2 + d^2\right)\sqrt{ex^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a*(3*x^2/((e*x^2 + d)^(3/2)*e) + 2*d/((e*x^2 + d)^(3/2)*e^2)) + b*inte
grate(x^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/((e^2*x^4 + 2*d*e*x^2 + d
^2)*sqrt(e*x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \left(a + b \operatorname{asin}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2 + d\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

```
[Out] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 (a + b \operatorname{acsc}(cx))}{(d + ex^2)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsc(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**3*(a + b*acsc(c*x))/(d + e*x**2)**(5/2), x)
```

3.158 $\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=138

$$-\frac{a+b\csc^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}+\frac{bcx\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3d^{3/2}e\sqrt{c^2x^2}}+\frac{bcx\sqrt{c^2x^2-1}}{3d\sqrt{c^2x^2}\left(c^2d+e\right)\sqrt{d+ex^2}}$$

```
[Out] 1/3*(-a-b*arccsc(c*x))/e/(e*x^2+d)^(3/2)+1/3*b*c*x*arctan((e*x^2+d)^(1/2)/d
^(1/2)/(c^2*x^2-1)^(1/2))/d^(3/2)/e/(c^2*x^2)^(1/2)+1/3*b*c*x*(c^2*x^2-1)^(
1/2)/d/(c^2*d+e)/(c^2*x^2)^(1/2)/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5237, 446, 96, 93, 204}

$$-\frac{a+b\csc^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}+\frac{bcx\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3d^{3/2}e\sqrt{c^2x^2}}+\frac{bcx\sqrt{c^2x^2-1}}{3d\sqrt{c^2x^2}\left(c^2d+e\right)\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*c*x*Sqrt[-1 + c^2*x^2])/(3*d*(c^2*d + e)*Sqrt[c^2*x^2]*Sqrt[d + e*x^2])
- (a + b*ArcCsc[c*x])/(3*e*(d + e*x^2)^(3/2)) + (b*c*x*ArcTan[Sqrt[d + e*x^
2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(3*d^(3/2)*e*Sqrt[c^2*x^2])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5237

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCsc[c*x]))/(2*e*(p + 1)), x
] + Dist[(b*c*x)/(2*e*(p + 1)*Sqrt[c^2*x^2]), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = -\frac{a+b\csc^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}} - \frac{(bcx)\int \frac{1}{x\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}\,dx}{3e\sqrt{c^2x^2}}$$

$$= -\frac{a+b\csc^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}} - \frac{(bcx)\,\text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}\,(d+ex)^{3/2}}\,dx, x, x^2\right)}{6e\sqrt{c^2x^2}}$$

$$= \frac{bcx\sqrt{-1+c^2x^2}}{3d\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{a+b\csc^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}} - \frac{(bcx)\,\text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x,$$

$$= \frac{bcx\sqrt{-1+c^2x^2}}{3d\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{a+b\csc^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}} - \frac{(bcx)\,\text{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}$$

$$= \frac{bcx\sqrt{-1+c^2x^2}}{3d\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{a+b\csc^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}} + \frac{bcx\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3d^{3/2}e\sqrt{c^2x^2}}$$

Mathematica [A] time = 0.21, size = 157, normalized size = 1.14

$$\frac{-ad\left(c^2d+e\right)+bcex\sqrt{1-\frac{1}{c^2x^2}}\left(d+ex^2\right)-bd\left(c^2d+e\right)\csc^{-1}(cx)}{3de\left(c^2d+e\right)\left(d+ex^2\right)^{3/2}} - \frac{bcx\sqrt{1-\frac{1}{c^2x^2}}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{c^2x^2-1}}{\sqrt{d+ex^2}}\right)}{3d^{3/2}e\sqrt{c^2x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (-(a*d*(c^2*d + e)) + b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2) - b*d*(c^2*
d + e)*ArcCsc[c*x])/(3*d*e*(c^2*d + e)*(d + e*x^2)^(3/2)) - (b*c*Sqrt[1 - 1
/(c^2*x^2)]*x*ArcTan[(Sqrt[d]*Sqrt[-1 + c^2*x^2])/Sqrt[d + e*x^2]])/(3*d^(3
/2)*e*Sqrt[-1 + c^2*x^2])
```

fricas [B] time = 1.06, size = 573, normalized size = 4.15

$$\left[-\frac{\left(bc^2d^3+\left(bc^2de^2+be^3\right)x^4+bd^2e+2\left(bc^2d^2e+bde^2\right)x^2\right)\sqrt{-d}\,\log\left(\frac{\left(c^4d^2-6\,c^2de+e^2\right)x^4-8\left(c^2d^2-de\right)x^2+4\,\sqrt{c^2x^2-1}\left(\left(c^2d-e\right)x\right.}{x^4}\right.}{12\left(c^2d^5e+d^4e^2+\left(c^2d^3e^3+d^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e +
 b*d*e^2)*x^2)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 -
 d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqr
t(-d) + 8*d^2)/x^4) + 4*(a*c^2*d^3 + a*d^2*e + (b*c^2*d^3 + b*d^2*e)*arccsc
(c*x) - (b*d*e^2*x^2 + b*d^2*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^2*d^
5*e + d^4*e^2 + (c^2*d^3*e^3 + d^2*e^4)*x^4 + 2*(c^2*d^4*e^2 + d^3*e^3)*x^2
), 1/6*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e +
 b*d*e^2)*x^2)*sqrt(d)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d
)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) - 2*(a
*c^2*d^3 + a*d^2*e + (b*c^2*d^3 + b*d^2*e)*arccsc(c*x) - (b*d*e^2*x^2 + b*d
^2*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^2*d^5*e + d^4*e^2 + (c^2*d^3*e
^3 + d^2*e^4)*x^4 + 2*(c^2*d^4*e^2 + d^3*e^3)*x^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x}{\left(ex^2 + d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x/(e*x^2 + d)^(5/2), x)
```

maple [F] time = 4.94, size = 0, normalized size = 0.00

$$\int \frac{x\left(a + b \operatorname{arccsc}(cx)\right)}{\left(e\,x^2 + d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\int \frac{x \arctan\left(1, \sqrt{cx+1}\,\sqrt{cx-1}\right)}{\left(e^2x^4 + 2\,dex^2 + d^2\right)\sqrt{ex^2+d}}\,dx - \frac{a}{3\left(ex^2+d\right)^{\frac{3}{2}}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] b*integrate(x*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/((e^2*x^4 + 2*d*e*x^2
 + d^2)*sqrt(e*x^2 + d)), x) - 1/3*a/((e*x^2 + d)^(3/2)*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2 + d\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

```
[Out] int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(a+b\operatorname{acsc}(cx)\right)}{\left(d+ex^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsc(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Integral(x*(a + b*acsc(c*x))/(d + e*x**2)**(5/2), x)
```

3.159 $\int \frac{a+b\csc^{-1}(cx)}{x(d+ex^2)^{5/2}}\,dx$

Optimal. Leaf size=26

$$\mathrm{Int}\left(\frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x/(e*x^2+d)^(5/2),x)
```

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^(5/2)),x]
```

```
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^(5/2)), x]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$$

Mathematica [A] time = 37.84, size = 0, normalized size = 0.00

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^(5/2)),x]
```

```
[Out] Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^(5/2)), x]
```

fricas [A] time = 0.82, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{ex^2+d}\,(b\,\mathrm{arccsc}\,(cx)+a)}{e^3x^7+3\,de^2x^5+3\,d^2ex^3+d^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2
*e*x^3 + d^3*x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\mathrm{arccsc}\,(cx)+a}{\left(ex^2+d\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/((e*x^2 + d)^(5/2)*x), x)
```

maple [A] time = 3.73, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arccsc}\left(cx\right)}{x\left(e\,x^{2}+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/x/(e*x^2+d)^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}\,a\left(\frac{3\,\mathrm{arsinh}\left(\frac{d}{\sqrt{de}\,|x|}\right)}{d^{\frac{5}{2}}}-\frac{3}{\sqrt{ex^{2}+d}\,d^{2}}-\frac{1}{\left(ex^{2}+d\right)^{\frac{3}{2}}d}\right)+b\int\frac{\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right)}{\left(e^{2}x^{5}+2\,dex^{3}+d^{2}x\right)\sqrt{ex^{2}+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a*(3*arcsinh(d/(sqrt(d*e)*abs(x)))/d^(5/2) - 3/(sqrt(e*x^2 + d)*d^2) -
 1/((e*x^2 + d)^(3/2)*d)) + b*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x -
 1))/((e^2*x^5 + 2*d*e*x^3 + d^2*x)*sqrt(e*x^2 + d)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x\,{\left(e\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^(5/2)),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

3.160 $\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

Rubi [A] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCsc[c*x])/(x^3*(d + e*x^2)^(5/2)),x]
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x^3*(d + e*x^2)^(5/2)), x]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$$

Mathematica [A] time = 46.56, size = 0, normalized size = 0.00

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x^3*(d + e*x^2)^(5/2)),x]
[Out] Integrate[(a + b*ArcCsc[c*x])/(x^3*(d + e*x^2)^(5/2)), x]
```

fricas [A] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2+d}\,(b\,\text{arccsc}\,(cx)+a)}{e^3x^9+3\,de^2x^7+3\,d^2ex^5+d^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/(e^3*x^9 + 3*d*e^2*x^7 + 3*d^2
*e*x^5 + d^3*x^3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arccsc}\,(cx)+a}{\left(ex^2+d\right)^{\frac{5}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/((e*x^2 + d)^(5/2)*x^3), x)
```

maple [A] time = 1.35, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{arccsc}\,(cx)}{x^3 \left(e\,x^2 + d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\,a\left(\frac{15\,e\,\mathrm{arsinh}\left(\frac{d}{\sqrt{de}\,|x|}\right)}{d^{\frac{7}{2}}} - \frac{15\,e}{\sqrt{ex^2+d}\,d^3} - \frac{5\,e}{\left(ex^2+d\right)^{\frac{3}{2}}d^2} - \frac{3}{\left(ex^2+d\right)^{\frac{3}{2}}dx^2}\right) + b\int \frac{\arctan\left(1, \sqrt{cx+1}\,\sqrt{cx-1}\right)}{\left(e^2x^7 + 2\,dex^5 + d^2x^3\right)\sqrt{ex^2+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*a*(15*e*arcsinh(d/(sqrt(d*e)*abs(x)))/d^(7/2) - 15*e/(sqrt(e*x^2 + d)*d
^3) - 5*e/((e*x^2 + d)^(3/2)*d^2) - 3/((e*x^2 + d)^(3/2)*d*x^2)) + b*integr
ate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/((e^2*x^7 + 2*d*e*x^5 + d^2*x^3
)*sqrt(e*x^2 + d)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^3\left(e\,x^2 + d\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x^3*(d + e*x^2)^(5/2)),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(x^3*(d + e*x^2)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x**3/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

3.161 $\int \frac{x^6\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{x^6\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}},x\right)$$

```
[Out] Unintegrable(x^6*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^6\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^6*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] Defer[Int][(x^6*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2), x]
```

Rubi steps

$$\int \frac{x^6\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{x^6\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

Mathematica [A] time = 14.83, size = 0, normalized size = 0.00

$$\int \frac{x^6\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^6*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] Integrate[(x^6*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2), x]
```

fricas [A] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^6\,\text{arccsc}\,(cx)+ax^6\right)\sqrt{ex^2+d}}{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^6*arccsc(c*x) + a*x^6)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x^4
 + 3*d^2*e*x^2 + d^3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arccsc}\,(cx)+a)x^6}{\left(ex^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^6/(e*x^2 + d)^(5/2), x)
```

maple [A] time = 8.81, size = 0, normalized size = 0.00

$$\int \frac{x^6 (a + b\,\mathrm{arccsc}\,(cx))}{(e\,x^2 + d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^6*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\left(\frac{3\,x^5}{(ex^2+d)^{\frac{3}{2}}e} + \frac{5\,dx\left(\frac{3\,x^2}{(ex^2+d)^{\frac{3}{2}}e} + \frac{2\,d}{(ex^2+d)^{\frac{3}{2}}e^2}\right)}{e} + \frac{5\,dx}{\sqrt{ex^2+d}\,e^3} - \frac{15\,d\,\mathrm{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{e^{\frac{7}{2}}}\right)a + b\int \frac{x^6 \arctan\left(1, \sqrt{cx+1}\,\sqrt{\vphantom{x}}\right.}{\left(e^2x^4 + 2\,dex^2 + d^2\right)\sqrt{\vphantom{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(3*x^5/((e*x^2 + d)^(3/2)*e) + 5*d*x*(3*x^2/((e*x^2 + d)^(3/2)*e) + 2*d
/((e*x^2 + d)^(3/2)*e^2))/e + 5*d*x/(sqrt(e*x^2 + d)*e^3) - 15*d*arcsinh(e*
x/sqrt(d*e))/e^(7/2))*a + b*integrate(x^6*arctan2(1, sqrt(c*x + 1)*sqrt(c*x
 - 1))/((e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(e*x^2 + d)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^6 \left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2 + d\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

```
[Out] int((x^6*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(a+b*acsc(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

3.162 $\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{x^4\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}},x\right)$$

```
[Out] Unintegrable(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^4\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] Defer[Int][(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2), x]
```

Rubi steps

$$\int \frac{x^4\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{x^4\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

Mathematica [A] time = 12.52, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] Integrate[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2), x]
```

fricas [A] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4\,\text{arccsc}\,(cx)+ax^4\right)\sqrt{ex^2+d}}{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arccsc(c*x) + a*x^4)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x^4
 + 3*d^2*e*x^2 + d^3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arccsc}\,(cx)+a)x^4}{\left(ex^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^4/(e*x^2 + d)^(5/2), x)
```

maple [A] time = 6.30, size = 0, normalized size = 0.00

$$\int \frac{x^{4} \left(a+b \,\mathrm{arccsc}\left(c x\right)\right)}{\left(e \,x^{2}+d\right)^{\frac{5}{2}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3} \left(x \left(\frac{3 \, x^{2}}{{\left(e x^{2} + d\right)}^{\frac{3}{2}} e} + \frac{2 \, d}{{\left(e x^{2} + d\right)}^{\frac{3}{2}} e^{2}}\right) + \frac{x}{\sqrt{e x^{2} + d} e^{2}} - \frac{3 \, \operatorname{arsinh}\left(\frac{e x}{\sqrt{d e}}\right)}{e^{\frac{5}{2}}}\right) a + b \int \frac{x^{4} \arctan\left(1, \sqrt{c x + 1} \sqrt{c x - 1}\right)}{{\left(e^{2} x^{4} + 2 \, d e x^{2} + d^{2}\right)} \sqrt{e x^{2} + d}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(x*(3*x^2/((e*x^2 + d)^(3/2)*e) + 2*d/((e*x^2 + d)^(3/2)*e^2)) + x/(sq
rt(e*x^2 + d)*e^2) - 3*arcsinh(e*x/sqrt(d*e))/e^(5/2))*a + b*integrate(x^4*
arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/((e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(e
*x^2 + d)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^4\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

```
[Out] int((x^4*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acsc(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

3.163 $\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=276

$$\frac{x^3\left(a+b\csc^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{bcx^2\sqrt{c^2x^2-1}}{3d\sqrt{c^2x^2}\left(c^2d+e\right)\sqrt{d+ex^2}}+\frac{bx\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\,F\left(\sin^{-1}(cx)|-\frac{e}{c^2d}\right)}{3de\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{bc^2x\sqrt{1-c^2x^2}}{3de\sqrt{c^2x^2}\sqrt{}}$$

```
[Out] 1/3*x^3*(a+b*arccsc(c*x))/d/(e*x^2+d)^(3/2)+1/3*b*c*x^2*(c^2*x^2-1)^(1/2)/d
/(c^2*d+e)/(c^2*x^2)^(1/2)/(e*x^2+d)^(1/2)-1/3*b*c^2*x*EllipticE(c*x,(-e/c^
2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d/e/(c^2*d+e)/(c^2*x^2)^(1/2
)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)+1/3*b*x*EllipticF(c*x,(-e/c^2/d)^(1/2
))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/e/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/
2)/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {264, 5239, 12, 471, 423, 427, 426, 424, 421, 419}

$$\frac{x^3\left(a+b\csc^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{bcx^2\sqrt{c^2x^2-1}}{3d\sqrt{c^2x^2}\left(c^2d+e\right)\sqrt{d+ex^2}}+\frac{bx\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\,F\left(\sin^{-1}(cx)|-\frac{e}{c^2d}\right)}{3de\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{bc^2x\sqrt{1-c^2x^2}}{3de\sqrt{c^2x^2}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*c*x^2*Sqrt[-1 + c^2*x^2])/(3*d*(c^2*d + e)*Sqrt[c^2*x^2]*Sqrt[d + e*x^2]
) + (x^3*(a + b*ArcCsc[c*x]))/(3*d*(d + e*x^2)^(3/2)) - (b*c^2*x*Sqrt[1 - c
^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(3*d*e*(c^2*d
 + e)*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) + (b*x*Sqrt[1 -
 c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(3*d*e*
Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{x^3\left(a+b\csc^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{(bcx)\int \frac{x^2}{3d\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}\,dx}{\sqrt{c^2x^2}}$$

$$= \frac{x^3\left(a+b\csc^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{(bcx)\int \frac{x^2}{\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}\,dx}{3d\sqrt{c^2x^2}}$$

$$= \frac{bcx^2\sqrt{-1+c^2x^2}}{3d\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x^3\left(a+b\csc^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} - \frac{(bcx)\int \frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\,dx}{3d\left(c^2d+e\right)\sqrt{c^2x^2}}$$

$$= \frac{bcx^2\sqrt{-1+c^2x^2}}{3d\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x^3\left(a+b\csc^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{(bcx)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{3de\sqrt{c^2x^2}}$$

$$= \frac{bcx^2\sqrt{-1+c^2x^2}}{3d\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x^3\left(a+b\csc^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} - \frac{\left(bc^3x\sqrt{1-c^2x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{1-c^2x}}}{3de\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{-1+}}$$

$$= \frac{bcx^2\sqrt{-1+c^2x^2}}{3d\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x^3\left(a+b\csc^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} - \frac{\left(bc^3x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\right.}{3de\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{-1+}}$$

$$= \frac{bcx^2\sqrt{-1+c^2x^2}}{3d\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x^3\left(a+b\csc^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} - \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\,E}{3de\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{-1}}$$

Mathematica [A] time = 0.29, size = 185, normalized size = 0.67

$$\frac{x^2\left(ax\left(c^2d+e\right)+bc\sqrt{1-\frac{1}{c^2x^2}}\left(d+ex^2\right)+bx\left(c^2d+e\right)\csc^{-1}(cx)\right)}{3d\left(c^2d+e\right)\left(d+ex^2\right)^{3/2}} - \frac{bcx\sqrt{1-\frac{1}{c^2x^2}}\sqrt{\frac{ex^2}{d}+1}\,E\left(\sin^{-1}\left(\sqrt{-\frac{e}{d}}\,x\right)\right|}{3d\sqrt{1-c^2x^2}\sqrt{-\frac{e}{d}}\left(c^2d+e\right)\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (x^2*(a*(c^2*d + e)*x + b*c*Sqrt[1 - 1/(c^2*x^2)]*(d + e*x^2) + b*(c^2*d +
e)*x*ArcCsc[c*x]))/(3*d*(c^2*d + e)*(d + e*x^2)^(3/2)) - (b*c*Sqrt[1 - 1/(c
^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*EllipticE[ArcSin[Sqrt[-(e/d)]*x], -((c^2*d)/
e)])/(3*d*Sqrt[-(e/d)]*(c^2*d + e)*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^2\,\text{arccsc}\,(cx)+ax^2\right)\sqrt{ex^2+d}}{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccsc(c*x) + a*x^2)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x^4
 + 3*d^2*e*x^2 + d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x^2}{(ex^2 + d)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)*x^2/(e*x^2 + d)^(5/2), x)
```

maple [F] time = 6.91, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + b \operatorname{arccsc}(cx))}{(e x^2 + d)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
[Out] int(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3} a \left(\frac{x}{(ex^2 + d)^{\frac{3}{2}} e} - \frac{x}{\sqrt{ex^2 + d}\, de} \right) + b \int \frac{x^2 \arctan\left(1, \sqrt{cx + 1}\sqrt{cx - 1}\right)}{(e^2x^4 + 2\, dex^2 + d^2)\sqrt{ex^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
[Out] -1/3*a*(x/((e*x^2 + d)^(3/2)*e) - x/(sqrt(e*x^2 + d)*d*e)) + b*integrate(x^
2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/((e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt
(e*x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \left(a + b \operatorname{asin}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2 + d)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2),x)
[Out] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsc(c*x))/(e*x**2+d)**(5/2),x)
[Out] Timed out
```

3.164 $\int \frac{a+b\csc^{-1}(cx)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=296

$$\frac{2x\left(a+b\csc^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\csc^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{bcex^2\sqrt{c^2x^2-1}}{3d^2\sqrt{c^2x^2}\left(c^2d+e\right)\sqrt{d+ex^2}}+\frac{2bx\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\,F\left(\sin^{-1}(cx)\right.}{3d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}$$

```
[Out] 1/3*x*(a+b*arccsc(c*x))/d/(e*x^2+d)^(3/2)+2/3*x*(a+b*arccsc(c*x))/d^2/(e*x^
2+d)^(1/2)-1/3*b*c*e*x^2*(c^2*x^2-1)^(1/2)/d^2/(c^2*d+e)/(c^2*x^2)^(1/2)/(e
*x^2+d)^(1/2)+1/3*b*c^2*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2
)*(e*x^2+d)^(1/2)/d^2/(c^2*d+e)/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/
d)^(1/2)+2/3*b*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^
2/d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {192, 191, 5229, 12, 527, 524, 427, 426, 424, 421, 419}

$$\frac{2x\left(a+b\csc^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\csc^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{bcex^2\sqrt{c^2x^2-1}}{3d^2\sqrt{c^2x^2}\left(c^2d+e\right)\sqrt{d+ex^2}}+\frac{2bx\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\,F\left(\sin^{-1}(cx)\right.}{3d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(d + e*x^2)^(5/2),x]
```

```
[Out] -(b*c*e*x^2*Sqrt[-1 + c^2*x^2])/(3*d^2*(c^2*d + e)*Sqrt[c^2*x^2]*Sqrt[d + e
*x^2]) + (x*(a + b*ArcCsc[c*x]))/(3*d*(d + e*x^2)^(3/2)) + (2*x*(a + b*ArcC
sc[c*x]))/(3*d^2*Sqrt[d + e*x^2]) + (b*c^2*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x
^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(3*d^2*(c^2*d + e)*Sqrt[c^2*x^2]*
Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) + (2*b*x*Sqrt[1 - c^2*x^2]*Sqrt[1 +
 (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(3*d^2*Sqrt[c^2*x^2]*Sqrt
[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 5229

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCsc[c*x], u, x]
+ Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1])
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1/2,
0])
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{x\left(a+b\csc^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\csc^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} + \frac{(bcx)\int \frac{3d+2ex^2}{3d^2\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}\,dx}{\sqrt{c^2x^2}}$$

$$= \frac{x\left(a+b\csc^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\csc^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} + \frac{(bcx)\int \frac{3d+2ex^2}{\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}\,dx}{3d^2\sqrt{c^2x^2}}$$

$$= -\frac{bcex^2\sqrt{-1+c^2x^2}}{3d^2\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x\left(a+b\csc^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\csc^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} + \frac{(bcx)\,\cdots}{3d^3\cdots}$$

$$= -\frac{bcex^2\sqrt{-1+c^2x^2}}{3d^2\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x\left(a+b\csc^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\csc^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} + \frac{(2bcx)\cdots}{\cdots}$$

$$= -\frac{bcex^2\sqrt{-1+c^2x^2}}{3d^2\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x\left(a+b\csc^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\csc^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} + \frac{\left(bc^3x\cdots\right.}{3d^2\left(c^2\cdots\right.}$$

$$= -\frac{bcex^2\sqrt{-1+c^2x^2}}{3d^2\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x\left(a+b\csc^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\csc^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} + \frac{\left(bc^3x\cdots\right.}{3d^2\left(c^2\cdots\right.}$$

$$= -\frac{bcex^2\sqrt{-1+c^2x^2}}{3d^2\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x\left(a+b\csc^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\csc^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} + \frac{bc^2x\sqrt{\cdots}}{3d^2\left(\cdots\right.}$$

Mathematica [C] time = 0.56, size = 249, normalized size = 0.84

$$\frac{x\left(a\left(c^2d+e\right)\left(3d+2ex^2\right)-bcex\sqrt{1-\frac{1}{c^2x^2}}\left(d+ex^2\right)+b\left(c^2d+e\right)\csc^{-1}(cx)\left(3d+2ex^2\right)\right)}{3d^2\left(c^2d+e\right)\left(d+ex^2\right)^{3/2}} + \frac{ibcx\sqrt{1-\frac{1}{c^2x^2}}\sqrt{\frac{ex^2}{d}\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(d + e*x^2)^(5/2),x]
```

```
[Out] (x*(-(b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2)) + a*(c^2*d + e)*(3*d + 2*e
*x^2) + b*(c^2*d + e)*(3*d + 2*e*x^2)*ArcCsc[c*x]))/(3*d^2*(c^2*d + e)*(d +
 e*x^2)^(3/2)) + ((I/3)*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^
2*d*EllipticE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))] + 2*(c^2*d + e)*Ellipt
icF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-c^2]*d^2*(c^2*d + e)*Sq
rt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{ex^2+d}\,(b\,\mathrm{arccsc}\,(cx)+a)}{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2
*e*x^2 + d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arccsc}(cx) + a}{\left(ex^2 + d\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/(e*x^2 + d)^(5/2), x)
```

maple [F] time = 4.32, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arccsc}(cx)}{\left(e x^2 + d\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\, a \left(\frac{2\, x}{\sqrt{ex^2 + d}\, d^2} + \frac{x}{\left(ex^2 + d\right)^{\frac{3}{2}} d} \right) + b \int \frac{\arctan\left(1, \sqrt{cx + 1}\, \sqrt{cx - 1}\right)}{\left(e^2 x^4 + 2\, dex^2 + d^2\right) \sqrt{ex^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*(2*x/(sqrt(e*x^2 + d)*d^2) + x/((e*x^2 + d)^(3/2)*d)) + b*integrate(a
rctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/((e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(e*
x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{asin}\left(\frac{1}{c\, x}\right)}{\left(e\, x^2 + d\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x^2)^(5/2),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(d + e*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

3.165 $\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\csc^{-1}(cx)\right) dx$

Optimal. Leaf size=585

$$\frac{d^3(fx)^{m+1}\left(a+b\csc^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\csc^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\csc^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\right.}{f^7(m+}$$

```
[Out] d^3*(f*x)^(1+m)*(a+b*arccsc(c*x))/f/(1+m)+3*d^2*e*(f*x)^(3+m)*(a+b*arccsc(c
*x))/f^3/(3+m)+3*d*e^2*(f*x)^(5+m)*(a+b*arccsc(c*x))/f^5/(5+m)+e^3*(f*x)^(7
+m)*(a+b*arccsc(c*x))/f^7/(7+m)+b*(c^6*d^3*(2+m)*(4+m)*(6+m)/(1+m)+e*(1+m)*
(e^2*(m^2+8*m+15)^2+3*c^2*d*e*(3+m)^2*(m^2+13*m+42)+3*c^4*d^2*(m^4+22*m^3+1
79*m^2+638*m+840))/(m^3+15*m^2+71*m+105))*x*(f*x)^(1+m)*hypergeom([1/2, 1/2
+1/2*m],[3/2+1/2*m],c^2*x^2)*(-c^2*x^2+1)^(1/2)/c^5/f/(1+m)/(2+m)/(4+m)/(6+
m)/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)+b*e*(e^2*(m^2+8*m+15)^2+3*c^2*d*e*(3+m
)^2*(m^2+13*m+42)+3*c^4*d^2*(m^4+22*m^3+179*m^2+638*m+840))*x*(f*x)^(1+m)*(
c^2*x^2-1)^(1/2)/c^5/f/(6+m)/(m^2+6*m+8)/(m^3+15*m^2+71*m+105)/(c^2*x^2)^(1
/2)+b*e^2*(e*(5+m)^2+3*c^2*d*(m^2+13*m+42))*x*(f*x)^(3+m)*(c^2*x^2-1)^(1/2)
/c^3/f^3/(4+m)/(5+m)/(6+m)/(7+m)/(c^2*x^2)^(1/2)+b*e^3*x*(f*x)^(5+m)*(c^2*x
^2-1)^(1/2)/c/f^5/(6+m)/(7+m)/(c^2*x^2)^(1/2)
```

Rubi [A] time = 2.40, antiderivative size = 566, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {270, 5239, 1809, 1267, 459, 365, 364}

$$\frac{3d^2e(fx)^{m+3}\left(a+b\csc^{-1}(cx)\right)}{f^3(m+3)}+\frac{d^3(fx)^{m+1}\left(a+b\csc^{-1}(cx)\right)}{f(m+1)}+\frac{3de^2(fx)^{m+5}\left(a+b\csc^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\right.}{f^7(m+}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)^3*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*e*(e^2*(15 + 8*m + m^2)^2 + 3*c^2*d*e*(3 + m)^2*(42 + 13*m + m^2) + 3*c^
4*d^2*(840 + 638*m + 179*m^2 + 22*m^3 + m^4))*x*(f*x)^(1 + m)*Sqrt[-1 + c^2
*x^2])/(c^5*f*(2 + m)*(3 + m)*(4 + m)*(5 + m)*(6 + m)*(7 + m)*Sqrt[c^2*x^2]
) + (b*e^2*(e*(5 + m)^2 + 3*c^2*d*(42 + 13*m + m^2))*x*(f*x)^(3 + m)*Sqrt[-
1 + c^2*x^2])/(c^3*f^3*(4 + m)*(5 + m)*(6 + m)*(7 + m)*Sqrt[c^2*x^2]) + (b*
e^3*x*(f*x)^(5 + m)*Sqrt[-1 + c^2*x^2])/(c*f^5*(6 + m)*(7 + m)*Sqrt[c^2*x^2
]) + (d^3*(f*x)^(1 + m)*(a + b*ArcCsc[c*x]))/(f*(1 + m)) + (3*d^2*e*(f*x)^(
3 + m)*(a + b*ArcCsc[c*x]))/(f^3*(3 + m)) + (3*d*e^2*(f*x)^(5 + m)*(a + b*A
rcCsc[c*x]))/(f^5*(5 + m)) + (e^3*(f*x)^(7 + m)*(a + b*ArcCsc[c*x]))/(f^7*(
7 + m)) + (b*c*(d^3/(1 + m)^2 + (e*(e^2*(15 + 8*m + m^2)^2 + 3*c^2*d*e*(3 +
 m)^2*(42 + 13*m + m^2) + 3*c^4*d^2*(840 + 638*m + 179*m^2 + 22*m^3 + m^4))
)/(c^6*(2 + m)*(3 + m)*(4 + m)*(5 + m)*(6 + m)*(7 + m)))*x*(f*x)^(1 + m)*Sq
rt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(f*S
qrt[c^2*x^2]*Sqrt[-1 + c^2*x^2])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
```

```
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] && !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\csc^{-1}(cx)\right) dx = \frac{d^3(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)} + \frac{3d^2e(fx)^{3+m}\left(a+b\csc^{-1}(cx)\right)}{f^3(3+m)} + \frac{3de}{}$$

$$= \frac{be^3x(fx)^{5+m}\sqrt{-1+c^2x^2}}{cf^5(6+m)(7+m)\sqrt{c^2x^2}} + \frac{d^3(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)} + \frac{3d^2e(fx}{}$$

$$= \frac{be^2\left(e(5+m)^2+3c^2d\left(42+13m+m^2\right)\right)x(fx)^{3+m}\sqrt{-1+c^2x^2}}{c^3f^3(4+m)(5+m)(6+m)(7+m)\sqrt{c^2x^2}} + \frac{b}{cf}$$

$$= \frac{be\left(e^2\left(15+8m+m^2\right)^2+3c^2de(3+m)^2\left(42+13m+m^2\right)+3c^4d^2\right.}{c^5f(2+m)(3+m)(4+m)(5+}$$

$$= \frac{be\left(e^2\left(15+8m+m^2\right)^2+3c^2de(3+m)^2\left(42+13m+m^2\right)+3c^4d^2\right.}{c^5f(2+m)(3+m)(4+m)(5+}$$

$$= \frac{be\left(e^2\left(15+8m+m^2\right)^2+3c^2de(3+m)^2\left(42+13m+m^2\right)+3c^4d^2\right.}{c^5f(2+m)(3+m)(4+m)(5+}$$

Mathematica [F] time = 0.22, size = 0, normalized size = 0.00

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\csc^{-1}(cx)\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m*(d + e*x^2)^3*(a + b*ArcCsc[c*x]),x]
[Out] Integrate[(f*x)^m*(d + e*x^2)^3*(a + b*ArcCsc[c*x]), x]
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ae^3x^6+3\,ade^2x^4+3\,ad^2ex^2+ad^3+\left(be^3x^6+3\,bde^2x^4+3\,bd^2ex^2+bd^3\right)\text{arccsc}\left(cx\right)\right)\left(fx\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccsc(c*x)),x, algorithm="fricas")
[Out] integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 +
3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arccsc(c*x))*(f*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2+d\right)^3 \left(b\,\text{arccsc}\left(cx\right)+a\right)\left(fx\right)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccsc(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^3*(b*arccsc(c*x) + a)*(f*x)^m, x)
```

maple [F] time = 12.13, size = 0, normalized size = 0.00

$$\int \left(fx\right)^{m} \left(e\,x^{2}+d\right)^{3} \left(a+b\,\mathrm{arccsc}\left(cx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)^3*(a+b*arccsc(c*x)),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)^3*(a+b*arccsc(c*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ae^{3}f^{m}x^{7}x^{m}}{m+7}+\frac{3\,ade^{2}f^{m}x^{5}x^{m}}{m+5}+\frac{3\,ad^{2}ef^{m}x^{3}x^{m}}{m+3}+\frac{\left(fx\right)^{m+1}ad^{3}}{f(m+1)}+\frac{\left(\left(be^{3}f^{m}m^{3}\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right)+9\,be^{3}f^{m}m^{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] a*e^3*f^m*x^7*x^m/(m + 7) + 3*a*d*e^2*f^m*x^5*x^m/(m + 5) + 3*a*d^2*e*f^m*x
^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d^3/(f*(m + 1)) + (((b*e^3*f^m*m^3*arctan2
(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 9*b*e^3*f^m*m^2*arctan2(1, sqrt(c*x + 1)
*sqrt(c*x - 1)) + 23*b*e^3*f^m*m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) +
15*b*e^3*f^m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*x^7 + 3*(b*d*e^2*f^m*
m^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 11*b*d*e^2*f^m*m^2*arctan2(1,
 sqrt(c*x + 1)*sqrt(c*x - 1)) + 31*b*d*e^2*f^m*m*arctan2(1, sqrt(c*x + 1)*s
qrt(c*x - 1)) + 21*b*d*e^2*f^m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*x^5
 + 3*(b*d^2*e*f^m*m^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 13*b*d^2*e*
f^m*m^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 47*b*d^2*e*f^m*m*arctan2(
1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 35*b*d^2*e*f^m*arctan2(1, sqrt(c*x + 1)*s
qrt(c*x - 1)))*x^3 + (b*d^3*f^m*m^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))
 + 15*b*d^3*f^m*m^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 71*b*d^3*f^m*
m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 105*b*d^3*f^m*arctan2(1, sqrt(c
*x + 1)*sqrt(c*x - 1)))*x)*x^m + (m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*inte
grate(-(b*d^3*f^m*m^3 + 15*b*d^3*f^m*m^2 + (b*e^3*f^m*m^3 + 9*b*e^3*f^m*m^2
 + 23*b*e^3*f^m*m + 15*b*e^3*f^m)*x^6 + 71*b*d^3*f^m*m + 105*b*d^3*f^m + 3*
(b*d*e^2*f^m*m^3 + 11*b*d*e^2*f^m*m^2 + 31*b*d*e^2*f^m*m + 21*b*d*e^2*f^m)*
x^4 + 3*(b*d^2*e*f^m*m^3 + 13*b*d^2*e*f^m*m^2 + 47*b*d^2*e*f^m*m + 35*b*d^2
*e*f^m)*x^2)*sqrt(c*x + 1)*sqrt(c*x - 1)*x^m/(m^4 + 16*m^3 - (c^2*m^4 + 16*
c^2*m^3 + 86*c^2*m^2 + 176*c^2*m + 105*c^2)*x^2 + 86*m^2 + 176*m + 105), x)
)/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(f\,x\right)^{m}\,\left(e\,x^{2}+d\right)^{3}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)^3*(a + b*asin(1/(c*x))),x)
```

```
[Out] int((f*x)^m*(d + e*x^2)^3*(a + b*asin(1/(c*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)**3*(a+b*acsc(c*x)),x)
```

```
[Out] Timed out
```

3.166 $\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right)\,dx$

Optimal. Leaf size=371

$$\frac{d^2(fx)^{m+1}\left(a+b\csc^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\csc^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\csc^{-1}(cx)\right)}{f^5(m+5)}+\frac{be^2x\sqrt{c^2x^2-1}\,(fx}{cf^3(m+4)(m+5)}$$

```
[Out] d^2*(f*x)^(1+m)*(a+b*arccsc(c*x))/f/(1+m)+2*d*e*(f*x)^(3+m)*(a+b*arccsc(c*x
))/f^3/(3+m)+e^2*(f*x)^(5+m)*(a+b*arccsc(c*x))/f^5/(5+m)+b*(c^4*d^2*(2+m)*(
3+m)*(4+m)*(5+m)+e*(1+m)^2*(e*(3+m)^2+2*c^2*d*(m^2+9*m+20)))*x*(f*x)^(1+m)*
hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)*(-c^2*x^2+1)^(1/2)/c^3/f/(1
+m)^2/(2+m)/(3+m)/(4+m)/(5+m)/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)+b*e*(e*(3+m
)^2+2*c^2*d*(m^2+9*m+20))*x*(f*x)^(1+m)*(c^2*x^2-1)^(1/2)/c^3/f/(4+m)/(5+m)
/(m^2+5*m+6)/(c^2*x^2)^(1/2)+b*e^2*x*(f*x)^(3+m)*(c^2*x^2-1)^(1/2)/c/f^3/(4
+m)/(5+m)/(c^2*x^2)^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 352, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {270, 5239, 12, 1267, 459, 365, 364}

$$\frac{d^2(fx)^{m+1}\left(a+b\csc^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\csc^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\csc^{-1}(cx)\right)}{f^5(m+5)}+\frac{bcx\sqrt{1-c^2x^2}\,(fx)^m}{}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*e*(e*(3 + m)^2 + 2*c^2*d*(20 + 9*m + m^2))*x*(f*x)^(1 + m)*Sqrt[-1 + c^2
*x^2])/(c^3*f*(2 + m)*(3 + m)*(4 + m)*(5 + m)*Sqrt[c^2*x^2]) + (b*e^2*x*(f*
x)^(3 + m)*Sqrt[-1 + c^2*x^2])/(c*f^3*(4 + m)*(5 + m)*Sqrt[c^2*x^2]) + (d^2
*(f*x)^(1 + m)*(a + b*ArcCsc[c*x]))/(f*(1 + m)) + (2*d*e*(f*x)^(3 + m)*(a +
 b*ArcCsc[c*x]))/(f^3*(3 + m)) + (e^2*(f*x)^(5 + m)*(a + b*ArcCsc[c*x]))/(f
^5*(5 + m)) + (b*c*(d^2/(1 + m)^2 + (e*(e*(3 + m)^2 + 2*c^2*d*(20 + 9*m + m
^2)))/(c^4*(2 + m)*(3 + m)*(4 + m)*(5 + m)))*x*(f*x)^(1 + m)*Sqrt[1 - c^2*x
^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(f*Sqrt[c^2*x^2]
*Sqrt[-1 + c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right) dx = \frac{d^2(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)} + \frac{2de(fx)^{3+m}\left(a+b\csc^{-1}(cx)\right)}{f^3(3+m)} + \frac{e^2(fx)^5}{}$$

$$= \frac{d^2(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)} + \frac{2de(fx)^{3+m}\left(a+b\csc^{-1}(cx)\right)}{f^3(3+m)} + \frac{e^2(fx)^5}{}$$

$$= \frac{be^2x(fx)^{3+m}\sqrt{-1+c^2x^2}}{cf^3(4+m)(5+m)\sqrt{c^2x^2}} + \frac{d^2(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)} + \frac{2de(fx)^{3+m}}{f}$$

$$= \frac{be\left(e(3+m)^2+2c^2d\left(20+9m+m^2\right)\right)x(fx)^{1+m}\sqrt{-1+c^2x^2}}{c^3f(2+m)(4+m)\left(15+8m+m^2\right)\sqrt{c^2x^2}} + \frac{be^2x(f}{cf^3(4+}$$

$$= \frac{be\left(e(3+m)^2+2c^2d\left(20+9m+m^2\right)\right)x(fx)^{1+m}\sqrt{-1+c^2x^2}}{c^3f(2+m)(4+m)\left(15+8m+m^2\right)\sqrt{c^2x^2}} + \frac{be^2x(f}{cf^3(4+}$$

$$= \frac{be\left(e(3+m)^2+2c^2d\left(20+9m+m^2\right)\right)x(fx)^{1+m}\sqrt{-1+c^2x^2}}{c^3f(2+m)(4+m)\left(15+8m+m^2\right)\sqrt{c^2x^2}} + \frac{be^2x(f}{cf^3(4+}$$

Mathematica [F] time = 0.14, size = 0, normalized size = 0.00

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m*(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
[Out] Integrate[(f*x)^m*(d + e*x^2)^2*(a + b*ArcCsc[c*x]), x]
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ae^2x^4+2\,adex^2+ad^2+\left(be^2x^4+2\,bdex^2+bd^2\right)\text{arccsc}\left(cx\right)\right)\left(fx\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="fricas")
[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^
2)*arccsc(c*x))*(f*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2+d\right)^2 (b\,\text{arccsc}\left(cx\right)+a)\left(fx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^2*(b*arccsc(c*x) + a)*(f*x)^m, x)
```

maple [F] time = 9.32, size = 0, normalized size = 0.00

$$\int \left(fx\right)^m \left(e\,x^2+d\right)^2 (a+b\,\text{arccsc}\left(cx\right))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)^2*(a+b*arccsc(c*x)),x)
[Out] int((f*x)^m*(e*x^2+d)^2*(a+b*arccsc(c*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ae^2f^mx^5x^m}{m+5}+\frac{2\,adef^mx^3x^m}{m+3}+\frac{\left(fx\right)^{m+1}ad^2}{f(m+1)}+\frac{\left(\left(be^2f^mm^2\arctan\left(1,\sqrt{cx+1}\sqrt{cx-1}\right)+4\,be^2f^mm\arctan\left(1,\sqrt{cx\right.\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="maxima")
[Out] a*e^2*f^m*x^5*x^m/(m + 5) + 2*a*d*e*f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d
^2/(f*(m + 1)) + (((b*e^2*f^m*m^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) +
 4*b*e^2*f^m*m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 3*b*e^2*f^m*arctan
2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*x^5 + 2*(b*d*e*f^m*m^2*arctan2(1, sqrt(c
*x + 1)*sqrt(c*x - 1)) + 6*b*d*e*f^m*m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x -
1)) + 5*b*d*e*f^m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*x^3 + (b*d^2*f^m
*m^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 8*b*d^2*f^m*m*arctan2(1, sqr
t(c*x + 1)*sqrt(c*x - 1)) + 15*b*d^2*f^m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x
- 1)))*x)*x^m + (m^3 + 9*m^2 + 23*m + 15)*integrate(-(b*d^2*f^m*m^2 + 8*b*d
^2*f^m*m + (b*e^2*f^m*m^2 + 4*b*e^2*f^m*m + 3*b*e^2*f^m)*x^4 + 15*b*d^2*f^m
```

```
 + 2*(b*d*e*f^m*m^2 + 6*b*d*e*f^m*m + 5*b*d*e*f^m)*x^2)*sqrt(c*x + 1)*sqrt(
c*x - 1)*x^m/(m^3 - (c^2*m^3 + 9*c^2*m^2 + 23*c^2*m + 15*c^2)*x^2 + 9*m^2 +
 23*m + 15), x))/(m^3 + 9*m^2 + 23*m + 15)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (f\,x)^m\,\left(e\,x^2+d\right)^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)^2*(a + b*asin(1/(c*x))),x)
```

```
[Out] int((f*x)^m*(d + e*x^2)^2*(a + b*asin(1/(c*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m\,(a+b\,\mathrm{acsc}\,(cx))\left(d+ex^2\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)**2*(a+b*acsc(c*x)),x)
```

```
[Out] Integral((f*x)**m*(a + b*acsc(c*x))*(d + e*x**2)**2, x)
```

3.167 $\int (fx)^m \left(d+ex^2\right) \left(a+b\csc^{-1}(cx)\right) dx$

Optimal. Leaf size=215

$$\frac{d(fx)^{m+1}\left(a+b\csc^{-1}(cx)\right)}{f(m+1)}+\frac{e(fx)^{m+3}\left(a+b\csc^{-1}(cx)\right)}{f^3(m+3)}+\frac{bx\sqrt{1-c^2x^2}\,(fx)^{m+1}\left(c^2d(m+2)(m+3)+e(m+1)^2\right.}{cf(m+1)^2(m+2)(m+3)\sqrt{c^2x^2}\,\sqrt{c}}$$

```
[Out] d*(f*x)^(1+m)*(a+b*arccsc(c*x))/f/(1+m)+e*(f*x)^(3+m)*(a+b*arccsc(c*x))/f^3
/(3+m)+b*(e*(1+m)^2+c^2*d*(2+m)*(3+m))*x*(f*x)^(1+m)*hypergeom([1/2, 1/2+1/
2*m],[3/2+1/2*m],c^2*x^2)*(-c^2*x^2+1)^(1/2)/c/f/(1+m)^2/(2+m)/(3+m)/(c^2*x
^2)^(1/2)/(c^2*x^2-1)^(1/2)+b*e*x*(f*x)^(1+m)*(c^2*x^2-1)^(1/2)/c/f/(m^2+5*
m+6)/(c^2*x^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 202, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {14, 5239, 12, 459, 365, 364}

$$\frac{d(fx)^{m+1}\left(a+b\csc^{-1}(cx)\right)}{f(m+1)}+\frac{e(fx)^{m+3}\left(a+b\csc^{-1}(cx)\right)}{f^3(m+3)}+\frac{bcx\sqrt{1-c^2x^2}\,(fx)^{m+1}\left(\frac{e}{c^2(m+2)(m+3)}+\frac{d}{(m+1)^2}\right)\,{}_2F_1\left(\frac{1}{2},\right.}{f\sqrt{c^2x^2}\,\sqrt{c^2x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*e*x*(f*x)^(1 + m)*Sqrt[-1 + c^2*x^2])/(c*f*(6 + 5*m + m^2)*Sqrt[c^2*x^2]
) + (d*(f*x)^(1 + m)*(a + b*ArcCsc[c*x]))/(f*(1 + m)) + (e*(f*x)^(3 + m)*(a
 + b*ArcCsc[c*x]))/(f^3*(3 + m)) + (b*c*(d/(1 + m)^2 + e/(c^2*(2 + m)*(3 +
m)))*x*(f*x)^(1 + m)*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3
 + m)/2, c^2*x^2])/(f*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
```

```
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)dx = \frac{d(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)} + \frac{e(fx)^{3+m}\left(a+b\csc^{-1}(cx)\right)}{f^3(3+m)} + \frac{(bcx)\int \frac{(fx)^{m}}{(1+}}{}$$

$$= \frac{d(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)} + \frac{e(fx)^{3+m}\left(a+b\csc^{-1}(cx)\right)}{f^3(3+m)} + \frac{(bcx)\int \frac{(fx)^{m}}{}}{(3+4m}$$

$$= \frac{bex(fx)^{1+m}\sqrt{-1+c^2x^2}}{cf\left(6+5m+m^2\right)\sqrt{c^2x^2}} + \frac{d(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)} + \frac{e(fx)^{3+m}(a+}{f^3(3}$$

$$= \frac{bex(fx)^{1+m}\sqrt{-1+c^2x^2}}{cf\left(6+5m+m^2\right)\sqrt{c^2x^2}} + \frac{d(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)} + \frac{e(fx)^{3+m}(a+}{f^3(3}$$

$$= \frac{bex(fx)^{1+m}\sqrt{-1+c^2x^2}}{cf\left(6+5m+m^2\right)\sqrt{c^2x^2}} + \frac{d(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)} + \frac{e(fx)^{3+m}(a+}{f^3(3}$$

Mathematica [F] time = 0.11, size = 0, normalized size = 0.00

$$\int (fx)^m \left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
[Out] Integrate[(f*x)^m*(d + e*x^2)*(a + b*ArcCsc[c*x]), x]
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(aex^2+ad+\left(bex^2+bd\right)\operatorname{arccsc}(cx)\right)(fx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsc(c*x))*(f*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2+d\right)\left(b\operatorname{arccsc}(cx)+a\right)(fx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)*(b*arccsc(c*x) + a)*(f*x)^m, x)
```

maple [F] time = 7.52, size = 0, normalized size = 0.00

$$\int (fx)^{m} \left(e\,x^{2}+d\right) (a+b\,\mathrm{arccsc}\,(cx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(a+b*arccsc(c*x)),x)
[Out] int((f*x)^m*(e*x^2+d)*(a+b*arccsc(c*x)),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (f\,x)^{m}\, \left(e\,x^{2}+d\right) \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)*(a + b*asin(1/(c*x))),x)
[Out] int((f*x)^m*(d + e*x^2)*(a + b*asin(1/(c*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^{m} (a+b\,\mathrm{acsc}\,(cx)) \left(d+ex^{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(a+b*acsc(c*x)),x)
[Out] Integral((f*x)**m*(a + b*acsc(c*x))*(d + e*x**2), x)
```

3.168 $\int \frac{(fx)^m(a+b\csc^{-1}(cx))}{d+ex^2}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{d+ex^2},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d),x)
```

Rubi [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{d+ex^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2),x]
[Out] Defer[Int][((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2), x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{d+ex^2}\,dx=\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{d+ex^2}\,dx$$

Mathematica [A] time = 1.99, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{d+ex^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2),x]
[Out] Integrate[((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2), x]
```

fricas [A] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b\,\text{arccsc}\,(cx)+a)\left(fx\right)^m}{ex^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="fricas")
[Out] integral((b*arccsc(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arccsc}\,(cx)+a)\left(fx\right)^m}{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

maple [A] time = 3.59, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^{m}\left(a+b\,\mathrm{arccsc}\left(cx\right)\right)}{e\,x^{2}+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d),x)
```

```
[Out] int((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\mathrm{arccsc}\left(cx\right)+a\right)\left(fx\right)^{m}}{ex^{2}+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] integrate((b*arccsc(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(f\,x\right)^{m}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{e\,x^{2}+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(a + b*asin(1/(c*x))))/(d + e*x^2),x)
```

```
[Out] int(((f*x)^m*(a + b*asin(1/(c*x))))/(d + e*x^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^{m}\left(a+b\,\mathrm{acsc}\left(cx\right)\right)}{d+ex^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acsc(c*x))/(e*x**2+d),x)
```

```
[Out] Integral((f*x)**m*(a + b*acsc(c*x))/(d + e*x**2), x)
```

3.169 $\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^2},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^2,x)
```

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
[Out] Defer[Int][((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2, x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

Mathematica [A] time = 6.45, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
[Out] Integrate[((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2, x]
```

fricas [A] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b\,\text{arccsc}\,(cx)+a)\left(fx\right)^m}{e^2x^4+2\,dex^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*arccsc(c*x) + a)*(f*x)^m/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arccsc}\,(cx)+a)\left(fx\right)^m}{\left(ex^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)
```

maple [A] time = 6.05, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^{m} \left(a+b\,\mathrm{arccsc}\left(cx\right)\right)}{\left(e\,x^{2}+d\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^2,x)
[Out] int((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\mathrm{arccsc}\left(cx\right)+a\right)\left(fx\right)^{m}}{\left(ex^{2}+d\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
[Out] integrate((b*arccsc(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(f\,x\right)^{m}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^{2}+d\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(a + b*asin(1/(c*x))))/(d + e*x^2)^2,x)
[Out] int(((f*x)^m*(a + b*asin(1/(c*x))))/(d + e*x^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acsc(c*x))/(e*x**2+d)**2,x)
[Out] Timed out
```

3.170 $\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right) dx$

Optimal. Leaf size=28

$$\text{Int}\left(\left(d+ex^2\right)^{3/2}(fx)^m\left(a+b\csc^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (fx)^m\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)dx$$

Verification is Not applicable to the result.

```
[In] Int[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
[Out] Defer[Int][(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]), x]
```

Rubi steps

$$\int (fx)^m\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)dx=\int (fx)^m\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)dx$$

Mathematica [A] time = 1.16, size = 0, normalized size = 0.00

$$\int (fx)^m\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
[Out] Integrate[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]), x]
```

fricas [A] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(aex^2+ad+\left(bex^2+bd\right)\text{arccsc}\,(cx)\right)\sqrt{ex^2+d}\,\left(fx\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsc(c*x))*sqrt(e*x^2 + d)*(f*x
)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2+d\right)^{\frac{3}{2}}(b\,\text{arccsc}\,(cx)+a)\left(fx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)*(f*x)^m, x)
```

maple [A] time = 4.78, size = 0, normalized size = 0.00

$$\int \left(fx\right)^{m} \left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\,\mathrm{arccsc}\left(cx\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
[Out] int((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^{2}+d\right)^{\frac{3}{2}}\left(b\,\mathrm{arccsc}\left(cx\right)+a\right)\left(fx\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)*(f*x)^m, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \left(f\,x\right)^{m}\,\left(e\,x^{2}+d\right)^{3/2}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))),x)
[Out] int((f*x)^m*(d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)**(3/2)*(a+b*acsc(c*x)),x)
[Out] Timed out
```

3.171 $\int (fx)^m \sqrt{d+ex^2} \left(a+b\csc^{-1}(cx)\right) dx$

Optimal. Leaf size=28

$$\text{Int}\left(\sqrt{d+ex^2}\,(fx)^m\left(a+b\csc^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)dx$$

Verification is Not applicable to the result.

```
[In] Int[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
[Out] Defer[Int][(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]), x]
```

Rubi steps

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)dx=\int (fx)^m\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)dx$$

Mathematica [A] time = 0.12, size = 0, normalized size = 0.00

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
[Out] Integrate[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]), x]
```

fricas [A] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{ex^2+d}\,(b\,\text{arccsc}\,(cx)+a)\left(fx\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)*(f*x)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex^2+d}\,(b\,\text{arccsc}\,(cx)+a)\left(fx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)*(f*x)^m, x)
```

maple [A] time = 4.88, size = 0, normalized size = 0.00

$$\int \left(fx\right)^m(a+b\,\text{arccsc}\,(cx))\sqrt{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
[Out] int((f*x)^m*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex^2+d}\,(b\operatorname{arccsc}(cx)+a)\left(fx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)*(f*x)^m, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \left(f\,x\right)^m\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))),x)
[Out] int((f*x)^m*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^m\,(a+b\operatorname{acsc}(cx))\sqrt{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acsc(c*x))*(e*x**2+d)**(1/2),x)
[Out] Integral((f*x)**m*(a + b*acsc(c*x))*sqrt(d + e*x**2), x)
```

3.172 $\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
[Out] Defer[Int][((f*x)^m*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

Mathematica [A] time = 1.49, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
[Out] Integrate[((f*x)^m*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2], x]
```

fricas [A] time = 0.78, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(b\,\mathrm{arccsc}\,(cx)+a)\left(fx\right)^m}{\sqrt{ex^2+d}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b*arccsc(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arccsc}\,(cx)+a)\left(fx\right)^m}{\sqrt{ex^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)
```

maple [A] time = 6.56, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + b \operatorname{arccsc}(cx))}{\sqrt{e x^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
[Out] int((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arccsc}(cx) + a)(fx)^m}{\sqrt{ex^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] integrate((b*arccsc(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(f\,x)^m \left(a + b \operatorname{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2),x)
[Out] int(((f*x)^m*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + b \operatorname{acsc}(cx))}{\sqrt{d + ex^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acsc(c*x))/(e*x**2+d)**(1/2),x)
[Out] Integral((f*x)**m*(a + b*acsc(c*x))/sqrt(d + e*x**2), x)
```

3.173 $\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
[Out] Defer[Int][((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [A] time = 1.67, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
[Out] Integrate[((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2), x]
```

fricas [A] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2+d}\,(b\,\text{arccsc}\,(cx)+a)\left(fx\right)^m}{e^2x^4+2\,dex^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)*(f*x)^m/(e^2*x^4 + 2*d*e*x^2 +
 d^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arccsc}\,(cx)+a)\left(fx\right)^m}{\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)*(f*x)^m/(e*x^2 + d)^(3/2), x)
```

maple [A] time = 6.27, size = 0, normalized size = 0.00

$$\int \frac{(fx)^{m} (a + b\,\mathrm{arccsc}\,(cx))}{(e\,x^{2} + d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
[Out] int((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arccsc}\,(cx) + a)\,(fx)^{m}}{(ex^{2} + d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
[Out] integrate((b*arccsc(c*x) + a)*(f*x)^m/(e*x^2 + d)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(f\,x)^{m}\,\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^{2} + d)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2),x)
[Out] int(((f*x)^m*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acsc(c*x))/(e*x**2+d)**(3/2),x)
[Out] Timed out
```

3.174 $\int \frac{x^{11}\left(a+b\csc^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx$

Optimal. Leaf size=401

$$-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{10c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\left(a+b\csc^{-1}(cx)\right)}{2c^{12}}-\frac{b\sqrt{1-c^2x^2}\left(c^2x^2\right.}{90c^{13}x\sqrt{1-\frac{}{c^2}}}$$

```
[Out] 1/3*(-c^4*x^4+1)^(3/2)*(a+b*arccsc(c*x))/c^12-1/10*(-c^4*x^4+1)^(5/2)*(a+b*
arccsc(c*x))/c^12+7/90*b*(c^2*x^2+1)^(3/2)*(-c^2*x^2+1)^(1/2)/c^13/x/(1-1/c
^2/x^2)^(1/2)-13/150*b*(c^2*x^2+1)^(5/2)*(-c^2*x^2+1)^(1/2)/c^13/x/(1-1/c^2
/x^2)^(1/2)+3/70*b*(c^2*x^2+1)^(7/2)*(-c^2*x^2+1)^(1/2)/c^13/x/(1-1/c^2/x^2
)^(1/2)-1/90*b*(c^2*x^2+1)^(9/2)*(-c^2*x^2+1)^(1/2)/c^13/x/(1-1/c^2/x^2)^(1
/2)+4/15*b*arctanh((c^2*x^2+1)^(1/2))*(-c^2*x^2+1)^(1/2)/c^13/x/(1-1/c^2/x^
2)^(1/2)-4/15*b*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(1/2)/c^13/x/(1-1/c^2/x^2)^(
1/2)-1/2*(a+b*arccsc(c*x))*(-c^4*x^4+1)^(1/2)/c^12
```

Rubi [A] time = 2.53, antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {266, 43, 5247, 12, 6721, 6742, 848, 50, 63, 208, 783}

$$-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{10c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\left(a+b\csc^{-1}(cx)\right)}{2c^{12}}-\frac{b\sqrt{1-c^2x^2}\left(c^2x^2\right.}{90c^{13}x\sqrt{1-\frac{}{c^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(a + b*ArcCsc[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] (-4*b*Sqrt[1 - c^2*x^2]*Sqrt[1 + c^2*x^2])/(15*c^13*Sqrt[1 - 1/(c^2*x^2)]*x
) + (7*b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(3/2))/(90*c^13*Sqrt[1 - 1/(c^2*x^
2)]*x) - (13*b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(5/2))/(150*c^13*Sqrt[1 - 1/
(c^2*x^2)]*x) + (3*b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(7/2))/(70*c^13*Sqrt[1
 - 1/(c^2*x^2)]*x) - (b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(9/2))/(90*c^13*Sqr
t[1 - 1/(c^2*x^2)]*x) - (Sqrt[1 - c^4*x^4]*(a + b*ArcCsc[c*x]))/(2*c^12) +
((1 - c^4*x^4)^(3/2)*(a + b*ArcCsc[c*x]))/(3*c^12) - ((1 - c^4*x^4)^(5/2)*(
a + b*ArcCsc[c*x]))/(10*c^12) + (4*b*Sqrt[1 - c^2*x^2]*ArcTanh[Sqrt[1 + c^2
*x^2]])/(15*c^13*Sqrt[1 - 1/(c^2*x^2)]*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 783

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)*(a/d + (c*x)/e)^p, x] /; F
reeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 5247

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(b^IntPart[p]*(
a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[
p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^{11}\left(a+b\csc^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = -\frac{\sqrt{1-c^4x^4}\left(a+b\csc^{-1}(cx)\right)}{2c^{12}} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3c^{12}} - \frac{\left(1-c^4x^4\right)^{5/2}\left(a+\right.}{10c^{12}}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\csc^{-1}(cx)\right)}{2c^{12}} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3c^{12}} - \frac{\left(1-c^4x^4\right)^{5/2}\left(a+\right.}{10c^{12}}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\csc^{-1}(cx)\right)}{2c^{12}} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3c^{12}} - \frac{\left(1-c^4x^4\right)^{5/2}\left(a+\right.}{10c^{12}}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\csc^{-1}(cx)\right)}{2c^{12}} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3c^{12}} - \frac{\left(1-c^4x^4\right)^{5/2}\left(a+\right.}{10c^{12}}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\csc^{-1}(cx)\right)}{2c^{12}} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3c^{12}} - \frac{\left(1-c^4x^4\right)^{5/2}\left(a+\right.}{10c^{12}}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\csc^{-1}(cx)\right)}{2c^{12}} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3c^{12}} - \frac{\left(1-c^4x^4\right)^{5/2}\left(a+\right.}{10c^{12}}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\csc^{-1}(cx)\right)}{2c^{12}} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3c^{12}} - \frac{\left(1-c^4x^4\right)^{5/2}\left(a+\right.}{10c^{12}}$$

$$= -\frac{4b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{15c^{13}\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{\sqrt{1-c^4x^4}\left(a+b\csc^{-1}(cx)\right)}{2c^{12}} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\csc^{-1}\right.}{3c^{12}}$$

$$= -\frac{4b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{15c^{13}\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{7b\sqrt{1-c^2x^2}\left(1+c^2x^2\right)^{3/2}}{90c^{13}\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{13b\sqrt{1-c^2x^2}\left(1+c^2x^2\right)^{5/}}{150c^{13}\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= -\frac{4b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{15c^{13}\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{7b\sqrt{1-c^2x^2}\left(1+c^2x^2\right)^{3/2}}{90c^{13}\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{13b\sqrt{1-c^2x^2}\left(1+c^2x^2\right)^{5/}}{150c^{13}\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

Mathematica [A] time = 0.29, size = 194, normalized size = 0.48

$$-\frac{105a\sqrt{1-c^4x^4}\left(3c^8x^8+4c^4x^4+8\right)+105b\sqrt{1-c^4x^4}\left(3c^8x^8+4c^4x^4+8\right)\csc^{-1}(cx)+840b\tan^{-1}\left(\frac{cx\sqrt{1-\frac{1}{c^2x^2}}}{\sqrt{1-c^4x^4}}\right)+}{3150c^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(a + b*ArcCsc[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -1/3150*(105*a*Sqrt[1 - c^4*x^4]*(8 + 4*c^4*x^4 + 3*c^8*x^8) + (b*c*Sqrt[1
- 1/(c^2*x^2)]*x*Sqrt[1 - c^4*x^4]*(768 + 36*c^2*x^2 + 78*c^4*x^4 + 5*c^6*x
```

```
^6 + 35*c^8*x^8))/(-1 + c^2*x^2) + 105*b*Sqrt[1 - c^4*x^4]*(8 + 4*c^4*x^4 +
 3*c^8*x^8)*ArcCsc[c*x] + 840*b*ArcTan[(c*Sqrt[1 - 1/(c^2*x^2)]*x)/Sqrt[1 -
 c^4*x^4]])/c^12
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError >> the translation of the FriCAS obje
ct sage2 to sage is not yet implemented
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 13.07, size = 0, normalized size = 0.00

$$\int \frac{x^{11} \left(a + b \,\mathrm{arccsc}\left(c x\right)\right)}{\sqrt{-c^4 x^4 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] int(x^11*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{30}\, a \left(\frac{3\left(-c^4x^4+1\right)^{\frac{5}{2}}}{c^{12}} - \frac{10\left(-c^4x^4+1\right)^{\frac{3}{2}}}{c^{12}} + \frac{15\sqrt{-c^4x^4+1}}{c^{12}}\right) + \frac{\left(c^{12}\int \frac{\left(3c^{10}x^{11}+3c^8x^9+4c^6x^7+4c^4x^5+8c^2x^3+8x\right)e^{\left(-\frac{1}{2}\log(\cdots\right.}}{(cx+1)(cx-1)\sqrt{-cx+1}\,c^{10}+\sqrt{-cx+1}\,c\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/30*a*(3*(-c^4*x^4 + 1)^(5/2)/c^12 - 10*(-c^4*x^4 + 1)^(3/2)/c^12 + 15*sq
rt(-c^4*x^4 + 1)/c^12) + 1/30*(30*c^12*integrate(1/30*(3*c^10*x^11 + 3*c^8*
x^9 + 4*c^6*x^7 + 4*c^4*x^5 + 8*c^2*x^3 + 8*x)*e^(-1/2*log(c^2*x^2 + 1) + 1
/2*log(c*x - 1))/(c^10*e^(log(c*x + 1) + log(c*x - 1) + 1/2*log(-c*x + 1))
+ sqrt(-c*x + 1)*c^10), x) - (3*c^8*x^8*arctan2(1, sqrt(c*x + 1)*sqrt(c*x -
 1)) + 4*c^4*x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 8*arctan2(1, sqr
t(c*x + 1)*sqrt(c*x - 1)))*sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1))*
b/c^12
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{11} \left(a + b \,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{1 - c^4\,x^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(a + b*asin(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)
[Out] int((x^11*(a + b*asin(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(a+b*acsc(c*x))/(-c**4*x**4+1)**(1/2),x)
[Out] Timed out
```

3.175 $\int \frac{x^7\left(a+b\csc^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx$

Optimal. Leaf size=268

$$\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{6c^8}-\frac{\sqrt{1-c^4x^4}\left(a+b\csc^{-1}(cx)\right)}{2c^8}-\frac{b\sqrt{1-c^2x^2}\left(c^2x^2+1\right)^{5/2}}{30c^9x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{b\sqrt{1-c^2x^2}\left(c^2x^2+1\right.}{18c^9x\sqrt{1-\frac{1}{c^2x^2}}}$$

```
[Out] 1/6*(-c^4*x^4+1)^(3/2)*(a+b*arccsc(c*x))/c^8+1/18*b*(c^2*x^2+1)^(3/2)*(-c^2
*x^2+1)^(1/2)/c^9/x/(1-1/c^2/x^2)^(1/2)-1/30*b*(c^2*x^2+1)^(5/2)*(-c^2*x^2+
1)^(1/2)/c^9/x/(1-1/c^2/x^2)^(1/2)+1/3*b*arctanh((c^2*x^2+1)^(1/2))*(-c^2*x
^2+1)^(1/2)/c^9/x/(1-1/c^2/x^2)^(1/2)-1/3*b*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^
(1/2)/c^9/x/(1-1/c^2/x^2)^(1/2)-1/2*(a+b*arccsc(c*x))*(-c^4*x^4+1)^(1/2)/c^
8
```

Rubi [A] time = 2.03, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {266, 43, 5247, 12, 6721, 6742, 848, 50, 63, 208, 783}

$$\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{6c^8}-\frac{\sqrt{1-c^4x^4}\left(a+b\csc^{-1}(cx)\right)}{2c^8}-\frac{b\sqrt{1-c^2x^2}\left(c^2x^2+1\right)^{5/2}}{30c^9x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{b\sqrt{1-c^2x^2}\left(c^2x^2+1\right.}{18c^9x\sqrt{1-\frac{1}{c^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(a + b*ArcCsc[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -(b*Sqrt[1 - c^2*x^2]*Sqrt[1 + c^2*x^2])/(3*c^9*Sqrt[1 - 1/(c^2*x^2)]*x) +
(b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(3/2))/(18*c^9*Sqrt[1 - 1/(c^2*x^2)]*x)
- (b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(5/2))/(30*c^9*Sqrt[1 - 1/(c^2*x^2)]*x
) - (Sqrt[1 - c^4*x^4]*(a + b*ArcCsc[c*x]))/(2*c^8) + ((1 - c^4*x^4)^(3/2)*
(a + b*ArcCsc[c*x]))/(6*c^8) + (b*Sqrt[1 - c^2*x^2]*ArcTanh[Sqrt[1 + c^2*x^
2]])/(3*c^9*Sqrt[1 - 1/(c^2*x^2)]*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 783

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)*(a/d + (c*x)/e)^p, x] /; F
reeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 5247

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(b^IntPart[p]*(
a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[
p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^7\left(a+b\csc^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = -\frac{\sqrt{1-c^4x^4}\left(a+b\csc^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{6c^8} + \frac{b\int \frac{\left(-2-c^4x^4\right)\sqrt{1-}}{6c^8\sqrt{1-\frac{1}{c^2x^2}}}}{c}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\csc^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{6c^8} + \frac{b\int \frac{\left(-2-c^4x^4\right)\sqrt{1-}}{\sqrt{1-\frac{1}{c^2x^2}}\,x}}{6c^9}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\csc^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{6c^8} + \frac{\left(b\sqrt{1-c^2x^2}\right)\int}{6c^9\sqrt{}}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\csc^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{6c^8} - \frac{\left(b\sqrt{1-c^2x^2}\right)\mathrm{S}}{}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\csc^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{6c^8} - \frac{\left(b\sqrt{1-c^2x^2}\right)\mathrm{S}}{}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\csc^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{6c^8} - \frac{\left(b\sqrt{1-c^2x^2}\right)\mathrm{S}}{6c}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\csc^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{6c^8} - \frac{\left(b\sqrt{1-c^2x^2}\right)\mathrm{S}}{6c^9}$$

$$= -\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{3c^9\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{\sqrt{1-c^4x^4}\left(a+b\csc^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\csc\right.}{6c^8}$$

$$= -\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{3c^9\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{b\sqrt{1-c^2x^2}\left(1+c^2x^2\right)^{3/2}}{18c^9\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{b\sqrt{1-c^2x^2}\left(1+c^2x^2\right)^{5/2}}{30c^9\sqrt{1-\frac{1}{c^2x^2}}\,x} -$$

$$= -\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{3c^9\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{b\sqrt{1-c^2x^2}\left(1+c^2x^2\right)^{3/2}}{18c^9\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{b\sqrt{1-c^2x^2}\left(1+c^2x^2\right)^{5/2}}{30c^9\sqrt{1-\frac{1}{c^2x^2}}\,x} -$$

Mathematica [A] time = 0.41, size = 159, normalized size = 0.59

$$-\frac{15a\sqrt{1-c^4x^4}\left(c^4x^4+2\right)+15b\sqrt{1-c^4x^4}\left(c^4x^4+2\right)\csc^{-1}(cx)+\frac{bcx\sqrt{1-\frac{1}{c^2x^2}}\sqrt{1-c^4x^4}\left(3c^4x^4+c^2x^2+28\right)}{c^2x^2-1}+30b\tan^{-1}}{90c^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(a + b*ArcCsc[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -1/90*(15*a*Sqrt[1 - c^4*x^4]*(2 + c^4*x^4) + (b*c*Sqrt[1 - 1/(c^2*x^2)]*x*
Sqrt[1 - c^4*x^4]*(28 + c^2*x^2 + 3*c^4*x^4))/(-1 + c^2*x^2) + 15*b*Sqrt[1
```

```
- c^4*x^4]*(2 + c^4*x^4)*ArcCsc[c*x] + 30*b*ArcTan[(c*Sqrt[1 - 1/(c^2*x^2)]
*x)/Sqrt[1 - c^4*x^4]])/c^8
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError >> the translation of the FriCAS obje
ct sage2 to sage is not yet implemented
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 9.29, size = 0, normalized size = 0.00

$$\int \frac{x^{7} \left(a + b \,\mathrm{arccsc}\left(c x\right)\right)}{\sqrt{-c^{4} x^{4}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] int(x^7*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\, a \left(\frac{\left(-c^{4} x^{4}+1\right)^{\frac{3}{2}}}{c^{8}} - \frac{3 \sqrt{-c^{4} x^{4}+1}}{c^{8}}\right) + \frac{\left(c^{8} x^{8} \arctan\left(1, \sqrt{c x+1} \sqrt{c x-1}\right) + \sqrt{c^{2} x^{2}+1} \sqrt{c x+1} \sqrt{-c x+1}\, c^{8} \int \frac{\left(c^{6} x^{7}+\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*a*((-c^4*x^4 + 1)^(3/2)/c^8 - 3*sqrt(-c^4*x^4 + 1)/c^8) + 1/6*(c^8*x^8*
arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 6*sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)
*sqrt(-c*x + 1)*c^8*integrate(1/6*(c^6*x^7 + c^4*x^5 + 2*c^2*x^3 + 2*x)*e^(
-1/2*log(c^2*x^2 + 1) + 1/2*log(c*x - 1))/(c^6*e^(log(c*x + 1) + log(c*x -
1) + 1/2*log(-c*x + 1)) + sqrt(-c*x + 1)*c^6), x) + c^4*x^4*arctan2(1, sqrt
(c*x + 1)*sqrt(c*x - 1)) - 2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*b/(sq
rt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*c^8)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7 \left(a+b \,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{1-c^4\,x^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(a + b*asin(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)
[Out] int((x^7*(a + b*asin(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(a+b*acsc(c*x))/(-c**4*x**4+1)**(1/2),x)
[Out] Timed out
```

3.176 $\int \frac{x^3\left(a+b\csc^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx$

Optimal. Leaf size=126

$$-\frac{\sqrt{1-c^4x^4}\left(a+b\csc^{-1}(cx)\right)}{2c^4}-\frac{bx\sqrt{1-c^4x^4}}{2c^3\sqrt{c^2x^2}\sqrt{c^2x^2-1}}+\frac{bx\tan^{-1}\left(\frac{\sqrt{1-c^4x^4}}{\sqrt{c^2x^2-1}}\right)}{2c^3\sqrt{c^2x^2}}$$

```
[Out] 1/2*b*x*arctan((-c^4*x^4+1)^(1/2)/(c^2*x^2-1)^(1/2))/c^3/(c^2*x^2)^(1/2)-1/
2*(a+b*arccsc(c*x))*(-c^4*x^4+1)^(1/2)/c^4-1/2*b*x*(-c^4*x^4+1)^(1/2)/c^3/(
c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 135, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {261, 5247, 12, 1572, 1252, 848, 50, 63, 208}

$$-\frac{\sqrt{1-c^4x^4}\left(a+b\csc^{-1}(cx)\right)}{2c^4}-\frac{b\sqrt{1-c^2x^2}\sqrt{c^2x^2+1}}{2c^5x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{b\sqrt{1-c^2x^2}\tanh^{-1}\left(\sqrt{c^2x^2+1}\right)}{2c^5x\sqrt{1-\frac{1}{c^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCsc[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -(b*Sqrt[1 - c^2*x^2]*Sqrt[1 + c^2*x^2])/(2*c^5*Sqrt[1 - 1/(c^2*x^2)]*x) -
(Sqrt[1 - c^4*x^4]*(a + b*ArcCsc[c*x]))/(2*c^4) + (b*Sqrt[1 - c^2*x^2]*ArcT
anh[Sqrt[1 + c^2*x^2]])/(2*c^5*Sqrt[1 - 1/(c^2*x^2)]*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1572

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_)*((a_) + (c_.)*(x_)^(n2_.))^(
p_.), x_Symbol] :> Dist[(e^IntPart[q]*(d + e*x^mn)^FracPart[q])/(x^(mn*Frac
Part[q])*(1 + d/(x^mn*e))^FracPart[q]), Int[x^(m + mn*q)*(1 + d/(x^mn*e))^q
*(a + c*x^n2)^p, x], x] /; FreeQ[{a, c, d, e, m, mn, p, q}, x] && EqQ[n2, -
2*mn] &&  !IntegerQ[p] &&  !IntegerQ[q] && PosQ[n2]
```

Rule 5247

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{x^3 \left(a+b\csc^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = -\frac{\sqrt{1-c^4x^4}\,\left(a+b\csc^{-1}(cx)\right)}{2c^4} + \frac{b\int -\frac{\sqrt{1-c^4x^4}}{2c^4\sqrt{1-\frac{1}{c^2x^2}}\,x^2}\,dx}{c}$$

$$= -\frac{\sqrt{1-c^4x^4}\,\left(a+b\csc^{-1}(cx)\right)}{2c^4} - \frac{b\int \frac{\sqrt{1-c^4x^4}}{\sqrt{1-\frac{1}{c^2x^2}}\,x^2}\,dx}{2c^5}$$

$$= -\frac{\sqrt{1-c^4x^4}\,\left(a+b\csc^{-1}(cx)\right)}{2c^4} - \frac{\left(b\sqrt{1-c^2x^2}\right)\int \frac{\sqrt{1-c^4x^4}}{x\sqrt{1-c^2x^2}}\,dx}{2c^5\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= -\frac{\sqrt{1-c^4x^4}\,\left(a+b\csc^{-1}(cx)\right)}{2c^4} - \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sqrt{1-c^4x^2}}{x\sqrt{1-c^2x}}\,dx,x,x^2\right)}{4c^5\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= -\frac{\sqrt{1-c^4x^4}\,\left(a+b\csc^{-1}(cx)\right)}{2c^4} - \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sqrt{1+c^2x}}{x}\,dx,x,x^2\right)}{4c^5\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= -\frac{b\sqrt{1-c^2x^2}\,\sqrt{1+c^2x^2}}{2c^5\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{\sqrt{1-c^4x^4}\,\left(a+b\csc^{-1}(cx)\right)}{2c^4} - \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\cdots}{x\sqrt{\cdots}}\cdots\right.}{4c^5\sqrt{1-\frac{1}{c^2x^2}\cdots}}$$

$$= -\frac{b\sqrt{1-c^2x^2}\,\sqrt{1+c^2x^2}}{2c^5\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{\sqrt{1-c^4x^4}\,\left(a+b\csc^{-1}(cx)\right)}{2c^4} - \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\cdots}{-\frac{1}{c^2}\cdots}\cdots\right.}{2c^7\sqrt{1-\cdots}}$$

$$= -\frac{b\sqrt{1-c^2x^2}\,\sqrt{1+c^2x^2}}{2c^5\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{\sqrt{1-c^4x^4}\,\left(a+b\csc^{-1}(cx)\right)}{2c^4} + \frac{b\sqrt{1-c^2x^2}\,\tanh^{-1}\left(\sqrt{1+\cdots}\right.}{2c^5\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

Mathematica [A] time = 0.18, size = 138, normalized size = 1.10

$$\frac{\sqrt{1-c^4x^4}\left(-ac^2x^2+a-bcx\sqrt{1-\frac{1}{c^2x^2}}\right)+\left(b-bc^2x^2\right)\tan^{-1}\left(\frac{cx\sqrt{1-\frac{1}{c^2x^2}}}{\sqrt{1-c^4x^4}}\right)-b\left(c^2x^2-1\right)\sqrt{1-c^4x^4}\,\csc^{-1}(cx)}{2c^4\left(c^2x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCsc[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] ((a - b*c*Sqrt[1 - 1/(c^2*x^2)]*x - a*c^2*x^2)*Sqrt[1 - c^4*x^4] - b*(-1 +
c^2*x^2)*Sqrt[1 - c^4*x^4]*ArcCsc[c*x] + (b - b*c^2*x^2)*ArcTan[(c*Sqrt[1 -
 1/(c^2*x^2)]*x)/Sqrt[1 - c^4*x^4]])/(2*c^4*(-1 + c^2*x^2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError >> the translation of the FriCAS obje
ct sage2 to sage is not yet implemented
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arccsc}(cx) + a)x^3}{\sqrt{-c^4x^4 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)*x^3/sqrt(-c^4*x^4 + 1), x)
```

maple [F] time = 6.28, size = 0, normalized size = 0.00

$$\int \frac{x^3 (a + b \operatorname{arccsc}(cx))}{\sqrt{-c^4x^4 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x)
[Out] int(x^3*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(c^4 \int \frac{(c^2x^3+x)e^{\left(-\frac{1}{2}\log(c^2x^2+1)+\frac{1}{2}\log(cx-1)\right)}}{(cx+1)(cx-1)\sqrt{-cx+1}\,c^2+\sqrt{-cx+1}\,c^2}\, dx - \sqrt{c^2x^2+1}\,\sqrt{cx+1}\,\sqrt{-cx+1}\,\arctan\left(1, \sqrt{cx+1}\,\sqrt{cx-1}\right)\right)b}{2\,c^4} - \frac{\sqrt{-c^4x^4}}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
[Out] 1/2*(2*c^4*integrate(1/2*(c^2*x^3 + x)*e^(-1/2*log(c^2*x^2 + 1) + 1/2*log(c
*x - 1))/(c^2*e^(log(c*x + 1) + log(c*x - 1) + 1/2*log(-c*x + 1)) + sqrt(-c
*x + 1)*c^2), x) - sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(1
, sqrt(c*x + 1)*sqrt(c*x - 1)))*b/c^4 - 1/2*sqrt(-c^4*x^4 + 1)*a/c^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \left(a + b \operatorname{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{1 - c^4\,x^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asin(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)
[Out] int((x^3*(a + b*asin(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 (a + b \operatorname{acsc}(cx))}{\sqrt{-(cx-1)(cx+1)\left(c^2x^2+1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsc(c*x))/(-c**4*x**4+1)**(1/2),x)
[Out] Integral(x**3*(a + b*acsc(c*x))/sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1)),
 x)
```

3.177 $\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{a+b\csc^{-1}(cx)}{x\sqrt{1-c^4x^4}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x/(-c^4*x^4+1)^(1/2),x)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCsc[c*x])/(x*Sqrt[1 - c^4*x^4]),x]
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x*Sqrt[1 - c^4*x^4]), x]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$$

Mathematica [A] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x*Sqrt[1 - c^4*x^4]),x]
[Out] Integrate[(a + b*ArcCsc[c*x])/(x*Sqrt[1 - c^4*x^4]), x]
```

fricas [A] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^4x^4+1}\,(b\,\text{arccsc}\,(cx)+a)}{c^4x^5-x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-c^4*x^4 + 1)*(b*arccsc(c*x) + a)/(c^4*x^5 - x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arccsc}\,(cx)+a}{\sqrt{-c^4x^4+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)/(sqrt(-c^4*x^4 + 1)*x), x)
```

maple [A] time = 6.20, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{arccsc}\,(cx)}{x\sqrt{-c^4x^4 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/x/(-c^4*x^4+1)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{4}\,a\left(\log\left(\sqrt{-c^4x^4 + 1} + 1\right) - \log\left(\sqrt{-c^4x^4 + 1} - 1\right)\right) + b\int \frac{\arctan\left(1, \sqrt{cx + 1}\,\sqrt{cx - 1}\right)}{\sqrt{c^2x^2 + 1}\,\sqrt{cx + 1}\,\sqrt{-cx + 1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*a*(log(sqrt(-c^4*x^4 + 1) + 1) - log(sqrt(-c^4*x^4 + 1) - 1)) + b*inte
grate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(sqrt(c^2*x^2 + 1)*sqrt(c*x +
 1)*sqrt(-c*x + 1)*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x\,\sqrt{1 - c^4\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x*(1 - c^4*x^4)^(1/2)),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(x*(1 - c^4*x^4)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{acsc}\,(cx)}{x\sqrt{-(cx - 1)(cx + 1)\left(c^2x^2 + 1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] Integral((a + b*acsc(c*x))/(x*sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1))),
x)
```

3.178 $\int \frac{a+b \csc^{-1}(cx)}{x^5 \sqrt{1-c^4 x^4}} \, dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{a+b\csc^{-1}(cx)}{x^5\sqrt{1-c^4x^4}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x^5\sqrt{1-c^4x^4}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCsc[c*x])/(x^5*Sqrt[1 - c^4*x^4]),x]
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x^5*Sqrt[1 - c^4*x^4]), x]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x^5\sqrt{1-c^4x^4}} \, dx = \int \frac{a+b\csc^{-1}(cx)}{x^5\sqrt{1-c^4x^4}} \, dx$$

Mathematica [A] time = 3.58, size = 0, normalized size = 0.00

$$\int \frac{a+b\csc^{-1}(cx)}{x^5\sqrt{1-c^4x^4}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x^5*Sqrt[1 - c^4*x^4]),x]
[Out] Integrate[(a + b*ArcCsc[c*x])/(x^5*Sqrt[1 - c^4*x^4]), x]
```

fricas [A] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^4x^4+1}\,(b\,\text{arccsc}\,(cx)+a)}{c^4x^9-x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-c^4*x^4 + 1)*(b*arccsc(c*x) + a)/(c^4*x^9 - x^5), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arccsc}\,(cx)+a}{\sqrt{-c^4x^4+1}\,x^5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)/(sqrt(-c^4*x^4 + 1)*x^5), x)
```

maple [A] time = 4.24, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{arccsc}\,(cx)}{x^5\sqrt{-c^4x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{8}\left(c^4\log\left(\sqrt{-c^4x^4+1}+1\right)-c^4\log\left(\sqrt{-c^4x^4+1}-1\right)+\frac{2\sqrt{-c^4x^4+1}}{x^4}\right)a+b\int\frac{\arctan\left(1,\sqrt{cx+1}\sqrt{cx-1}\right)}{\sqrt{c^2x^2+1}\sqrt{cx+1}\sqrt{-cx+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*(c^4*log(sqrt(-c^4*x^4 + 1) + 1) - c^4*log(sqrt(-c^4*x^4 + 1) - 1) + 2
*sqrt(-c^4*x^4 + 1)/x^4)*a + b*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x
- 1))/(sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^5), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^5\,\sqrt{1-c^4\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x^5*(1 - c^4*x^4)^(1/2)),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(x^5*(1 - c^4*x^4)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{acsc}\,(cx)}{x^5\sqrt{-(cx-1)(cx+1)\left(c^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x**5/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] Integral((a + b*acsc(c*x))/(x**5*sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1))
), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```